PERFORMANCE OF CODED MULTISTAGE DETECTION IN CORRELATED RAYLEIGH CHANNELS

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ABSTRACT
In this paper, we estimate the performance of coded multistage detectors operating over a correlated Rayleigh fading channel without interleaving. We derive analytical estimates for the single-cell capacity of such systems by estimating the effective signal-to-noise ratio (SNR) after each stage and by calculating the exact symbol error probabilities of the channel decoder in a correlated Rayleigh fading channel. These analytical estimates are compared with the capacity estimates achieved through simulations.

1 INTRODUCTION
In the conventional multiuser detection research, the channel coding has largely been ignored in the analysis and algorithm design. Recently, there has been a growing interest for an integrated approach, where the channel decoding is considered jointly with the multiuser detection. Many multiuser receiver algorithms have been proposed for coded CDMA systems ([1]–[7]). The results obtained in these articles indicate that major performance gains can be achieved when coding is utilized in multiuser receivers. Naturally, the increased complexity of the receivers reduces the attractiveness of the proposals. Additionally, some kind of interleaving is usually needed for channel codes used in fading channels. The proposed algorithms for coded CDMA systems can handle interleaving only with the cost of additional delays. These delays may be unacceptable for low-latency service classes as well as for some vital feedback mechanisms in practical cellular systems, such as power control or automatic repeat request (ARQ) mechanisms.

In this paper, we study a multistage detector for a coded CDMA system, where the channel coding is utilized for the tentative decisions that are used in interference cancellation. We estimate the single-cell capacity of such a system when operating over a correlated Rayleigh fading channel without interleaving, which is the case if one wishes to avoid the deinterleaving delays in the interference cancellation. These analytical estimates are compared with simulation results.

2 SYSTEM MODEL
The CDMA system modeled in this paper is the uplink synchronous direct sequence DS-CDMA communication system with \( K \) users. We assume BPSK modulation. The model uses channel coding to improve the BER performance of the system. The channel is modeled as a Rayleigh-fading channel where Gaussian noise with zero mean and variance \( \sigma^2 \) is added.

The matched filter output at time \( i \) can be expressed as

\[
y(i) = R(i)A(i)x(i) + n(i) \tag{1}
\]

where \( x(i) = (\sqrt{E_b}b_{1}(i), \ldots, \sqrt{E_b}b_{K}(i))^T \) is the coded data vector containing the transmitted data symbols and the transmission energies of every user and \( n(i) \) is a Gaussian random vector. Furthermore, \( R(i) \) is the correlation matrix and \( A(i) \) is the channel matrix, that is

\[
R(i) = \begin{pmatrix}
1 & \rho_{12}(i) & \cdots & \rho_{1K}(i) \\
\rho_{21}(i) & 1 & \cdots & \rho_{2K}(i) \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{K1}(i) & \rho_{K2}(i) & \cdots & 1
\end{pmatrix} \tag{2}
\]

\[
A(i) = \text{diag}(a_1(i), \ldots, a_K(i)) \tag{3}
\]

where \( \rho_{jk}(i) \) is the cross-correlation between users \( j \) and \( k \). We will assume long random codes and thus we have \( E(\rho_{jk}(i)) = 0 \) and \( E(\rho_{jk}(i)^2) = 1/G \), where \( G \) is the processing gain. The \( a_k(i) \) in the channel matrix \( A \) are Rayleigh distributed random variables with normalized variance \( \sigma_a = 1/2 \). We will assume that the channel coefficients of different users are independent and that for each \( k \) the auto-correlation of \( a_k(i) \) is given by

\[
\rho_a(m) = \frac{1}{2}E[a_k(i)a_k(i+m)] = E_kJ_0(2\pi mf_DT), \tag{4}
\]

where \( f_D \) is the maximum Doppler frequency, \( T \) is the symbol period, and \( J_0 \) is the zero-order Bessel function.

We consider a receiver with multistage interference cancellation. At each stage \( m \) we use the channel decoder symbol decisions \( \hat{b}_k^{(m)}(i) \) to form the tentative symbol estimates \( \hat{c}_k^{(m)}(i) = \sqrt{E_b}\hat{b}_k^{(m)}(i) \). These estimates are used for interference cancellation, where the
modified MF samples that are used in the next stage of the receiver are obtained as

\[ \hat{y}^{(m)}(i) = y(i) - (R(i) - I)A(i)x^{(m)}(i), \]

where I is the identity matrix and \( x^{(m)}(i) \) is the vector containing the estimates.

3 RESIDUAL NOISE IN MULTISTAGE DETECTORS

In this section, we will estimate the amount of residual noise after the \( m \)th IC stage. Instead of the usual Gaussian approximation, we will use a slightly weaker condition on the performance of the tentative symbol estimator used for the soft or hard tentative decisions.

If we define \( \hat{x}^{(0)}(i) = 0 \), then modified the matched filter output sample after IC stage \( m \geq 0 \) for user \( k \) at time \( i \) is

\[ \hat{y}^{(m)}(i) = a_k(i)x_k(i) + I_k^{(m)}(i) + n_k(i) \]

where

\[ I_k^{(m)}(i) = \sum_{j \neq k} \rho_{jk}(i)a_j(i) \left( x_j(i) - \hat{x}_j^{(m)}(i)) \right) \]

In the conventional Gaussian approximation it is assumed that \( \hat{x}^{(m)}(i) \) are Gaussian distributed and that, subsequently, the mean and the variance describe these random variable fully. The assumption used here is a slightly weaker one, namely, that the performance of the tentative symbol estimator should only depend on the total mean noise power \( N_{j^{(m−1)}}(i) = E((I_k^{(m)}(i) + n_k(i))^2) \), not on the higher moments. Specifically, we will assume that the mean-square error of the tentative symbol estimator can for all \( k \) be presented by the same function \( f \) of the SNR, that is

\[ E(\Delta_k^{(m)}(i)^2) = f(E_k/N_k^{(m−1)}(i)) \]

where \( \Delta_j^{(m)}(i) = x_j(i) - \hat{x}_j^{(m)}(i) \) and \( E_k \) is the symbol energy for user \( k \).

The mean residual noise power naturally depends on the mean-square error of the tentative estimator. If we assume that the crosscorrelations between different users are independent we obtain

\[ E(I_k^{(m)}(i)^2) = \sum_{j \neq k} \frac{1}{G} E(\Delta_j^{(m)}(i)^2) \]

since \( E(a_j(i))^2 = 1 \) and \( E(\rho_{jk}(i))^2 = 1/G \). If the received power levels are equal \( (E_k = E_s \) for all \( k \)), then at each stage \( m \) the mean noise power is the same for all users. Thus the mean square error of the estimator is the same for all \( k \) and we obtain

\[ E(I_k^{(m)}(i)^2) = \frac{K-1}{G} f(E_s/N^{(m−1)}(i)) \]

Now the total mean noise power \( N^{(m−1)}(i) \) does not depend on \( i \) and we may drop the index obtaining

\[ N^{(m)} = \frac{K-1}{G} f(E_s/N^{(m−1)}(i)) + \sigma^2. \]

For any number of users, we can iterate Equation (11) to calculate the amount of noise present in the system after \( m \) iterations. If we are given a target signal-to-noise ratio \( \gamma_r \) and the number of stages \( m \) in the IC receiver, we can calculate the single-cell capacity of the system by determining the largest \( K \), where the mean noise power level is still less than \( E_s/\gamma_r \).

A successful calculation of the capacity requires a good estimate for the function \( f \). In the next section, we will derive \( f \) for an uninterleaved Rayleigh fading channel.

4 PERFORMANCE IN CORRELATED RAYLEIGH CHANNELS

In this section we will study a receiver, where tentative symbol decisions are based on the soft-decision decoder output, and which is operating over a correlated Rayleigh fading channel without interleaving. The mean square error of the estimator is \( 4E_sP_E \), where \( P_E \) is the probability for channel symbol error. The exact computation of \( P_E \) is not simple in a correlated Rayleigh fading channel. In general, the pairwise error probability in a correlated Rayleigh fading channel depends not only on the distance between the codewords, but on the actual codewords compared. Following [8, 9] we can present the pairwise error probability as \( P(\mathbf{c}_h \rightarrow \mathbf{c}_k) = P(\mathbf{z}^T \mathbf{D} \mathbf{z} < 0) \), where

\[ \mathbf{D} = \begin{pmatrix} 0 & C_h - C_k \\ (C_h - C_k)^T & C_k^H C_k - C_h^H C_h \end{pmatrix}, \]

where \( C_h = \text{diag} (\mathbf{c}_h) \), \( C_k = \text{diag} (\mathbf{c}_k) \) and \( \mathbf{z} = (y_k(1), \ldots, y_k(M), a_k(1), \ldots, a_k(M))^T \). Then the pairwise error probability as derived in [10] is

\[ P_2(\mathbf{c}_h \rightarrow \mathbf{c}_k) = \sum_{g_j < 0} \prod_{n=1}^N \frac{1}{1 - g_n/g_j} \]

where \( g_j \) are the eigenvalues of \( E(\mathbf{z}^H \mathbf{z}) \mathbf{D} \). Naturally, those depend on the effective signal-to-noise ratio \( E_s/N^{(m−1)}(i) \) at each stage \( m \). Now when the channel code is linear, the channel symbol error probability is

\[ P_E = \sum_{\mathbf{c}_i} P_2(0 \rightarrow \mathbf{c}_i) \]

and thus

\[ f = 4E_s \sum_{\mathbf{c}_i} P_2(0 \rightarrow \mathbf{c}_i) \]
5 NUMERICAL RESULTS

In this section, we estimate the impact of the correlated Rayleigh fading channel on the performance of the IC receiver using uninterleaved coding for the generation of tentative decisions. Figure 1 shows the analytically estimated performance of both 2- and 4-stage IC receivers with and without coding as a function of the normalized Doppler frequency $f_D T$. The processing gain is $G = 63$, the channel SNR is $\gamma = 20\text{dB}$ and the target SNR is $\gamma_r = 15\text{dB}$. An uninterleaved $(31,6)$ BCH code is used for the coded IC.

In order to evaluate the accuracy of our analytical results, we performed a number of simulations to calculate the effective SNR for the desired user with different cell loads. In the analysis, the channel coefficient were assumed to be uncorrelated between different users and thus the time-correlated Rayleigh fading channel coefficients were generated with a modified Jakes model described in [11].

In Figure 2, the effective SNR of the desired user is simulated with $f_D T = 0.03$ and with a varying number of interfering users, when a 2-stage receiver is used. The target SNR is indicated by a dashed line. The analytical estimate is a reasonably tight lower bound for the capacity. More generally, the estimate gives a tight lower bound for both coded and uncoded 2-stage receiver regardless of the value of the normalized Doppler frequency. This is illustrated in Figure 3, where the corresponding simulation results with $f_D T = 0.01$ are shown.

Unfortunately, the estimate is not accurate when the number of stages is increased. The simulations indicate that the analytical estimate is still a lower bound, but not a tight one. Figure 4 shows the simulated performance of both the coded and uncoded 4-stage IC receiver in a Rayleigh correlated fading channel with $f_D T = 0.01$. One can see that, in both cases, the actual capacity is more than the one predicted by the analysis. The same phenomena occurs also with different values of the normalized Doppler frequency. The accuracy degrades as the number of stages is increased due to the cumulating error in the iteration of Equation 11.

6 CONCLUSIONS

In this paper, we have estimated the capacity of a CDMA system with coded multistage detectors operating over a correlated Rayleigh fading channel without interleaving. The resulting analytical estimates were compared with some simulation results. The analytical estimates acquired were found to be accurate lower bounds for the capacity when the number of stages was small. With 4 or more stages, the analytical estimation gives only a loose lower bound.
Figure 4: Simulated Capacity for 4-stage Receivers with $f_{DT} = 0.01$

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References


