ON EXTENDED SOURCE LOCALIZATION IN MULTIBASELINE AND MULTIFREQUENCY SAR INTERFEROMETRY

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ABSTRACT
Baseline or frequency diversity have been recently proposed to reduce problems of interferometric phase ambiguity and data noise in synthetic aperture radar interferometry (InSAR) for topographic mapping. This paper presents a general model-based multibaseline multifrequency (MBMF) InSAR framework. A statistical model for the MBMF SAR-processed echoes from extended natural targets is presented, then the Cramèr-Rao lower bound on the MBMF-InSAR phase accuracy is derived. The maximum likelihood estimator for MBMF-InSAR is introduced, and its simulated performance is compared to the bound for the common cases of dual baseline InSAR, dual frequency InSAR, and for a novel joint dual baseline dual frequency InSAR configuration. Performance of these configurations and improvement over conventional interferometry is analyzed.

1 INTRODUCTION
Cross-track SAR interferometry (InSAR) is a modern technique to derive high-resolution elevation maps of the land surface from SAR images [1, and references therein]. It has increasing applications in topographic mapping, geophysical remote sensing, forestry, surface change detection and classification. Waveform modulation and synthetic aperture processing exploiting platform motion provide the desired horizontal resolution. Two SAR images are taken from two antennas slightly separated by a cross-track baseline. This can be obtained either from a repeat-pass satellite-borne SAR or from a dual-antenna airborne interferometric SAR. After image registration, for each pixel corresponding to the same area of ground in both images the terrain height is determined from the elevation angle, the known pixel slant-range distance and the interferometer attitude and altitude. The elevation angle is determined very accurately from the estimated phase center from it; the known pixel slant-range distance and the interferometer attitude and altitude. The elevation angle is determined very accurately from the estimated phase difference at the end-to-end baseline. These baselines are normalized to the critical baseline for topographic mapping problem can be cast as a direction of arrival estimation problem for extended sources, being typical natural (geological) targets distributed and the SAR spatial resolution finite (see Fig.1). Despite its importance, the array processing literature on distributed source localization is sparse. Some work has been carried out, e.g., in the field of mobile communications (see [5]). Here we apply parameter estimation theory to a practical problem that has some resemblance with the latter, which is of particular interest for the radar and remote sensing community and has not been analyzed before.

Also, in the paper the recently proposed [6] maximum likelihood (ML) phase estimator for MB-InSAR is recalled, and the ML estimator for MBMF-InSAR is derived, so extending [1,4,6]. The small sample simulated performance of ML dual baseline, dual frequency, and joint dual baseline dual frequency InSAR is compared to the corresponding CRBLs and to conventional interferometry.

2 MBMF-INSAR MODEL FOR SPATIALLY DISTRIBUTED SOURCE
Consider M radar carrier frequencies $f_u$, $u = 1, \ldots, M$, with $f_1 = f$ the highest frequency. For each $u$-th carrier frequency, $K_u$ complex SAR images are obtained by processing the echoes from an array of $K_u$ aligned phase centers $u k, k=1, \ldots, K_u$. An example is in Fig.1 for $M=2$, $K_1 = K_2 = 3$. For each $u$-th frequency array, taking the first phase center as a reference, we define the spacing $B_{uK}$ as the distance of the $k$-th phase center from it: $B_{uK} = B_k$ is the end-to-end baseline. These baselines are normalized to the critical baseline $B_{cK} = B_c$, namely the baseline for which the signals at the baseline extremes are completely decorrelated [1] for the highest carrier frequency $f_1$. We call $\varphi$ the interferometric phase difference at the end-to-end...
baseline of the 1st frequency array (master baseline $B_i = B$), related to the nominal terrain elevation angle [1]. Under the far-field assumption, the corresponding phase difference at the elements $ul,um$ of the $u$-th frequency array is given by the fraction $f_u(B_{ul} - B_{um})/(f_u B_i)$ of it. As usual in SAR interferometry, in each image taken at frequency $f_u$ we consider $N_u$ independent and identically distributed looks, i.e. multiple adjacent pixels of equal nominal elevation, that will be combined to reduce statistical variations [1]. For each $u$-th frequency, and each $i$-th look, $i = 1, ..., N_u$, the complex amplitudes $P_{ul,i}^{(0)}$, $k = 1, ..., K_u$, of the pixels corresponding to the same area on ground, observed in the $K_u$ SAR images, are arranged in the vector $P_u^{(0)} = [P_{ul,1}^{(0)}, P_{ul,2}^{(0)}, ... , P_{ul,K_u}^{(0)}]^T$, where $\tilde{\cdot}$ denotes complex conjugate.

We now generalize the conventional InSAR statistical model proposed in [1] for SAR processed echoes from surfaces with spatially white reflectivity. We can characterize each vector $P_u^{(0)}$ as a complex Gaussian random vector with zero mean value and covariance matrix

$$C_u = E[P_u^{(0)} P_u^{(0)*}] = (SNR_u + 1) R_u$$

with $H$ denotes hermitian, and $SNR_u$ is the pixel signal to noise power ratio for the $u$-th frequency. The known thermal noise power has been assumed unitary without loss of generality, and $R_u$ is the normalized covariance matrix. The extended source of the echoes is incoherently distributed and its signal spatial density is related, through SAR processing, to the squared Sinc SAR impulse response that acts as an illumination function. This is sketched in Fig.1. The signal density also depends on the illumination geometry (SAR look angle, local terrain slope), and on possible volumetric scattering distribution (rough terrain, vegetation). Extending the Fourier integral-based derivation in [1], it is shown that for locally flat terrain $\hat{\rho}_{um}$, the off-diagonal element $(l,m)$ of $R_u$, is

$$\hat{\rho}_{um} = \exp\left[j \Phi_{ul}^{(0)}(B_{ul} - B_{um})/(f_u B_i)\right] f_{ul,um}(1/f_{ul,ul})SNR_u/(SNR_u + 1)$$

(1)

for $f_{ul,ul}(B_{ul} - B_{um})/(f_{ul,ul}) \leq 1$, 0 otherwise, where $r_u$, $r_l$ are the SAR slant-range resolutions for frequency $f_u$ and $f_l$, respectively. Its modulus is a triangular function of the displacement between elements $ul,um$ of the $u$-th frequency array, and is determined by the decorrelation produced by the different speckle patterns (multiplicative noise) observed at the two phase centers (baseline decorrelation effect) and by the signal to thermal noise ratio [1]. Other typical InSAR decorrelation causes such as pixel misregistration, temporal decorrelation, volumetric scattering [1] can be promptly included in (1). Being the source incoherently distributed, it is easily shown that the array data Gaussian vectors for different carrier frequencies are independent each other, since $E[P_u^{(0)} P_{w}^{(0)*}] = 0$ for $u \neq w$, provided that the SAR bands centred at the frequencies $f_u,f_w$ are separated enough. Because of the spectral shift effect peculiar of interferometric physics, that couples frequency and spatial shifts [7], the necessary band spacing for independence is related to the useful parameter to be estimated. In this paper we deal with the clairvoyant estimation, assuming that the speckle covariance matrices $R_u$ and the signal to noise ratios $SNR_u, u=1, ..., M$, are known. This is reasonable in some land surveillance applications. In other applications these signal model parameters can be estimated on-line from the acquired data (see [6]).

A compact expression for the CRLB on the MBMF clairvoyant phase estimation error variance is derived. From the pdf (2), exploiting the decomposition of the covariance matrices $C_u$, the log-likelihood function of the MBMF data is given by

$$L = -\sum_{u=1}^{M}(SNR_u + 1)^{-1} \sum_{i=1}^{N_u} \Phi_{ul}^{(0)}(\Phi_{ul}^{(0)} R_u \Phi_{ul}^{(0)} + \sum_{u=1}^{M}N_u \ln(\pi(SNR_u + 1))^{-1} |R_u|^{-1})$$

(3)

Taking account of the expressions of the steering matrices $\Phi_{ul}$ and setting $B_u = diag\{0, B_{ul}/B_{ul,k}, B_{ul,k}/B_{ul}\}, u=1, ..., M$, application of a result for general Gaussian CRLB [8] and some manipulations produce

$$CRLB_u = \left\{\sum_{u=1}^{M}2N_u(1-t_u)^2 \left[tr[R_u B_u R_u B_u] - tr[B_u^2]\right]\right\}$$

(4)

where $1 - t_u = (f_u B_i)/(f_i B_i), \text{viz.} \ t_u$ is the relative difference between the master baseline and the end-to-end baseline of the $u$-th frequency array, each measured in the corresponding wavelength unit, and $B_u$ codes the corresponding array structure. The CRLB (4) is the extension of the conventional single-baseline single-frequency InSAR CRLB in [1]. For the MB case, it coincides with the ML asymptotic variance expression in [9]. The bound (4) should be useful in practical applications and theoretical studies to compare the performance of a given MB (e.g. [2,3]), MF (e.g. [4]) or MBMF estimator to the ultimate achievable performance. Also, it can be exploited for feasibility studies and design optimization of multiparametric InSAR systems.
4 MBMF-INSAR ML ESTIMATOR

The ML estimator of the interferometric phase $\phi$ in MBMF-INSAR is also readily obtained as the maximizer of the log-likelihood function (3), where the constant rightmost summation can be dropped. It is the generalization of the conventional ML estimator for single frequency and two phase centers [1]. It produces the asymptotically optimum combination of the echoes received at the $\sum_{k_u} K_u$ phase centers, after SAR processing. The unambiguous phase range resulting from baseline and/or frequency diversity depends on the structure and end-to-end baseline of the arrays and on the relation among the different frequencies. It can be derived analyzing the identifiability of the problem. Since a number of interferometric airborne and spaceborne SAR systems with dual baseline and/or dual frequency capabilities are presently in operation (see, e.g., [4,10], and the InSAR Space Shuttle experiment), examples are focused on these cases.

For the single-baseline MF-INSAR with two carrier frequencies, we obtain from (3) the ML dual frequency (DF) estimator as the maximizer of the objective function

$$J(\phi) = \frac{1}{(SNR_1 + 1) \det[R_1]} \left[ m_{tu} \left\{ \rho_{t_u} \exp \left[ -j\phi(t-1) \sum_{n_1} \tilde{p}_{n_1}^{(0)} \tilde{p}_{t_2}^{(0)} \right] \right\} + \frac{1}{(SNR_2 + 1) \det[R_2]} \left[ m_{tu} \left\{ \rho_{t_u} \exp \left[ -j\phi(t-1) \sum_{n_2} \tilde{p}_{n_2}^{(0)} \tilde{p}_{t_2}^{(0)} \right] \right\} + \frac{1}{(SNR_3 + 1) \det[R_3]} \left[ m_{lu} \left\{ \rho_{l_u} \exp \left[ -j\phi(t-1) \sum_{n_3} \tilde{p}_{n_3}^{(0)} \tilde{p}_{l_2}^{(0)} \right] \right\} + \frac{1}{(SNR_4 + 1) \det[R_4]} \left[ m_{lu} \left\{ \rho_{l_u} \exp \left[ -j\phi(t-1) \sum_{n_4} \tilde{p}_{n_4}^{(0)} \tilde{p}_{l_2}^{(0)} \right] \right\} + \frac{1}{(SNR_5 + 1) \det[R_5]} \left[ m_{lu} \left\{ \rho_{l_u} \exp \left[ -j\phi(t-1) \sum_{n_5} \tilde{p}_{n_5}^{(0)} \tilde{p}_{l_2}^{(0)} \right] \right\} + \frac{1}{(SNR_6 + 1) \det[R_6]} \left[ m_{lu} \left\{ \rho_{l_u} \exp \left[ -j\phi(t-1) \sum_{n_6} \tilde{p}_{n_6}^{(0)} \tilde{p}_{l_2}^{(0)} \right] \right\} \right] \right\} \right\} \right]$$

where subscripts $H$ and $L$ denote the higher and the lower frequency, respectively, $m_{tu} = \rho_{t_u}$ is the element (1,2) of the cofactor matrix of $R_{t_u}$, and $m_{lu} = \rho_{l_u}$, $u=U,L,H$. Subscript of $t$ is dropped since there is only one additional phase. For $t=m/n$, with $m,n$ coprime integers, the unambiguous range for the phase estimate is $2\pi n$. The explicit expression (5) allows both the ML MF-estimator to be interpreted and implemented with efficient algorithms.

The expected symmetry is moreover apparent in the structure of the estimator. The explicit expression of the ML estimator for the single-frequency MB-INSAR with three phase centers (dual baseline, DB) is in [6,9]. For the dual baseline InSAR with two carrier frequencies, from (3) we get the ML dual baseline dual frequency (DBDF) estimator as the maximizer of

$$J(\phi) = \frac{1}{(SNR_1 + 1) \det[R_1]} \left[ m_{tu} \left\{ \rho_{t_u} \exp \left[ -j\phi(t-1) \sum_{n_1} \tilde{p}_{n_1}^{(0)} \tilde{p}_{t_2}^{(0)} \right] \right\} + \frac{1}{(SNR_2 + 1) \det[R_2]} \left[ m_{tu} \left\{ \rho_{t_u} \exp \left[ -j\phi(t-1) \sum_{n_2} \tilde{p}_{n_2}^{(0)} \tilde{p}_{t_2}^{(0)} \right] \right\} + m_{lu} \left\{ \rho_{l_u} \exp \left[ -j\phi(t-1) \sum_{n_3} \tilde{p}_{n_3}^{(0)} \tilde{p}_{l_2}^{(0)} \right] \right\} + \frac{1}{(SNR_3 + 1) \det[R_3]} \left[ m_{lu} \left\{ \rho_{l_u} \exp \left[ -j\phi(t-1) \sum_{n_4} \tilde{p}_{n_4}^{(0)} \tilde{p}_{l_2}^{(0)} \right] \right\} \right] \right\} \right] \right]$$

where $m_{tu} = \rho_{t_u} - \rho_{t_u} \rho_{t_u}$ is the element (v,w) of the cofactor matrix of $R_{t_u}$ with $v,w=1, 2, 3$ and $z \neq v,w, u=L,H$. Here det $[R_{t_u}] = 1 - \rho_{t_u}^2 - \rho_{t_u}^2 - \rho_{t_u}^2 + 2 \rho_{t_u} \rho_{t_u} \rho_{t_u}$, and $p_u = (B_u - B_{u-1})/B_u$, viz. $p_u$ is the relative difference between the end-to-end baseline and the baseline for the intermediate phase center of the $u$-th frequency array.

5 MBMF-INSAR PERFORMANCE ANALYSIS

Case studies of DB-, DF-, and joint DBDF-INSAR are presented. For the DB-INSAR, $p = 1/2$, for DF $t = 1/2$, $N_L = N_H = N$. $r_1 = r_2$, $SNR_1 = SNR_2 = SNR$, and for DBDF $p_L = p_H = 1/2$, the other parameters as for DF.

Fig.2 reports the standard deviation of the phase error for the three ML multiparametric InSAR estimators as a function of the number of looks $N$, obtained by Monte Carlo simulation according to model (1-2) (100,000 trials), together with the performance of conventional interferometry with baseline equal to the master baseline. The corresponding CRLBs are also shown (dotted lines). The value of $B = B_1$, the master baseline normalized to the critical baseline, is $B = 0.4$, and $SNR = 12dB$. All the ML estimators result to be unbiased modulo the unambiguous range. As expected, they are asymptotically efficient. More interestingly, they almost achieve the CRLB for the small number of looks typically employed in practical applications ($N = 8 \pm 16$). Note that the ultimate performance given by the CRLB for $B = 0.4$ improves employing in the order conventional, DB, DF, DBDF interferometry.

Fig.3 shows the simulated performance as a function of $B$, for $N = 8$, $SNR = 12dB$. The CRLB for DBDF-INSAR is also reported (dotted line), showing that the bound can be reliably used for ML performance prediction for varying $B$. Performance of all the estimators degrades with increasing $B$, because of the increasing signal decorrelation. However, good accuracy is obtained by ML multiparametric interferometry over a wide range of values of $B$, which indicates higher system flexibility than conventional InSAR. In fact, $B$ varies with the terrain slope through the critical baseline [1], and a larger range of admissible $B$ corresponds to a wider range of operating terrain slopes. Also, a large $B$ obtained through a large master baseline can produce higher height accuracy, because of the higher interferometer sensitivity [1]. Note that conventional interferometry cannot usefully operate beyond $B = 1$ (complete signal decorrelation at the single available baseline). This limit is moved to $B = 2$ for DB and DF, and to $B = 4$ for DBDF. This is obtained jointly with a 2- and 4-fold magnification of the unambiguous phase range, respectively, and consequently of the equivocuation height [3,4,6] for a given master baseline. Note that rankings of phase accuracy of DB and DF for $B > 0.5$ are exchanged compared to lower $B$ (see the inset). DF is better than DB for low $B$, since it has independent information from the two baselines with different frequencies. Conversely, it is worse for high $B$, since it has then less information (for $B > 1$ just the baseline operating at $f_1$ has correlated phase centers) than DB (which has two couples of correlated phase centers in the array). DBDF always exhibits the best performance.

Fig.4 reports the simulated performance for $N = 8$, $SNR = 6dB$. The low $SNR$ affects performance of DF-INSAR by a larger extent than DB and DBDF, this results in a low accuracy gain of DF over conventional InSAR.
This is due to the threshold effect. DB and DBDF exhibit a more robust solution of the phase ambiguity (relying completely [6] or partially on correlated multibaseline data) and hence much better accuracy.

6 Conclusions

This paper applies parameter estimation theory to a general MBMF-InSAR framework. Benchmarks for MBMF-InSAR estimators and performance prediction tools, and asymptotically optimal estimators are produced. Case studies of dual baseline, dual frequency, and joint dual baseline dual frequency InSAR are presented. Work is planned to extend the MBMF theory to the case of unknown signal model parameters, possibly exploiting covariance matching estimation techniques, and on the validation of the theory with real data.

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References


