2-D WOLD DECOMPOSITION: NEW PARAMETER ESTIMATION APPROACH TO EVANESCENT FIELD SPECTRAL SUPPORTS

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ABSTRACT

In the context of parametric model-based methods for image processing, this paper deals with image modeling, when the image is considered as a two-dimensional (2-D) random process. The model we are searching on is based on the 2-D Wold-type decomposition. The core of the contribution of this paper is to provide with a new estimation algorithm of the so called "evanescent field" in the 2-D Wold decomposition framework. This new method is based on a projection approach and requires a set of projection directions which is obtained by using Farey's series. The algorithm performances are illustrated and investigated using Monte Carlo simulations.

1 INTRODUCTION

Many natural images can be described as a finite ensemble of patches of uniform textures. Therefore, the ability to exactly parameterize texture patches and to have an efficient parameter estimation methods are of great importance in the fields of image understanding, image analysis and synthesis. To obtain an exact texture parameterization is linked to an appropriate choice of model. The use of Wold-based modeling which is the core of this paper provides an appropriate characterization of the image randomness, its periodicity and directionality.

On the basis of the 2-D Wold-like decomposition for homogeneous random fields reported in [1], the texture field, considered as a realization of random fields, can be decomposed into a sum of two mutually orthogonal components: a purely non deterministic field and a deterministic one. Furthermore, the deterministic component can be expressed as a sum of two orthogonal components: a sum of harmonic fields and a countable number of evanescent fields. In many applications, the deterministic part of the field contains either harmonic components or evanescent components. As a consequence, we only consider in this paper the estimation of evanescent components combined with a purely non deterministic field. When considering the spectral support of the evanescent field, this arises as peaks which form continuous lines in the spectral domain [2]. These lines are parameterized by their slopes. The estimation of the slope parameters is the focus of this paper.

A previous estimation method of these slope parameters have been proposed by Francos using Hough Transformation [2] [3]. The main drawback of this approach is to only consider the number of summed pixels. As a consequence, this leads to a bad estimation accuracy when there is a small number of considered pixels in the transformation. The "*projection approach*" proposed in this paper considers either the number of summed pixels and the value of these pixels. This method consists in orthogonally projecting the Fourier magnitude image in several directions. A set of projection directions is then required. Each projection is performed by adding all encountered pixel values along a projection direction. A criteria of line detection will then be proposed in this paper using the different projection values.

This paper is organized as follows: In part II, we briefly recall the 2-D Wold decomposition models. In part III, a five-step procedure based on "projection approach" is described in order to estimate the evanescent field slope parameters. Therefore, an algorithm to obtain a set of projection directions is proposed. In the last section, we provide experimental results.

2 THE TEXTURE MODEL

The linear prediction theory of a 2-D random field requires the definition of a total order and a support region in the 2-D plane.

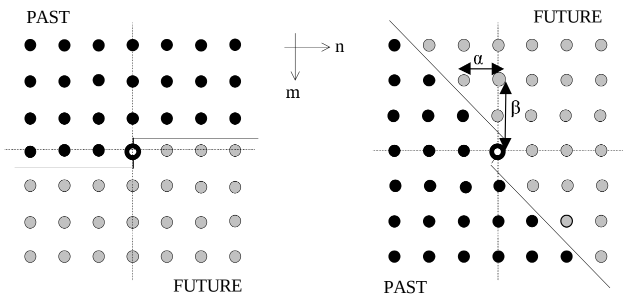
A total order can be defined for the samples of a random field $\{y(n,m),(n,m)\in Z^2\}$ as

$(i,j)\prec(s,t)$ iff

$$(i,j)\in\{(k,l)/\,l=t,k<s\}\cup\{(k,l)/\,l<t,-\infty<k<\infty\} \quad \textbf{(1)}$$

Based on the total-order definition, *a totally ordered, non-symmetric half plane* (NSHP) support can be defined as follows: given the $(n,m)^{th}$ sample as "present", all $(i,j)\prec(n,m)$ are in the "past", and the rest is in the "future" (Fig. 1(a)).

Let α and β be coprime integers and $\alpha\neq 0$. A new total-order and NSHP support can be defined on the original grid by rotating the NSHP total ordering counterclockwise by an angle



(a) **(b)**

Fig. 1 (a) NSHP total order support,
(**b**) RNSHP total order support with (α,β) = (-1,2).

$$\theta = \tan^{-1}\frac{\beta}{\alpha} \qquad \textbf{(2)}$$

about the origin of its coordinate system, (Fig. 1(b)). This new support is called *the rational non-symmetric half-plane* (RNSHP) support since its boundary line is of rational slope. Denote the set of all possible total order and RNSHP support defined in this manner by

$$\mathcal{O} = \{o \,/\, o = (\alpha, \beta); \quad \alpha, \beta \text{ are coprime integers}\}.$$

According to [2], when no harmonic component is present, the field $\{y(n,m)\}$ can be represented by a sum of a purely non deterministic field denoted by $\{w(n,m)\}$, and a countable number of evanescent fields $\{e_o(n,m)\}$. The latter is performed with respect to the RNSHP total order support $o \in \mathcal{O}$.

$$y(n,m) = w(n,m) + \sum_{o \in \mathcal{O}} e_o(n,m). \qquad \textbf{(3)}$$

Figure 2 shows the spectral representation of the field *y*.

Under some assumptions and approximations [2], Francos et al. have proposed the following models:

a- $\{w(n,m)\}$ can be represented by a 2-D AR model driven by a 2-D white innovation fields $\{u(n,m)\}$ with variance σ_1^2.

$$w(n,m) = -\sum_{(k,l)\in S_{N,M}} b(k,l)\, w(n-k, m-l) + u(n,m), \qquad \textbf{(4)}$$

where $\forall(k,l)$, $b(k,l)$ are the prediction coefficients of the AR model of NSHP support defined for $(N,M) \in Z^2$ as

$$S_{N,M} = \{(k,l) \,/\, l = 0, 0 \le k \le N; 0 < l \le M, -N \le k \le N\} \qquad \textbf{(5)}$$

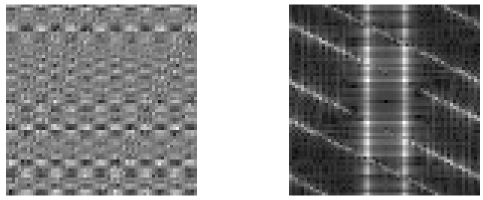


Fig. 2 (Left) Synthetic texture with two evanescent components of slope parameters (1,0) et (1,-2). **(Right)** its corresponding spectral representation.

b- The evanescent field model is defined w. r. t. the RNSHP total order supports. For a given $(\alpha,\beta) \in \mathcal{O}$, the associated evanescent component is modeled as follows:

$$e_{(\alpha,\beta)}(n,m) = \sum_{i=1}^{I}\Big[s_i^{\alpha,\beta}(n\alpha - m\beta)\cos\Big(2\pi \frac{\nu_i}{\alpha^2+\beta^2}(n\beta + m\alpha)\Big) + t_i^{\alpha,\beta}(n\alpha - m\beta)\sin\Big(2\pi \frac{\nu_i}{\alpha^2+\beta^2}(n\beta + m\alpha)\Big)\Big] \qquad \textbf{(6)}$$

where

- I is the number of evanescent components with (α,β) slope,
- ν_i is the frequency of the i^{th} evanescent component,
- $\{s_i(n\alpha - m\beta)\}$, $\{t_i(n\alpha - m\beta)\}$ are mutually orthogonal 1-D purely non deterministic processes. Both can be modeled with finite orders 1-D AR models.

When considering the decomposition reported in (3), a combination of evanescent components with different slope parameters can be made. For each slope, the corresponding evanescent component has the form of (6).

3 ESTIMATION OF THE SLOPE PARAMETERS

In this section, let us consider a field $\{y(n,m), (n,m) \in D\}$ considered as a realization of a 2-D random field where the region $D = \{(n,m): 0 \le n, m \le T-1\}$. Let us recall that each evanescent component of the field y with slope parameters $(\alpha_\rho, \beta_\rho)$ arises in the spectral domain as lines oriented in $(\alpha_\rho, \beta_\rho)^\perp$; this latter is the orthogonal of $(\alpha_\rho, \beta_\rho)$ (Fig. 2).

The estimation of the slope parameter (α,β) requires the following five step procedures:

a- Computation of A_r, $0 < r < T$, a set of projection vectors comprising all integer couples (a_i, b_i) such that

$$A_r = \Big\{(a_i, b_i), 0 \le a_i \le b_i < r, \ \mathrm{GCD}(a_i, b_i) = 1, \ \theta_i = \tan\frac{b_i}{a_i} \in [0, \pi]\Big\} \qquad \textbf{(7)}$$

A_r is obtained by using Farey's series [5]. By definition, a rth order Farey's series is F_r, the set of irreducible ratio p/q in

$[0,1]$ ($q \le r$) ranked in increasing order. F_r have the following properties:

- Two consecutive elements p/q and p'/q' satisfy the unimodular relation $p'q - pq' = 1$.
- The first two elements are $0/1$ and $1/r$.
- If p/q and p'/q' are two consecutive elements of F_r, the next element will be defined as follows

$$p'' = \lfloor (q+r)/q' \rfloor p' - p, \text{ and}$$
$$q'' = \lfloor (q+r)/q' \rfloor q' - q,$$

where $\lfloor .. \rfloor$ is the floor function (if $n \le x \le n+1$, then $\lfloor x \rfloor = n$).

These properties allow a direct construction of the given r^th order Farey's series.

The corresponding (p,q) set describes the first octant of A_r and the three others can be deduced by symmetries. Then,

$$A_r = \{(p,q),(q,p),(p,-q),(q,-p) \ / \ (p,q) \in F_r\}. \quad \textbf{(8)}$$

b- Let Y be the Discrete Fourier Transform of the field y. We compute $Q(\eta,\zeta)$ the magnitude of Y, where Y is given by

$$Y(\eta,\varsigma) = \sum_{m=0}^{T-1}\sum_{n=0}^{T-1} y(n,m) e^{-j\frac{2\pi}{T}\eta n} e^{-j\frac{2\pi}{T}\varsigma m}, \quad (\eta,\varsigma) \in D. \quad \textbf{(9)}$$

c- For any (a_i, b_i) pairs belonging to A_r, we compute the projection of the Fourier magnitude image along the given (a_i, b_i) direction on a 1-D axis. This projection is given by

$$v_i(\lambda) = \sum_{\tau=-\infty}^{\infty} Q(\eta + \tau b_i, \varsigma + \tau a_i), \ (\eta,\varsigma) \in D \quad \textbf{(10)}$$

where $\lambda = -b_i\eta + a_i\varsigma$ [4].

d- Let $npix_i$ be the number of contributed pixels in this projection. For any (a_i, b_i) pairs belonging to A_r, we determine the maximum value of elements of the vector v_i: $\lambda^i_{\max} = \max_{\lambda}\{v_i(\lambda)\}$. A line of direction (a_ρ, b_ρ) is detected if and only if the following criteria is verified

$$\frac{\lambda^\rho_{\max}}{npix_\rho} = \max_i\left\{\frac{\lambda^i_{\max}}{npix_i}\right\}, i \in [0, card(A_r)]. \quad \textbf{(11)}$$

e- The estimated slope parameters is $(\alpha,\beta) = (a_\rho, b_\rho)^\perp$.

As far as computation cost is concerned, this method corresponds to the Fast Fourier Transform, that requires $O(T\log_2 T)$ operations. The algorithm that constructs A_r the set of projection directions is computed just before the projection algorithm and its computation cost is insignificant compared with this last one.

When only a single evanescent component is present in the field, one iteration is sufficient to obtain the estimated slope parameters. When there are multiple evanescent components, it is necessary to perform several iterations. For any iteration, if every parameter of one evanescent component is estimated, we next extract the estimated evanescent component from the field $\{y(n,m), (n,m) \in D\}$ and repeat this procedure iteratively until every evanescent component is extracted. The residual field is the purely non deterministic component.

4 EXPERIMENTAL RESULTS

In the following examples, we consider 64×64 synthetic textures. For each texture, we use the 3^th order Farey's series. The algorithm performance is illustrated by estimating direction error rates through Monte Carlo simulations. The experimental results are based on 1000 independent realizations of the purely non deterministic component and the modulating 1-D purely non deterministic processes of each evanescent field.

Example 1: We consider a field which consists of a sum of a purely non deterministic component and a single evanescent component. The purely non deterministic component is modeled by a 2-D AR model with support $S_{1,1}$ driven by a Gaussian white noise input process of variance σ^2_{PA}. The modulating 1-D purely non deterministic processes $s^{1,-2}$ and $t^{1,-2}$ of the evanescent component are independent second-order AR processes driven by a unit variance Gaussian white noise input process.

Example 2: In this example, we consider a field that is a sum of a purely non deterministic component with AR model support $S_{1,1}$ driven by a unit Gaussian white noise input process and two evanescent components. The parameters of the first evanescent component are chosen to be the same as in example1. The modulating 1-D purely non deterministic processes $s^{1,0}$ and $t^{1,0}$ of the second evanescent component are independent second-order AR process driven by a unit variance Gaussian white noise input process.

The noise sensitivity of this method using Monte Carlo simulations for example 1 is shown in Table1. Let us notice that when $\sigma^2_{PA} = 1$, the (α,β) error rate is equal to zero for 1000 experiments. Note from Table2 that, in presence of multiple evanescent components, the slope parameters of evanescent components that have a very closed energy are simultaneously estimated with different detection rate for 1000 experiments. The first slope parameter (1,-2) is obtained with 39,8% of detection rate, while the second (1,0) is located with 60,2% of detection rate. We do not encounter erroneous (α,β) pairs.

Example3: Let us consider four natural textures of size 256×256 from the Brodatz album [6] (Fig. 3). These textures contain evanescent components. The slope parameters and frequency for one component are accurately estimated; they are listed in table3. If all parameters of this one evanescent component are estimated, we next extract the estimated evanescent component from the observed signal $\{y(n,m), (n,m) \in D\}$ and repeat this procedure iteratively until all the evanescent components are extracted. The residual field is the purely indeterministic component.

Field parameters		**(α,β) error rate**
Example1 $\alpha = 1$ $\beta = -2$	$\sigma_{PA} = 1.0$	0 %
	$\sigma_{PA} = 1.5$	0,1 %
	$\sigma_{PA} = 2.0$	0,7 %
	$\sigma_{PA} = 2.5$	3,5 %
	$\sigma_{PA} = 3.0$	13 %

Table 1: Noise sensitivity of the method for example 1

Field parameters	**Estimated (α,β) pairs**	**(α,β) detection rate**
Example2 $\sigma_{PA} = 1.0$, $\alpha_1 = 1$, $\beta_1 = -2$ $\alpha_2 = 1$, $\beta_2 = 0$,	(1,-2)	39,8 %
	(1,0)	60,2 %

Table 2: Detection rate in case of existence of two evanescent components (example 2).

Original field	(a)	(b)	(c)	(d)
Estimated (α,β) pairs	(1,0)	(1,0)	(1,0)	(0,1)

Table 3 : Estimated (α,β) pairs for the four Brodatz textures in figure 3.

5 CONCLUSION

The estimation of the slope parameters is the key step in the estimation of the evanescent field parameters. This is due to the fact that erroneous estimates of the slope parameters will provide erroneous estimates of the other evanescent component parameters. Projection algorithm has two advantages: it first reduces the computation cost and secondly provides more accurate estimation due to a good choice of projection set.

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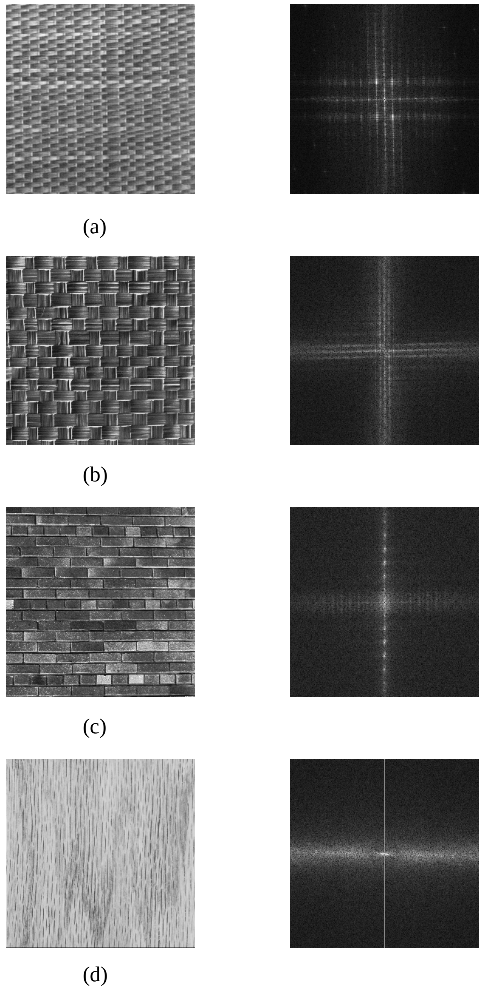

Fig. 3: Left column: Original Brodatz textures,
Right column: 2-D DFT magnitude.
(a) d55 with evanescent components,
(b) d64 with evanescent components, (c) d94,
(d) Wood grain having a strong evanescent component.

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