A GENERAL-PURPOSE OPTIMIZATION TECHNIQUE FOR DESIGNING TWO-CHANNEL FIR FILTER BANKS

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ABSTRACT

An efficient general-purpose optimization algorithm is proposed for designing two-channel FIR filter banks. This technique can be used for optimizing two-channel FIR filters in all the alias-free cases proposed in the literature. The generalized problem is to minimize the maximum of the stopband energies of the two analysis filters subject to the given passband and transition band constraints and the given allowable reconstruction error. Therefore, in addition to the perfect-reconstruction filter banks, nearly perfect-reconstruction banks can be optimized in a controlled manner. The optimization is carried out in two steps. In the first step, a good starting-point filter banks for further optimization is generated using an existing design scheme for the selected class of filter banks. The second step involves optimizing the filter bank with the aid of the proposed algorithm. Several examples are included illustrating the efficiency of the proposed approach.

1. INTRODUCTION

Since 1976, when Esteban and Galand [1] applied filter banks in a voice coding scheme, filter banks have found various applications in many different areas such as image compression, signal denoising, speech coding, adaptive signal processing, and transmission of signals through channels of different bandwidths [2]–[4]. To cope with diverse requirements that exist in different applications, various filter bank types have emerged. These types include two-cannel and multichannel (M-channel) filter banks. This contribution concentrates on synthesizing alias-free two-channel FIR filter banks. These filter banks can be divided into quadrature mirror filter (QMF) banks, orthogonal filter banks, and biorthogonal filter banks. Orthogonal and biorthogonal filter banks can be designed to have either the perfect-reconstruction (PR) or nearly perfect-reconstruction (NPR) property. Furthermore, QMF and biorthogonal filter banks can be designed to be either linear-phase or low-delay (nonlinear-phase) filter banks. The formal definitions of all these filter bank types will be given in Section 3. More details can be found in [5] where the authors of the present paper have given an overview on alias-free two-channel FIR filter banks. Multichannel filter banks can be easily obtained from these two-channel filter banks by using a tree structure or the half of a tree structure.

Designing efficient alias-free two-channel filter banks is a rather difficult task. The best way would be to design them analytically, but analytical design techniques result in the optimum solutions only for some special cases from the above-mentioned types. For other types, adequate iterative or optimization methods are need. A comprehensive reference list on existing design techniques can also be found in [5]. Nonlinear optimization techniques are also required because iterative methods are in general fast, but usually do not result in the best possible solution. Therefore, there is a need to develop a systematic approach based on the use of nonlinear optimization. As shown by the authors of this paper in [6] for designing biorthognal filter banks, it is beneficial to use the following two-step procedure. First, a rather good solution is generated using a simple technique. Second, this solution is further improved using a nonlinear optimization technique, giving the optimized solution.

The purpose of this paper is to generate, based on this two-step strategy, a general-purpose two-step optimization technique for designing all the above-mentioned filter bank types in the same manner. For the second step of the overall procedure, we propose an optimization method that is obtained by modifying the Dutta-Vidyasagar algorithm [7]. Combining the proposed method with a generalized optimization problem statement [5] allows us to use the same approach for designing all filter banks under consideration. Depending on the filter bank type, only slight modifications of the optimization problem are needed. Given the type of the filter bank, a starting-point filter bank is found using an existing synthesis technique from the literature.

The paper is organized as follows. Section 2 reviews some basic relations for two-channel filter banks and introduces a filter transform that simplifies the overall design procedure. In Section 3, a filter bank classification is performed. Section 4 states the general optimization problem. The proposed optimization algorithm is described in Section 5. Section 6 shows, by means of several examples, the efficiency of the proposed method.

2. TWO-CHANNEL FILTER BANKS

The block diagram for a two-channel filter bank is shown in Figure 1 [2]–[4]. It consists of an analysis filter bank followed by downsamplers, upsamplers, and a synthesis filter bank.

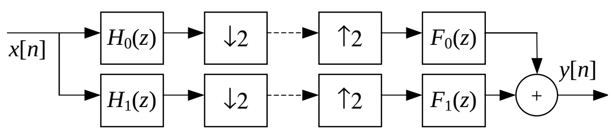

Figure 1. Two-channel filter bank.

It is well known that the relation between the output and input of this overall system is expressible as

$$Y(z) = T(z)X(z) + A(z)X(-z), \tag{1}$$

where the first term

$$T(z) = \frac{1}{2}\left[H_0(z)F_0(z) + H_1(z)F_1(z)\right] \tag{2a}$$

is the distortion transfer function and the second term

$$A(z) = \frac{1}{2}\left[H_0(-z)F_0(z) + H_1(-z)F_1(z)\right] \tag{2b}$$

is the aliasing transfer function. The last term can be made zero by selecting the synthesis filters as $F_0(z) = 2H_1(-z)$ and $F_1(z) = -2H_0(-z)$, giving the following input-output relation:

$$Y(z) = T(z)X(z), \tag{3a}$$

where

$$T(z) = H_0(z)H_1(-z) - H_1(z)H_0(-z). \tag{3b}$$

In order to simplify the optimization problem to be described in Section 4 we use, instead of $H_0(z)$ and $H_1(z)$, the following transfer functions:

$$G_0(z) = \sum_{n=0}^{N_0} g_0[n]z^{-n} \equiv H_0(z) = \sum_{n=0}^{N_0} h_0[n]z^{-n} \tag{4a}$$

$$G_1(z) = \sum_{n=0}^{N_1} g_1[n]z^{-n} \equiv H_1(-z) = \sum_{n=0}^{N_1} (-1)^n h_1[n]z^{-n}\ . \tag{4b}$$

$G_0(z)$ and $H_0(z)$ are thus identical, whereas $G_1(z) = H_1(-z)$. Therefore, $|G_1(e^{j\omega})| = |H_1(e^{j(\pi-\omega)})|$ so that the amplitude response of $G_1(z)$ is obtained from that of $H_1(z)$ using the substitution $\pi - \omega \to \omega$ and vice versa. If $\tilde{\omega}_p^{(1)}$ and $\tilde{\omega}_s^{(1)}$ are the passband and stopband edges of $H_1(z)$, then the corresponding edges for $G_1(z)$ are $\omega_p^{(1)} = \pi - \tilde{\omega}_p^{(1)}$ and $\omega_s^{(1)} = \pi - \tilde{\omega}_s^{(1)}$, respectively. Figure 2 exemplifies the above relations in additional to showing the constraints for $G_0(z)$ and $G_1(z)$ to be stated in the general optimization problem to be considered in Section 4. Hence, the conversion of the design of $H_1(z)$ to that of $G_1(z)$ is straightforward. Once $G_1(z)$ has been determined, the corresponding impulse-response values of $H_1(z)$ can be determined according to Equation (4b). The advantage of using $G_0(z)$ and $G_1(z)$ as primary transfer functions lies in the fact that both of them are transfer functions of lowpass filters, enabling us to treat them in the same way. In terms of $G_0(z)$ and $G_1(z)$, the transfer function $T(z)$ takes the following form:

$$T(z) = G_0(z)G_1(z) - G_0(-z)G_1(-z). \tag{5}$$

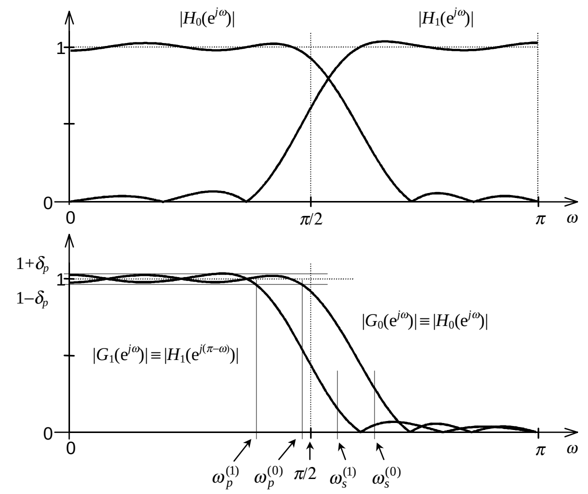

Figure 2. Specifications for $G_0(z)$ and $G_1(z)$ and the relations between $H_0(z)$ and $G_0(z)$ and $H_1(z)$ and $G_1(z)$.

3. TWO-CHANNEL FIR FILTER BANKS UNDER CONSIDERATION

The two-channel FIR filter banks are first classified into perfect-reconstruction (PR) and nearly perfect-reconstruction (NPR) banks. For PR banks, the overall transfer function, as given by Equation (5), satisfies $T(z) = z^{-K}$ with K being an integer, whereas for NPR banks, this condition is only approximately satisfied. The optimization technique to be described later in this paper can be used for designing the following alias-free two-channel PR and NPR FIR filter banks [5]:

1) NPR quadrature mirror filter (QMF) banks: $G_1(z) = G_0(z)$.
   a) $G_0(z)$ is a linear-phase FIR filter and $K = N_0$.
   b) $G_0(z)$ is a nonlinear-phase FIR filter and $K < N_0$.
2) PR orthogonal filter banks: $K = N_0$ and $G_1(z) = z^{-N_0} G_0(z^{-1})$.
3) PR biorthogonal filter banks.
   a) $G_0(z)$ and $G_1(z)$ are different linear-phase FIR filters and $K = (N_0+N_1)/2$.
   b) $G_0(z)$ and $G_1(z)$ are different nonlinear-phase FIR filters and $K < (N_0+N_1)/2$.
4) Generalized NPR filter banks.
   a) $G_0(z)$ and $G_1(z)$ are different linear-phase FIR filters and $K = (N_0+N_1)/2$.
   b) $G_0(z)$ and $G_1(z)$ are different nonlinear-phase FIR filters and $K < (N_0+N_1)/2$.

For a useful filter bank, the delay K is an odd integer and the sum of the orders of $G_0(z)$ and $G_1(z)$ is two times an odd integer. For linear-phase filters, this sum must be two times K.

4. OPTIMIZATION PROBLEM

Before stating the general optimization problem, we denote the transfer functions $G_0(z)$ and $G_1(z)$, as given by Equations (4a) and (4b), by $G_0(\mathbf{g}_0, z)$ and $G_1(\mathbf{g}_1, z)$, where $\mathbf{g}_0$ and $\mathbf{g}_1$ contain the adjustable parameters of $G_0(z)$ and $G_1(z)$, respectively. This paper concentrates on the following optimization problem: Given the class of two-channel filter banks, N_0, N_1, $\rho_p^{(k)}$ and $\rho_s^{(k)}$ for $k = 0, 1$, δ_a, and δ_p as well as K, find $\mathbf{g}_0$ and $\mathbf{g}_1$ to minimize

$$\varepsilon = \max(\varepsilon_0(\mathbf{g}_0), \varepsilon_1(\mathbf{g}_1)), \tag{6a}$$

where

$$\varepsilon_k(\mathbf{g}_k) = \int_{\omega_s^{(k)}}^{\pi} \left| G_k\left(\mathbf{g}_k, e^{j\omega}\right) \right|^2 d\omega \quad \text{for} \quad k = 0,1 \tag{6b}$$

subject to

$$\max_{\omega \in [0,\pi]} \left| T\left(\mathbf{g}_0, \mathbf{g}_1, e^{j\omega}\right) - e^{-jK\omega} \right| \le \delta_a, \tag{6c}$$

$$\max_{\omega \in \left[0, \omega_p^{(k)}\right]} \left| \left| G_k\left(\mathbf{g}_k, e^{j\omega}\right) \right| - 1 \right| \le \delta_p \quad \text{for} \quad k = 0,1, \tag{6d}$$

and

$$\max_{\omega \in \left(\omega_p^{(k)}, \omega_s^{(k)}\right)} \left| G_k\left(\mathbf{g}_k, e^{j\omega}\right) \right| - 1 \le \delta_p \quad \text{for} \quad k = 0,1, \tag{6e}$$

where

$$\begin{aligned} T\left(\mathbf{g}_0, \mathbf{g}_1, e^{j\omega}\right) = {} & G_0\left(\mathbf{g}_0, e^{j\omega}\right) G_1\left(\mathbf{g}_1, e^{j\omega}\right) \\ & - G_0\left(\mathbf{g}_0, e^{j(\omega+\pi)}\right) G_1\left(\mathbf{g}_1, e^{j(\omega+\pi)}\right) \end{aligned} \tag{6f}$$

and

$$\omega_s^{(k)} = (1+\rho_s^{(k)})\pi/2,\ \omega_p^{(k)} = (1-\rho_p^{(k)})\pi/2 \quad \text{for} \quad k = 0,1. \tag{6g}$$

In the above, the passband and stopband regions of $G_k(\mathbf{g}_k, z)$ for $k = 0, 1$ are $[0, \omega_p^{(k)}]$ and $[\omega_s^{(k)}, \pi]$, respectively. The goal is to minimize the maximum of the stopband energies of $G_0(\mathbf{g}_0, z)$ and

$G_1(\mathbf{g}_1, z)$, denoted by $\varepsilon_0(\mathbf{g}_0)$ and $\varepsilon_1(\mathbf{g}_1)$, subject to some constraints, as illustrated in Figure 2. First, the overall frequency response $T(\mathbf{g}_0, \mathbf{g}_1, e^{j\omega})$ should approximate on $[0, \pi]$ the delay term $e^{-jK\omega}$ with the given tolerance δ_a. For PR filter banks, δ_a is zero. Second, the passband amplitude responses of $G_0(\mathbf{g}_0, z)$ and $G_1(\mathbf{g}_1, z)$ have to stay within $1 \pm \delta_p$. Third, the transition band maximum values are restricted to be less than or equal to $1+\delta_p$.

The above optimization problem covers all the filter bank classes considered in Section 3. Depending the filter bank class under consideration, some of the above constraints can be dropped out. For QMF banks and PR orthogonal filter banks, $\varepsilon_0(\mathbf{g}_0) \equiv \varepsilon_1(\mathbf{g}_1)$.

5. OPTIMIZATION ALGORITHM

To solve the above problem conveniently, the region $[0, \pi/2]$ is discretized into the points $\omega_m \in [0, \pi/2]$ for $m = 1, 2, \cdots, M$, whereas the regions $[0, \omega_p^{(k)}]$ and $(\omega_p^{(k)}, \omega_s^{(k)})$ are discretized into the points $\omega_i^{(k)} \in [0, \omega_p^{(k)}]$ for $i = 1, 2, \cdots, L_p^{(k)}$ and $\omega_i^{(k)} \in (\omega_p^{(k)}, \omega_s^{(k)})$ for $i = L_p^{(k)}+1, L_p^{(k)}+2, \cdots, L_p^{(k)}+L_t^{(k)}$. The resulting discretized problem is to find $\mathbf{g}_0$ and $\mathbf{g}_1$ to minimize

$$\varepsilon = \max\left(\varepsilon_0(\mathbf{g}_0), \varepsilon_1(\mathbf{g}_1)\right) \tag{7a}$$

subject to

$$\left| T\left(\mathbf{g}_0, \mathbf{g}_1, e^{j\omega_m}\right) - e^{-jK\omega_m} \right| - \delta_a \le 0 \quad \text{for} \quad m = 1, 2, \cdots, M \tag{7b}$$

and for $k = 0,1$

$$\left| \left| G_k\left(\mathbf{g}_k, e^{j\omega_i^{(k)}}\right) \right| - 1 \right| - \delta_p \le 0 \quad \text{for} \quad i = 1, 2, \cdots, L_p^{(k)} \tag{7c}$$

and

$$\left| G_k\left(\mathbf{g}_k, e^{j\omega_i^{(k)}}\right) \right| - \left(1 + \delta_p\right) \le 0 \quad \text{for} \quad i = L_p^{(k)}+1, L_p^{(k)}+2, \cdots, L_p^{(k)}+L_t^{(k)}. \tag{7d}$$

In order to apply the algorithms of Dutta and Vidyasagar proposed in [7] with proper modifications, we consider the following objective function:

$$\begin{aligned} P(\mathbf{g}_0, \mathbf{g}_1, \phi) = {} & \left[\max\left(\varepsilon_0(\mathbf{g}_0) - \phi, 0\right)\right]^2 + \left[\max\left(\varepsilon_1(\mathbf{g}_1) - \phi, 0\right)\right]^2 \\ & + \sum_{\substack{m=1 \\ \left|T\left(\mathbf{g}_0,\mathbf{g}_1,e^{j\omega_m}\right) - e^{-jK\omega_m}\right| - \delta_a > 0}}^{M} w_m \left[\left| T\left(\mathbf{g}_0, \mathbf{g}_1, e^{j\omega_m}\right) - e^{-jK\omega_m} \right| - \delta_a \right]^2 \\ & + \sum_{k=0}^{1} \left\{ \sum_{\substack{i=1 \\ \left|\left|G_k\left(\mathbf{g}_k, e^{j\omega_i^{(k)}}\right)\right| - 1\right| - \delta_p > 0}}^{L_p^{(k)}} v_i^{(k)} \left[\left| \left| G_k\left(\mathbf{g}_k, e^{j\omega_i^{(k)}}\right) \right| - 1 \right| - \delta_p \right]^2 \right\} \\ & + \sum_{k=0}^{1} \left\{ \sum_{\substack{i=L_p^{(k)}+1 \\ \left|G_k\left(\mathbf{g}_k, e^{j\omega_i^{(k)}}\right)\right| - (1+\delta_p) > 0}}^{L_p^{(k)}+L_t^{(k)}} v_i^{(k)} \left[\left| G_k\left(\mathbf{g}_k, e^{j\omega_i^{(k)}}\right) \right| - \left(1 + \delta_p\right) \right]^2 \right\}. \end{aligned} \tag{8}$$

In the above summations only those terms not satisfying the given criteria are present. Furthermore, the first (second) term is present provided that $\varepsilon_0(\mathbf{g}_0) > \phi$ ($\varepsilon_1(\mathbf{g}_1) > \phi$). It should be pointed out that $\varepsilon_0(\mathbf{g}_0)$ and $\varepsilon_1(\mathbf{g}_1)$, as given by Equation (6b), are expressible in the closed form for all the filter bank classes under consideration. The w_m's and $v_i^{(k)}$'s are the weights given by the user. Usually they are selected to be equal to unity. For the PR case, δ_a is zero. In practice, $\delta_a = 10^{-13}$ is a good selection.

The main idea of introducing the additional constant ϕ in the objective function $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$ lies in the fact that it is desired to gradually find its minimum value for which the objective function optimized with respect $\mathbf{g}_0$ and $\mathbf{g}_1$ becomes zero. For this minimum value of ϕ, $\varepsilon = \phi$ is the minimum achievable maximum of the stopband energies of $G_0(\mathbf{g}_0, z)$ and $G_1(\mathbf{g}_1, z)$. If ϕ is too large, then $\mathbf{g}_0$ and $\mathbf{g}_1$ can be found to make $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$ equal to zero. If ϕ, in turn, is too small, then $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$ cannot be made equal to zero. The very attractive property in forming the objective function $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$ in the above form is the fact that all the error terms are raised to a power to two, making it well behaved.

In the original algorithms proposed by Dutta and Vidyasagar, ϕ is gradually increased until finding the value for which the objective function becomes practically equal to zero. For designing two-channel filter banks, it has been experimentally observed that it is more efficient to gradually decrease ϕ until finding its minimum value after which $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$ cannot be made zero.

The proposed algorithm is carried out in the following steps:

1) Find initial vectors for $\mathbf{g}_0$ and $\mathbf{g}_1$, denoted by $\mathbf{g}_0^{(0)}$ and $\mathbf{g}_1^{(0)}$, using an existing design scheme in the literature. Let $\phi^{(0)}$ be the maximum of the stopband energies for this solution. Set $\beta = 0.1$ and $l = 1$.
2) Set $\phi^{(l)} = (1-\beta)\phi^{(l-1)}$. Find $\hat{\mathbf{g}}_0$ and $\hat{\mathbf{g}}_1$ to minimize $P(\phi^{(l)}, \mathbf{g}_0, \mathbf{g}_1)$ using $\mathbf{g}_0^{(l-1)}$ and $\mathbf{g}_1^{(l-1)}$ as initial vectors.
3) If $P(\phi^{(l)}, \mathbf{g}_0^{(l)}, \mathbf{g}_1^{(l)}) = 0$, then set $\mathbf{g}_0^{(l)} = \hat{\mathbf{g}}_0$, $\mathbf{g}_1^{(l)} = \hat{\mathbf{g}}_1$, $l = l + 1$, and go to Step 2. Otherwise, go to the next step.
4) If $\beta < 10^{-9}$, then stop. Otherwise, set $\beta = \beta/10$ and go to Step 2.

The above algorithm starts by finding a starting-point suboptimal solution with the aid of an existing design scheme. For this design, the maximum of the two stopband energies is determined. This maximum value, denoted by $\Phi_{\min}^{(1)}$, is used as an initial value of ϕ for $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$. Then, ϕ is iteratively decreased by a factor of $(1-\beta)$ with $\beta = 10^{-1}$ until $P(\phi, \mathbf{g}_0, \mathbf{g}_1)$ cannot be made zero. The minimum value of ϕ for which the objective function becomes zero is denoted by $\Phi_{\min}^{(2)}$. In each iteration, $\mathbf{g}_0$ and $\mathbf{g}_1$ making the objective function equal to zero is found with the aid of Fletcher-Powell algorithm. $\mathbf{g}_0$ and $\mathbf{g}_1$ obtained in the previous iteration are used as initial vectors. Then, the same procedure is performed for $\Phi_{\min}^{(r)}$ and $\beta = 10^{-r}$ for $r = 2, 3, \cdots, 8$. The solution corresponding to $\phi = \Phi_{\min}^{(8)}$ is the desired final solution.

6. EXAMPLES

This section illustrates, by means of three examples, the efficiency of the proposed optimization method. To show the robustness of this optimization algorithm with respect to initial filters, FIR filters designed in the least-mean-square sense have been used. Of course, if the initial solution is selected in a better way, the optimization procedure becomes faster and more reliable.

Example 1. It is desired to design a low-delay PR biorthogonal filter bank for $N_0 = N_1 = 45$, $K = 31$, $\omega_p^{(0)} = \omega_p^{(1)} = 0.414\pi$, $\omega_s^{(0)} = \omega_s^{(1)} = 0.586\pi$, and $\delta_p = 0.01$. As initial filters low-delay filters designed in the least-mean-square sense have been used. Figure 3 shows the overall amplitude responses and passband details for the optimized analysis filters obtained by applying the proposed optimization scheme.

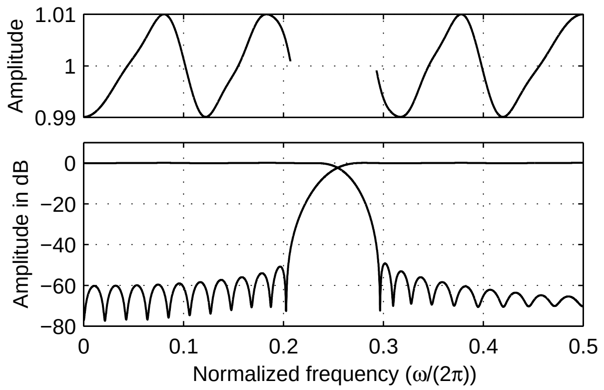

Figure 3. Amplitude responses for the analysis filters of Example 1.

Example 2. It is desired to design a NPR biorthogonal filter bank with linear-phase filters of orders $N_0 = 22$ and $N_1 = 40$ for $\omega_p^{(0)} = \omega_p^{(1)} = 0.414\pi$, $\omega_s^{(0)} = \omega_s^{(1)} = 0.586\pi$, $\delta_p = 0.01$, and $\delta_a = 10^{-4}$. Since both filters have linear phase, the overall delay is $K = (N_0 + N_1)/2 = 31$. As initial filters linear-phase filters designed in the least-mean-square sense have again been used. Some characteristics of the optimized filter bank are shown in Figure 4. In addition to the amplitude responses of the analysis filters, the zero-phase frequency response of the overall distortion error, denoted by $T(\omega)$–1, is depicted.

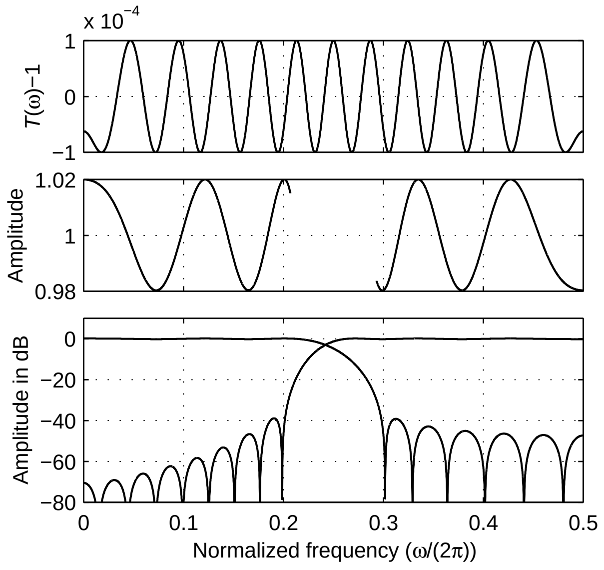

Figure 4. Some characteristics for the filter bank of Example 2.

Example 3. In order to show the flexibility of the proposed method, two low-delay QMF banks with $N_0 = 31$, $K = 15$, $\delta_a = 1.38 \cdot 10^{-4}$, $\omega_p^{(0)} = 0.31\pi$, $\omega_s^{(0)} = 0.69\pi$, and different passband ripples of $\delta_p = 0.087$ and $\delta_p = 10^{-4}$ have been designed. As initial filters, linear-phase filters designed in the least-mean-square sense have been used. Figure 5 shows the amplitude characteristics of the optimized analysis filters as well as the reconstruction errors for both filter banks. As expected, a smaller passband ripple results in lower stopband attenuation. In both cases, the resulting passband ripple is very small. Only in the transition band, the amplitude characteristics exhibit the value of $1+\delta_p$.

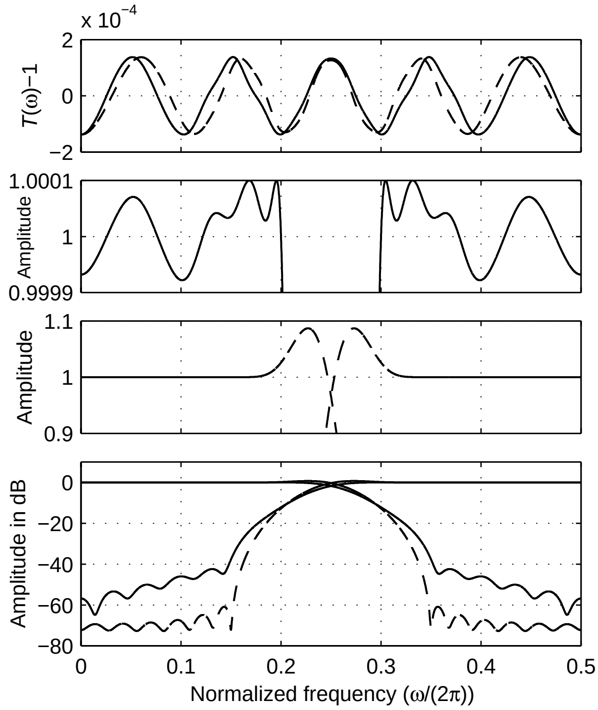

Figure 5. Some characteristics for the filter banks of Example 3.

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