Design of Digital Filters and Filter Banks by Optimization: A State of the Art Review

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ABSTRACT

The many advancements in the area of numerical optimization in conjunction with the ever-increasing power of computers have made optimization-based filter design an increasingly important field of research. In this paper, several recent optimization methods for the design of FIR and IIR digital filters and filter banks are reviewed.

1 Introduction

It has long been recognized that the design of finite-duration impulse-response (FIR) and infinite-duration impulse-response (IIR) digital filters can be formulated and carried out as unconstrained or constrained optimization problems [1]–[4]. The optimization-based methods used offer a design framework in which a variety of design criteria and specifications can be readily incorporated. Parallel to the development of these methods, the past several decades have witnessed the development of new optimization algorithms that have proven very efficient for the design of digital filters. These powerful optimization methods in conjunction with the ever-increasing power of computers have made optimization-based filter design an increasingly important field of research.

In this paper, several recently proposed optimization-based algorithms for the design of FIR and IIR filters and filter banks are reviewed. Well established algorithms such as the Remez exchange algorithm for linear-phase FIR filters and L_p designs for IIR filters can be found in the literature (see [2] for detailed accounts of these methods).

2 FIR Filters

2.1 Weighted least-squares design of FIR filters

Let the FIR filter to be designed be represented by the transfer function

$$H(z) = \sum_{n=0}^{N-1} h_n z^{-n} \tag{1}$$

In a weighted least-squares design, one seeks to find the impulse response coefficients $\{h_n,\ n = 0,\ 1,\ \ldots,\ N-1\}$ such that $H(z)$ in (1) minimizes the weighted least-squares objective function

$$e(\mathbf{x}) = \int_{-\pi}^{\pi} W(\omega)|H(e^{j\omega}) - H_d(\omega)|^2\, d\omega \tag{2}$$

where $W(\omega) \geq 0$ is a known weighting function, $H_d(\omega)$ is the desired frequency response, $\mathbf{x} = [h_0\ h_1\ \cdots\ h_{N-1}]^T$ is the N-dimensional variable vector, and ω is the normalized frequency varying from $-\pi$ to π.

In order to design an FIR digital filter with linear phase response, the impulse response of the filter is required to be symmetrical or antisymmetrical with respect to its midpoint [2], i.e.,

$$h_n = h_{N-1-n} \qquad \text{for } n = 0,\ 1,\ \ldots,\ N-1 \tag{3}$$

For N odd, (3) implies that

$$H(e^{j\omega}) = e^{-j\omega(N-1)/2}A(\omega) = e^{-j\omega(N-1)/2}\sum_{n=0}^{(N-1)/2} a_n \cos n\omega \tag{4}$$

For the frequency response $H(e^{j\omega})$ in (4) to approximate a linear-phase frequency response $H_d(\omega) = e^{-j\omega(N-1)/2}A_d(\omega)$, the least-squares objective function involved is reduced to

$$e_l(\mathbf{x}) = \int_{-\pi}^{\pi} W(\omega)[A(\omega) - A_d(\omega)]^2\, d\omega \tag{5}$$

where $\mathbf{x} = [a_0\quad a_1\quad \cdots\quad a_{(N-1)/2}]^T$. By defining vector $\mathbf{c}_l(\omega) = [1\quad \cos\omega\quad \cdots\quad \cos((N-1)\omega/2)]^T$, $A(\omega)$ can be written as $\mathbf{x}^T\mathbf{c}_l(\omega)$ and $e_l(\mathbf{x})$ in (5) can be expressed as

$$e_l(\mathbf{x}) = \mathbf{x}^T\mathbf{Q}_l\mathbf{x} - 2\mathbf{x}^T\mathbf{b}_l + \mu \tag{6}$$

where μ is a constant and

$$\mathbf{Q}_l = \int_0^{\pi} W(\omega)\mathbf{c}_l(\omega)\mathbf{c}_l^T(\omega)\, d\omega \tag{7}$$

$$\mathbf{b}_l = \int_0^{\pi} W(\omega)A_d(\omega)\mathbf{c}_l(\omega)\, d\omega \tag{8}$$

In communications applications, it is often desirable to design linear-phase FIR filters with equiripple passbands and peak-constrained least-square stopbands [5]. The design of this class of FIR filters can be accomplished using a quadratic programming (QP) approach as described below.

For the sake of simplicity, we consider the problem of designing a linear-phase, lowpass FIR filter of odd length N with normalized passband and stopband edges ω_p and ω_a, respectively. If we use the piecewise-constant weighting function $W(\omega)$ which assumes value 1 on $[0,\ \omega_p]$, γ on $[\omega_a,\ \pi]$, and 0 elsewhere, then the objective function $e(\mathbf{x})$ in (5) becomes

$$e(\mathbf{x}) = \int_0^{\omega_p}[A(\omega)-1]^2\, d\omega + \gamma\int_{\omega_a}^{\pi} A^2(\omega)\, d\omega \tag{9a}$$

$$= \mathbf{x}^T\mathbf{Q}_l\mathbf{x} - 2\mathbf{b}_l\mathbf{x} + \mu \tag{9b}$$

If the weight γ in (9a) is much greater than 1, then minimizing $e(\mathbf{x})$ would yield an FIR filter with a least-square stopband but the passband would not be constrained in any way. The problem can be fixed by imposing the constraint

$$|A(\omega) - 1| \leq \delta_p \qquad \text{for } \omega \in [0,\ \omega_p] \tag{10}$$

A realistic way to implement the constraint in (10) is to impose the constraint on a set of sample frequencies $\mathcal{S}_p = \{\omega_i^{(p)},\ i = 1,\ \ldots,\ M_p\}$ in the passband. These constraints can be put together in the form

$$\mathbf{A}_p\mathbf{x} \leq \mathbf{b}_p \tag{11}$$

Additional constraints can be imposed to ensure that the peak of the amplitude response in the stopband is well controlled. To this end, we can impose the constraint

$$|A(\omega)| \leq \delta_a \qquad \text{for } \omega \in [\omega_a,\ \pi] \tag{12}$$

A discretized version of (12) on $\mathcal{S}_a$, where $\mathcal{S}_a = \{\omega_i^{(a)},\ i = 1,\ \ldots,\ M_a\}$ is a set of sample frequencies in the stopband, can be expressed in matrix form as

$$\mathbf{A}_a\mathbf{x} \leq \mathbf{b}_a \tag{13}$$

The design problem can now be formulated as the minimization problem

$$\text{minimize} \quad e(\mathbf{x}) = \mathbf{x}^T\mathbf{Q}_l\mathbf{x} - 2\mathbf{b}_l\mathbf{x} + \mu \tag{14a}$$

$$\text{subject to:} \quad \begin{bmatrix} \mathbf{A}_p \\ \mathbf{A}_a \end{bmatrix} \mathbf{x} \leq \begin{bmatrix} \mathbf{b}_p \\ \mathbf{b}_a \end{bmatrix} \tag{14b}$$

Since matrix $\mathbf{Q}_l$ is positive definite, this is a convex QP problem which can be solved using the algorithms documented in [6][7].

2.2 Minimax design of FIR filters

The weighted-Chebyshev method can be applied for the design of linear-phase FIR digital filters as described in [2]. In this method, a weighted error function is minimized by using the Remez exchange algorithm. The minimax design of FIR filters can also be carried out using semidefinite programming (SDP). The SDP-based approach is applicable to the design of a general class of FIR filters with arbitrary amplitude and phase responses [8].

In a minimax design, one seeks to find a coefficient vector $\mathbf{h} = [h_0 \ \ h_1 \ \ \cdots \ \ h_{N-1}]^T$ that solves the problem

$$\underset{\mathbf{h}}{\text{minimize}}\{\underset{\omega \in \Omega}{\text{maximize}}[W(\omega)|H(e^{j\omega}) - H_d(\omega)|]\} \tag{15}$$

where Ω is a frequency range of interest. If δ denotes the upper bound of the squared weighted error, i.e.,

$$W^2(\omega)|H(e^{j\omega}) - H_d(\omega)|^2 \leq \delta \qquad \text{for } \omega \in \Omega \tag{16}$$

then the problem can be reformulated as

$$\text{minimize} \quad \delta \tag{17a}$$

$$\text{subject to:} \quad W^2(\omega)|H(e^{j\omega}) - H_d(\omega)|^2 \leq \delta \qquad \text{for } \omega \in \Omega \tag{17b}$$

Now let $H_r(\omega)$ and $-H_i(\omega)$ be the real and imaginary parts of $H_d(\omega)$, respectively, and compute

$$W^2(\omega)|H(e^{j\omega}) - H_d(\omega)|^2 = \alpha_1^2(\omega) + \alpha_2^2(\omega) \tag{18}$$

where

$$\begin{aligned} \alpha_1(\omega) &= W(\omega)[\mathbf{h}^T\mathbf{c}(\omega) - H_r(\omega)] \\ \alpha_2(\omega) &= W(\omega)[\mathbf{h}^T\mathbf{s}(\omega) - H_i(\omega)] \\ \mathbf{c}(\omega) &= [1 \ \ \cos\omega \ \ \cdots \ \ \cos(N-1)\omega]^T \\ \mathbf{s}(\omega) &= [0 \ \ \sin\omega \ \ \cdots \ \ \sin(N-1)\omega]^T \end{aligned}$$

Using (18), the constraint in (17b) becomes

$$\delta - \alpha_1^2(\omega) - \alpha_2^2(\omega) \geq 0 \qquad \text{for } \omega \in \Omega \tag{19}$$

which is equivalent to

$$\mathbf{D}(\omega) = \begin{bmatrix} \delta & \alpha_1(\omega) & \alpha_2(\omega) \\ \alpha_1(\omega) & 1 & 0 \\ \alpha_1(\omega) & 0 & 1 \end{bmatrix} \succeq \mathbf{0} \qquad \text{for } \omega \in \Omega \tag{20}$$

where $\mathbf{D}(\omega) \succeq 0$ denotes that matrix $\mathbf{D}(\omega)$ is positive semidefinite.

Now if

$$\mathbf{x} = \begin{bmatrix} \delta \\ \mathbf{h} \end{bmatrix} \tag{21}$$

then matrix $\mathbf{D}(\omega)$ is *affine* with respect to $\mathbf{x}$. If $\mathcal{S} = \{\omega_i,\ i = 1,\ 2,\ \ldots,\ M\} \subset \Omega$, then a discretized version of (20) is given by $\mathbf{F}(\mathbf{x}) \succeq \mathbf{0}$ where

$$\mathbf{F}(\mathbf{x}) = \text{diag}\{\mathbf{D}(\omega_1),\ \mathbf{D}(\omega_2),\ \ldots,\ \mathbf{D}(\omega_M)\} \tag{22}$$

and the minimization problem in (17) can be converted into

$$\text{minimize} \quad \mathbf{c}^T\mathbf{x} \tag{23a}$$

$$\text{subject to:} \quad \mathbf{F}(\mathbf{x}) \succeq \mathbf{0} \tag{23b}$$

where $\mathbf{c} = [1 \ \ 0 \ \ \cdots \ \ 0]^T$. This problem belongs to the class of SDP problems. Efficient interior-point algorithms and software are now available for the solution of SDP problems [7][9]. It should be pointed out that the weighted least-squares design of FIR filters formulated as a QP problem in (14) can also be solved in an SDP framework (see [8][10]).

3 IIR Filters

IIR digital filters offer improved selectivity, computation efficiency, and reduced system delay compared to what can be achieved with FIR filters of comparable approximation accuracy [2].

Consider the transfer function of an IIR filter

$$H(z) = \frac{A(z)}{\hat{B}(z)} \tag{24a}$$

where

$$A(z) = \sum_{n=0}^{N-1} a_n z^{N-1-n},\ \ \hat{B}(z) = z^{N-K}B(z) \tag{24b}$$

$$B(z) = \sum_{n=0}^{K-1} b_n z^{K-1-n} \qquad \text{with } b_0 = 1 \tag{24c}$$

and K is an integer between 1 and N. The denominator polynomial $\hat{B}(z)$ in (24b) has $N - K$ zeros at the origin which, as was recently observed [11], can be beneficial for the design of several types of digital filters. The basic weighted least-squares and minimax designs of stable IIR filters can be formulated as

$$\text{minimize } J_2(\mathbf{x}) = \int_{-\pi}^{\pi} W(\omega)|H(e^{j\omega}) - H_d(\omega)|^2 \, d\omega \tag{25a}$$

$$\text{subject to: } B(z) \neq 0 \qquad \text{for } |z| \geq 1 \tag{25b}$$

and

$$\text{minimize}\{\underset{-\pi\leq\omega\leq\pi}{\text{maximize}}[W(\omega)|H(e^{j\omega}) - H_d(\omega)|^2]\} \tag{26a}$$

$$\text{subject to: } \quad B(z) \neq 0 \qquad \text{for } |z| \geq 1 \tag{26b}$$

respectively. The constraints in (25b) and (26b) are imposed to ensure the filter's stability. Additional constraints may be imposed to design a filter that satisfies certain design specifications.

The optimization problems in (25) and (26) give rise to two technical issues that are not encountered in FIR designs. First, the presence of the denominator polynomial $B(z)$ makes (25) and (26) constrained *rational approximation* problems with a much higher nonlinearity compared to that in FIR filters. Second, the stability constraint as described in (25b) and (26b), confines the zeros of $B(z)$ to within the unit circle. Since the zeros of a high-order polynomial are related to the polynomial's coefficients in an implicit and highly nonlinear manner, dealing with the stability constraint turns out to be a technical challenge.

3.1 Modifications of object function

A typical treatment of the objective functions in (25) and (26) is to write

$$|H(e^{j\omega}) - H_d(\omega)| = \frac{|A(e^{j\omega}) - B(e^{j\omega})H_d(\omega)|}{|B(e^{j\omega})|} \tag{27}$$

and neglect the denominator $|B(e^{j\omega})|$ in the optimization. This transforms (25a) into a quadratic objective function. A problem with this approach is that neglecting $|B(e^{j\omega})|$ *alters* the objective function and, consequently, the solution obtained can at best be a suboptimal solution for the original problem.

A remedy for the problem is to treat the denominator $|B(e^{j\omega})|$ obtained from the preceding optimization iteration as a part of the weighting function [12]. For the objective function in (25a), this leads to a modified $J_2(\mathbf{x})$ as

$$J_{2k}(\mathbf{x}) \;=\; \int_{-\pi}^{\pi} W_k(\omega)|A(e^{j\omega}) - B(e^{j\omega})H_d(\omega)|^2 \, d\omega \tag{28a}$$

$$W_k(\omega) \;=\; \frac{W(\omega)}{|B_{k-1}(e^{j\omega})|} \tag{28b}$$

where $B_{k-1}(e^{j\omega})$ is the denominator polynomial obtained from iteration $k-1$. Note that $J_{2k}(\mathbf{x})$ in (28a) is a quadratic function of the filter coefficients and, therefore, least-squares designs of IIR filters can be obtained using iterative quadratic programming (QP) routines.

In control and adaptive filtering literature, the iterative modification of the weighting function in a least-squares approximation similar to (28) is known as the Steiglitz-McBride algorithm [13]. The above design concept, however, can be extended to cases other than least-squares design. For instance, the objective function in the minimax problem in (26) can be converted into an upper bound δ of the form

$$W_k^2(\omega)|A(e^{j\omega}) - B(e^{j\omega})H_d(\omega)|^2 \leq \delta \tag{29}$$

where $W_k(\omega)$ is defined by (28b). It can be shown that (29) is satisfied if and only if a matrix $\mathbf{\Gamma}_k(\mathbf{x},\ \delta)$, which is affine with respect to filter coefficient vector $\mathbf{x}$ and upper bound δ, is positive semidefinite [14].

Another approach for handling the nonlinearity of function $J_2(\mathbf{x})$ in (25a) is to approximate $H(e^{j\omega})$ in a neighborhood of the current iterate $\mathbf{x}_k$ as [11]

$$H(e^{j\omega}) \approx H_k(e^{j\omega}) + \nabla^T H_k(e^{j\omega})\boldsymbol{\delta}$$

where $H_k(e^{j\omega})$ and $\nabla H_k(e^{j\omega})$, denoting $H(e^{j\omega})$ and its gradient, are evaluated at $\mathbf{x}_k$. This reduces $J_2(\mathbf{x})$ to

$$\hat{J}_{2k}(\boldsymbol{\delta}) \;=\; \int_{-\pi}^{\pi} W(\omega)|\nabla^T H_k(e^{j\omega})\boldsymbol{\delta} - D_k(\omega)|^2 \, d\omega \tag{30}$$

where

$$D_k(\omega) \;=\; H_d(\omega) - H_k(e^{j\omega})$$

The reduced objective function $\hat{J}_{2k}(\boldsymbol{\delta})$ in (30) is a quadratic function with respect to $\boldsymbol{\delta}$. Once the optimal $\boldsymbol{\delta}^*$ is determined by the optimization, the next iterate is generated as $\mathbf{x}_{k+1} = \mathbf{x}_k + \boldsymbol{\delta}^*$ and so on.

3.2 Stability

A stability constraint, which depends on the filter's coefficient affinely, was proposed in [15] as

$$\text{Re}[B(e^{j\omega})] \geq \varepsilon \qquad \text{for } -\pi \leq \omega \leq \pi \tag{31}$$

where $\varepsilon > 0$ is a small tolerance. With (31) as the stability constraint and $J_{2k}(\mathbf{x})$ in (28a) or $\hat{J}_{2k}(\boldsymbol{\delta})$ in (30a) as the objective function, weighted least-squares designs can be obtained using QP [11][12]. A problem with the constraint in (31) is that it is a sufficient condition for stability, which is often found to be too restrictive in the sense that many good stable designs will be excluded.

An improved stability constraint based on Rouché's theorem was proposed in [11]. To explain the method, let $B(z,\ \mathbf{x})$ be a polynomial, whose zeros at $\mathbf{x} = \mathbf{x}_k$ are strictly inside the unit circle, and $\boldsymbol{\delta}$ be a vector increment for $\mathbf{x}_k$. According to Rouché's theorem, $B(z,\ \mathbf{x}_k)$ and $B(z,\ \mathbf{x}_k + \boldsymbol{\delta})$ have the same number of zeros inside the unit circle if

$$|B(e^{j\omega}, \boldsymbol{\delta})| \leq |B(e^{j\omega}, \mathbf{x}_k)| \qquad \text{for } -\pi \leq \omega \leq -\pi \tag{32}$$

In other words, the filter at $\mathbf{x}_k + \boldsymbol{\delta}$ is stable if it is stable at $\mathbf{x}_k$ and $\boldsymbol{\delta}$ satisfies (32). Although (32) is not an affine constraint on $\boldsymbol{\delta}$, it can readily be converted into a linear matrix inequality (LMI) so that the design can be formulated as an SDP problem.

A drawback of the constraints in (31) and (32) is their dependence on the frequency ω which varies from $-\pi$ to π. Discretizing (31) and (32) for $\omega \in [-\pi,\ \pi]$ in terms of a dense set of grid points yields a large number of constraints. This inevitably degrades the quality of the design and adds additional computational complexity to the design. This problem can be fixed with an LMI inequality based on the well-known Lyapunov stability theory [14]. Note that polynomial $B(z)$ in (24c) is of order $K-1$ and its zeros are the eigenvalues of the matrix

$$\mathbf{D} = \begin{bmatrix} -b_1 & -b_2 & \cdots & -b_{K-1} \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \tag{33}$$

Matrix $\mathbf{D}$ in (33) is said to be *stable* if the largest modulus of its eigenvalues is strictly less than one. Hence the IIR filter is stable if and only if $\mathbf{D}$ is a stable matrix. The Lyapunov theory [16] states that $\mathbf{D}$ is stable if and only if there exists a positive definite matrix $\mathbf{P}$ such that $\mathbf{P}-\mathbf{D}^T\mathbf{P}\mathbf{D}$ is positive definite. For a stable $B_{k-1}(z)$, there exists a positive definite $\mathbf{P}_{k-1}$ that solves the Lyapunov equation [16]

$$\mathbf{P}_{k-1}-\mathbf{D}_{k-1}^T\mathbf{P}_{k-1}\mathbf{D}_{k-1}=\mathbf{I} \tag{34}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{D}_{k-1}$ is the matrix in (33) with $-\mathbf{b}_{k-1}^T$ as its first row. Eq. (34) suggests a stability constraint for $B(z)$ as

$$\begin{bmatrix} \mathbf{P}_{k-1}^{-1} & \mathbf{D} \\ \mathbf{D}^T & \mathbf{P}_{k-1} \end{bmatrix} \succ \mathbf{0} \tag{35}$$

i.e., the above matrix is positive definite. Note that the LMI constraint in (35) is affine with respect to the filter's coefficients and is independent of frequency ω.

In [17], a least-squares algorithm was proposed where the filter's stability is assured by including a weighted sum of the squared impulse response in the objective function.

4 Filter Banks

The design of filter banks can also be considered in an optimization framework. For a 2-channel FIR filter bank with analysis lowpass filter H_0 and highpass filter H_1, and synthesis lowpass filter F_0 and highpass filter F_1, the input-output relation of the system is given by

$$\hat{X}(z) = T(z)X(z)+A(z)X(-z)$$

where

$$\begin{aligned} T(z) &= \tfrac{1}{2}[H_0(z)F_0(z)+H_1(z)F_1(z)] \\ A(z) &= \tfrac{1}{2}[H_0(-z)F_0(z)+H_1(-z)F_1(z)] \end{aligned}$$

and $H_i(z),\ F_i(z)$ are the transfer function of the various filters. By assuming that $H_1(z)=H_0(-z),\ F_1(z)=-H_0(-z)$, and $F_0(z)=H_0(z)$, then $A(z)$ becomes zero, the aliasing term is eliminated, and $T(z)$ becomes

$$T(z)=\tfrac{1}{2}[H_0^2(z)-H_0^2(-z)]$$

Perfect reconstruction requires that

$$H_0^2(z)-H_0^2(-z)=2z^{-k_d} \tag{36}$$

where k_d is the normalized reconstruction delay. Assuming a passband gain of $\sqrt{2}$ for the lowpass filter, the design problem at hand can be solved by minimizing the objective function

$$\Psi(\mathbf{x})=\int_0^{\omega_p}|H_0(e^{j\omega})-\sqrt{2}e^{-jk_d\omega/2}|^2\,d\omega+\int_{\omega_s}^{\pi}|H_0(e^{j\omega})|^2\,d\omega$$

subject to the constraint in (36). This constrained optimization problem can be converted into an unconstrained problem which can be solved by an iterative algorithm as suggested by several authors recently [18]–[20]. In [21], the design of 2-channel FIR filter banks was carried out by minimizing the stopband energy of the analysis filters subject to some passband and transition and constraints and allowable reconstruction error.

Acknowledgement The authors are grateful to Micronet and the Natural Science and Engineering Research Council of Canada for supporting this work.

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