A MODIFIED VITERBI ALGORITHM FOR NON-GAUSSIAN INTERFERENCE SCENARIOS

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ABSTRACT

Viterbi sequence detectors and Turbo decoders are widely used in the receivers of mobile communications systems. They offer an efficient means of implementing a near-maximum likelihood receiver. However, Viterbi sequence estimators are usually based on the assumption of Gaussian noise and/or interference statistics. It is becoming clear that for third generation and enhanced second generation cellular mobile packet systems that Gaussian-ness cannot be taken for granted and in such scenarios it is not clear what performance a Viterbi receiver will offer. This paper explores this issue and develops a modified Viterbi for just such situations and presents performance results.

INTRODUCTION

The drive towards providing multi-media wireless services has accelerated in recent years, fuelled by the rapid growth in the Internet and the world wide web. The arrival of 3rd generation cellular access communication systems by 2002 will result in multi-media services being offered over 3G cellular networks. These multimedia services are likely to range from streaming of MPEG coded video, teleconferencing and video-on-demand at the high bit rate end down to Internet browsing and non-real time file transfers. The likelihood is that TCP/IP protocols and packet transfers will pervade the cellular radio environment. Moreover the decision to use code division multiple access (CDMA) for 3G access, a modulation/coding format where the radio spectrum is a communal resource with widespread low-level mutual interference means that bursty RF interference effects will be present for large numbers of users. Indifferent adjacent channel protection in 3G cellular systems combined with high variability of transmitter EIRP’s for differing bit rates means that there is a high risk of intra- and inter-operator interference from all undesirable combinations of terminal to terminal, terminal to base and base to base couplings. Various combinations of bursty interference from adjacent cells and adjacent frequency channels which has a heavy tailed probability distribution plus interleaving in the receiver can result in the decoder being presented with a signal which may be fading and have additive non-Gaussian noise.

Viterbi sequence detectors and their close relatives the turbo decoders are widely used in the receivers of mobile communications systems. They are recognized as an efficient means of implementing a maximum likelihood sequence detector. Viterbi-based sequence estimators are traditionally based on the assumption that the noise and/or interference statistics are uncorrelated Gaussian noise. However, it is becoming clear that for third generation and enhanced second generation
cellular mobile systems this AWGN model may no longer be valid and it is not clear what performance the Viterbi receiver will offer. This paper explores this issue and develops modified Viterbi metrics for situations where the noise is non-Gaussian. Results are presented which show a significant BER performance improvement with only a low increase in computation count.

**A MODIFIED VITERBI ALGORITHM**

If the channel symbols expected in say a half rate coder are \( s_1, s_2 \) etc. and the observed symbols are \( x_1, x_2 \) then the probability of the transition is proportional to the appropriate displacement in the 2-dimensional Gaussian PDF \( \{ \text{eqn (1)} \} \)

\[
p(x_1, x_2 \mid s_1, s_2) = \frac{1}{2\pi\sigma^2} \exp \left( -\frac{(x_1 - s_1)^2}{2\sigma^2} \right) \exp \left( -\frac{(x_2 - s_2)^2}{2\sigma^2} \right)
\]

In log space these metrics accumulate additively over the sequence run rather than multiplicatively:

\[
\log \left\{ p(x_1, x_2 \mid s_1, s_2) \right\} = \log \left\{ p_G(x_1 - s_1) \right\} + \log \left\{ p_G(x_2 - s_2) \right\}
\]

The constant term and the noise standard deviation \( \sigma \) can be omitted and this is the form in which the algorithm is conventionally presented. When the conditional probability distribution of the noise is replaced by an arbitrary general (non-Gaussian) distribution function \( p_G(.) \) then eqn (1) is replaced by

\[
\log \left\{ p_G(x_1, x_2 \mid s_1, s_2) \right\} = \log \left\{ p_G(x_1 - s_1) \right\} + \log \left\{ p_G(x_2 - s_2) \right\}
\]

We have now lost the ability to use the simple quadratic accumulating running metric and may have to operate in probability space, but to compensate, we have gained the ability to model the noise accurately and expect the resulting sequence detector, with its normal conditional branching arrangements, to have maximum likelihood performance in relation to a known noise probability distribution.

Of particular interest is the behaviour of the detector in the presence of noise transients. This can be understood by the following simple example. If the likelihood ratio of the observed probabilities of a single symbol for binary user data \( \pm 1 \) are calculated at the receiver we have:

\[
LR(x) = \frac{P(x \mid s = +1)}{P(x \mid s = -1)} = \frac{p(x - 1)}{p(x + 1)}
\]

For the quadratic Gaussian metric assumption this likelihood ratio diverges indefinitely for large excursions of the sampled value:

\[
LR_2(x) = \frac{\exp \left( -\frac{(x - 1)^2}{2\sigma^2} \right)}{\exp \left( -\frac{(x + 1)^2}{2\sigma^2} \right)} = \frac{4x}{2\sigma^2}
\]

and the detection process is highly vulnerable to overload. By contrast, in the case of a non-quadratic metric the overload behaviour can be considerably modified to make it less sensitive. For example, if a first power law replaced the quadratic one we would get

\[
LR_1(x) = \begin{cases} 
\exp(2) & x > 1 \\
\exp(2x), & -1 < x < +1 \\
\exp(-2) & x < -1 
\end{cases}
\]

This metric has the unusual property of remaining constant for large excursions of \( x \) outside \( |x|=1 \). It resembles a limiter and does not inject unwanted large likelihood ratio samples into the sequential detection process. We would expect the final sequence decision to be relatively unbiased by the presence of impulses using this metric and also expect better performance than if a limiter was used at the front end of the receiver which is the intuitive place to install it.

These simple calculations show that (i) the Viterbi sequence estimator, when designed with a Gaussian assumption, will perform poorly in an environment where the noise is impulsive (ii) that when the noise distribution has a heavier tail than Gaussian noise we should use a slowly decaying long-tailed
metric in the likelihood ratio calculation. This is perhaps a rather counter-intuitive result. In the absence of precise knowledge of the noise distribution the metrics could take on the generic form,

\[ \exp\left(-|x - s|^\rho\right) \text{ where } \rho < 1 \]

Using such metrics causes the likelihood ratio to converge on unity, for large noise excursions, and to offer no opinion on the sign of the underlying data. Noise impulses will therefore bias the data sequence decision process only marginally though the corollary is that the detection performance in Gaussian noise may well be compromised. Gaussian-based detectors tend to rely on the larger excursions of the receiver signal plus noise samples.

**PERFORMANCE**

Simulation results are now presented of the performance of the modified metric in non-Gaussian and Gaussian interference scenarios. Fig. 1 shows the performance of the modified Viterbi sequence estimator against interleaved log-normal faded interference, of unknown level, using a variety of values for \( \rho \) in the equation above where \( \rho = 2 \) equates to the Gaussian assumption. There is a performance enhancement of up to 2 dB obtained from using lower \( \rho \) values. In such a situation the post-deinterleaving C/I ratio of each sample would sometimes be known, so the comparison is a little unfair. We are here merely using the fading as a convenient non-Gaussian noise source.

Next figure 2 shows the equivalent performance against Gaussian interference whose envelope is Rayleigh faded. Again there is an enhancement of one or two decibels.

Finally fig. 3 shows the performance of a non-linear receiver operating in standard AWGN. We see that the performance is slightly degraded when \( \rho \) is less than the Gaussian-assumption ideal of 2.
CONCLUSIONS

The conclusion drawn from the results presented in this paper is that Viterbi algorithms optimised for Gaussian noise do not work too well in bursty interference, whose statistics are better described by a heavy-tailed probability distribution. Such bursty interference is expected to be in evidence in the third generation cellular environment with its less than ideal inter-cell and intra-cell interference rejection performance combined with packet mode traffic causing bursty interference through fluctuations in EIRP. The presence of a heavy tailed noise pdf requires a corresponding “heavy-tailed” metric with long sidelobes in a maximum likelihood receiver. The Viterbi algorithm is easily modified to handle this so we can still have a maximum likelihood sequence detector while retaining the numerical efficiency of the sequential algorithm. The performance of this modified Viterbi can be up to 2 dB better even in relatively simple Rayleigh faded and lognormal interference. If the modified receiver is now used in an AWGN environment then it has become suboptimum so we must have some a-priori idea of the noise statistics whether in the form of a complete probability distribution or just a general description of its heavy tail characteristics.

REFERENCES

