LOCALIZATION PROPERTIES OF AN EEG SENSOR SYSTEM: LOWER BOUNDS AND OPTIMALITY

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ABSTRACT

Most studies concerning the EEG inverse problem focus on the properties of one or another specific inverse solution. Few studies approach the problem of the bounds imposed by the system itself, indifferently of the inversion method used.

We are interested in the localization properties of an EEG sensor system using a generic reconstruction procedure in the context of a Brain Computer Interface project. We investigate various perturbations: additive noise, electrode misplacement errors and external sources contributions. The estimation of errors uses the notions of normalized measurements and sensitivity functions in a deterministic framework, but our results closely link to the stochastic Cramér-Rao minimum bound.

We propose to modify the system, and more specifically the electrodes configuration, such as to minimize the forecasted errors, thus enhancing the robustness of the system. The configurations obtained through a hybrid Simulated Annealing – Gradient Descent approach show significant improvement when compared to normal setups.

1. INTRODUCTION

In the framework of a Brain Computer Interface (BCI) project, researchers are interested in properly distinguishing between different mental activities. One way of doing it is through identification of the main active areas of the brain which are activated according to the current mental activities. Precise localization is thus very important.

The source reconstruction is usually performed through linear inversion after uniform discretization of the solutions space ([1], [2]). Using the resolution kernel of such inversion operators one can establish, for a fixed electrode system, the spatial resolution and the accuracy in terms of amplitude ([3], [4]). However, this has to be applied for each inversion method and only supports linear constraints. We are interested in an analysis that should be independent of the inversion method chosen, and that should identify the limits of the reconstruction based solely on the physical properties of the system itself (physical equations and physical layout).

In sections 2 and 3 we provide a deterministic framework for analysing nuisances caused by any types of perturbations. This framework is rooted in the system through its sensitivity functions and considers an L2 metric for the quantification of errors. We chose the L2 norm instead of another one mainly for computational convenience, and because it is used in the vast majority of inversion procedures (e.g. linear inversion). Sections 4 and 5 include stochastic models of perturbations and show the direct link between our results and the Cramér-Rao bound when estimating the minimum statistical variances of inversion. We use the results from these sections for deriving optimality criteria (section 6) that should be used when building an electrode system. Simulation results based on the optimization method from section 7 are presented in section 8, comparing such optimal designs with normal configurations.

2. MEASUREMENT SPACE AND RESEMBLANCE FUNCTION

We construct our analysis on an obvious but necessary statement: the power to discriminate between two sources is not related to the distance between them in the source space, but solely to the distance in the measurements space after mapping. We thus need to choose a convenient measure of the distance in the measurements space.

In the EEG setup, the amplitudes of the sources (brain activity) are unknown. Therefore the localization of a source is completely defined by its normalized measurement, which we call the *form-factor*. Considering the classical relationship between the measurements $y(r_{el})$ and sources $x(r_x)$:

$$y(r_{el}) = \Sigma[h(r_{el},r_x)\cdot x(r_x)],$$

we define the *form-factor* associated to a source as:

$$F(r_x) = \frac{h(r_{el},r_x)}{\|h(r_{el},r_x)\|} \quad (1)$$

Using the L2 metric, the distance between two sources in the measurement space is (the brackets stand for scalar product):

$$dist_{1\Leftrightarrow 2} = \|F_1 - F_2\|^2 = 2\left(1 - \langle F_1, F_2 \rangle\right)$$

Thus different source configurations are indistinguishable if the scalar product of their form-factors (1) equals 1 (they are identical). In consequence we define the *resemblance function* as the scalar product between the form-factor indicated by the measurements F_m and the form-factor associated with a hypothetical source F_{hS}:

$$\rho(F_m, F_{hS}) = \langle F_m, F_{hS} \rangle \quad (2)$$

Considering one source, when searching in its vicinity, if the

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measurements are unperturbed, the maximum of the resemblance function will be reached when the hypothetical source is the original source and it will be equal to 1. But noise will modify the form-factor such that the resemblance function will no longer have its maximum on the real source position, but in a neighbouring position. This shift of the maximum of the resemblance function is denoted in this paper as the *resemblance bias*, and it is naturally adopted as the basis for the measure of the localization properties, in terms of bias and variance, of a perturbed generic sensor system.

3. RESEMBLANCE BIAS

This section is focused on finding the maximum of the resemblance bias for a system perturbed by errors caused through sensor misplacement, additive noise and external sources. We are interested in properly localizing a source in a specific region of the brain, thus all other sources are naturally nuisance sources. We build our model sequentially, integrating the different perturbations step by step.

3.1 Sensor misplacement

Suppose the sensors are misplaced by Δr_{el}, a vector of displacements for each sensor. Then, in the vicinity of the original source, the resemblance function (2) will be :

$$\rho(\Delta r_x, \Delta r_{el}) = \left\langle \underbrace{F(r_x, r_{el}+\Delta r_{el})}_{\text{measurement}}, \underbrace{F(r_x+\Delta r_x, r_{el})}_{\text{hypothetical source}} \right\rangle$$

And the measurement bias is :

$$\Delta r_{xbias} = \arg\max \rho(\Delta r_x, \Delta r_{el}) \qquad (3)$$

In order to solve this maximization, let's stick to the case of small displacements, where the following linear approximations hold :

$$\begin{aligned} F_m(r_x, r_{el}+\Delta r_{el}) &= F(r_x, r_{el}) + \frac{\partial F}{\partial r_{el}}\Delta r_{el} \overset{notation}{=} F + F_{el}\Delta r_{el} \\ F_{hS}(r_x+\Delta r_x, r_{el}) &= F(r_x, r_{el}) + \frac{\partial F}{\partial r_x}\Delta r_x \overset{notation}{=} F + F_x\Delta r_x \end{aligned} \qquad (4)$$

Equation (4) introduces the *sensitivity functions* F_x (source position) and F_{el} (sensor position) which ultimately define the system's robustness. Considering the geometry of the EEG problem, F_x is of size N by 3 (3D solution space), and F_{el} is of size N by 2N, as the electrodes are placed on a 2D surface, N being the number of electrodes.

Using and the normalization property of the form-factors:

$$1 = \left\| F(r_x, r_{el}+\Delta r_{el}) \right\|^2 = \|F\|^2 + 2\langle F, F_{el}\Delta r_{el}\rangle + \|F_{el}\Delta r_{el}\|^2$$

Thus:

$$\langle F, F_{el}\Delta r_{el}\rangle = -\frac{1}{2}\|F_{el}\Delta r_{el}\|^2 \qquad (5)$$

Similarly

$$\langle F, F_x\Delta r_x\rangle = -\frac{1}{2}\|F_x\Delta r_x\|^2 \qquad (6)$$

Substituting (4), (5) and (6) in (3):

$$\Delta r_{xbias} = \arg\max\left(\langle F_x\Delta r_x, F_{el}\Delta r_{el}\rangle - \frac{1}{2}\|F_x\Delta r_x\|^2 - \frac{1}{2}\|F_{el}\Delta r_{el}\|^2 \right)$$

Imposing the maximization condition through the derivative :

$$\frac{\partial}{\partial \Delta r_x}\left(\langle F_x\Delta r_x, F_{el}\Delta r_{el}\rangle - \frac{1}{2}\|F_x\Delta r_x\|^2 - \frac{1}{2}\|F_{el}\Delta r_{el}\|^2 \right) = 0 \Leftrightarrow$$

$$\Leftrightarrow -F_x^t F_x \Delta r_x + F_x^t F_{el}\Delta r_{el} = 0$$

This gives directly the measurement bias as :

$$\Delta r_{xbias} = \left(F_x^t F_x\right)^{-1} F_x^t F_{el}\Delta r_{el} \qquad (7)$$

Let's compute the distance between the form-factor associated with a hypothetical source situated at the position defined by the resemblance bias and the measurement form-factor. Using the relationships (4) to (7), one obtains:

$$\|F_m - F_{hS}\|^2 = \frac{1}{2}(F_{el}\Delta r_{el})^t \left(F_x\left(F_x^t F_x\right)^{-1} F_x^t - I\right)(F_{el}\Delta r_{el})$$

This distance is null if a linear relationship exists between the displacements influence and the source position sensitivity:

$$F_{el}\Delta r_{el} = F_x A,$$

A needing to be a 3 by 1 vector. Moreover, replacing in (7):

$$\Delta r_{xbias} = A \Rightarrow F_{el}\Delta r_{el} = F_x \Delta r_{xbias}$$

So if the disturbance is in the subspace spanned by F_x, the perturbed system will be completely equivalent to its non-perturbed version, but with a shifted source.

3.2 Additive noise

The same kind of reasoning can be applied to additive noise, denoted as n. The form-factor will now be expressed as :

$$F_m(r_x, r_{el}+\Delta r_{el}, n) = F + F_{el}\Delta r_{el} + F_n n$$

The total disturbance is now the sum of disturbances produced by the positioning sensitivity and the noise sensitivity F_n. Similarly to the previous computations :

$$\Delta r_{xbias} = \left(F_x^t F_x\right)^{-1} F_x^t \left(F_{el}\Delta r_{el} + F_n n\right) \qquad (8)$$

3.3 External sources

The contribution of additional sources to the measurement can be written as:

$$y_{ext} = \sum h_{ext} x_{ext}$$

Similarly to the previous sections, we can prove that the resemblance bias caused by this external source is:

$$\Delta r_{xbias} = \left(F_x^t F_x\right)^{-1} F_x^t \sum F_{ext} y_{ext\_x};\; y_{ext\_x} = \frac{\|h_{ext}\| x_{ext}}{\|h\| x} \qquad (9)$$

It can be noticed that this bias is proportional to the power of the external signal in respect to the signal produced by the source, but at the same time it strongly depends on the correlation between the external form-factor and the source position sensitivity function. If the signal produced by the external sources is orthogonal to the sensitivity function F_x, no deviation of the maximum is present.

Considering also additive noise and electrode misplacement:

$$\Delta r_{xbias} = \left(F_x^t F_x\right)^{-1} F_x^t \left(F_{el}\Delta r_{el} + F_n n + \sum F_{ext} y_{ext\_xp}\right) \qquad (10)$$

This time the strength of the signal of the external source is divided by the strength of the original perturbed signal.

4. LOCALIZATION VARIANCE

All the previous computations implicitly supposed a deterministic form of disturbances. Generally these are unknown parameters, which can be considered to be generated by a random process. In this case the resemblance bias will also be a random process, and we need to evaluate its parameters. The mean bias can be straightforward obtained from (10):

$$\overline{\Delta r_{xbias}} = \left(F_x^{\,t} F_x\right)^{-1} F_x^{\,t} \left(F_{el}\overline{\Delta r_{el}} + F_n \overline{n} + \sum F_{ext}\overline{y_{ext\_x}}\right) \quad (11)$$

When the perturbing processes are zero-mean, which naturally should be the case, the measurement bias is, on average, null. The same thing can not be asserted about its variance.

If the covariance matrix of the perturbations, R_{pp}, is known, then:

$$R_{\Delta r_{xbias}\Delta r_{xbias}} = \left(F_x^{\,t} F_x\right)^{-1} F_x^{\,t} \left(\sum F_p R_p F_p^{\,t}\right) F_x \left(F_x^{\,t} F_x\right)^{-1} \quad (12)$$

Equations (11) and (12) provide us with the localization power of a sensor system. Indeed, for a certain perturbation level, the system will be unable (on the average) to distinguish between different source positions that are inside a zone defined by the variance of the resemblance bias.

Next it will be shown how the formula (12) links to the Cramér-Rao bound.

5. SPATIAL RESOLUTION AND THE CRAMÉR-RAO BOUND

Finding the location of a source is equivalent to finding its form-factor. From this point of view, the problem can be reconsidered as the estimation of the position from the noisy measurement of its form-factor degraded either by the positioning error, additive noise or external sources.

Using the linear approximation, in case of positioning error the degradation process can be put in equation as (4):

$$F_m = F + F_{el}\Delta r_{el}$$

If the positioning error is a Gaussian process $N\left(0, R_{\Delta r_{el}\Delta r_{el}}\right)$, the perturbation $F_{el}\Delta r_{el}$ is also a Gaussian process and the minimum bound for the estimation is (using the general form of the Cramér-Rao bound [5]) :

$$R_{r_x r_x} = \left(\frac{\partial r_x}{\partial F}\right)\left(E\left(-\frac{\partial^2 \ln p(F_m \mid F)}{\partial x^2}\right)\right)^{-1}\left(\frac{\partial r_x}{\partial F}\right)^t$$
$$R_{r_x r_x} = \left(\frac{\partial r_x}{\partial F}\right) F_{el} R_{\Delta r_{el}\Delta r_{el}} F_{el}^t \left(\frac{\partial r_x}{\partial F}\right)^t \quad (13)$$

In the least square sense, and using (4):

$$\partial F = F_x \partial r_x \Rightarrow \partial r_x = \left(F_x^{\,t} F_x\right)^{-1} F_x^{\,t} \partial F$$

Replacing in (13) we find the same result as (12). Thus our deterministic resemblance bias model leads to the statistical Cramér-Rao bound for the case of Gaussian perturbations.

6. SYSTEM OPTIMALITY CRITERIA

We show in this section how the previous results can be used for optimal system design. First, based on (12) we define the resolution cell Δ of the system:

$$\Delta = diag\left(R_{\Delta r_{xbias}\Delta r_{xbias}}\right)$$

Considering that the regions of interest for source localization are known (which should be the case for a BCI oriented system), the sensors can be optimally placed such as to achieve maximum resolution power in that area, or equivalently minimize the resolution cell Δ under the constraints of the sensor system (e.g. number of sensors):

$$r_{el} = \arg\min\left(\Delta\right) \quad (14)$$

The minimization of this functional requires knowledge of perturbations statistics (noise, nuisance sources etc.). This minimization can be seen as a search for the system that possesses the minimum Cramér-Rao lower bound among all.

7. OPTIMIZATION

Placing N electrodes on the head surface is a continuous combinatorial-like problem. We want to minimize the diagonal terms of the covariance matrix (12).We define our minimization functional based on (14):

$$Err\Phi = Tr\left(R_{\Delta r_{xbias}\Delta r_{xbias}}\right) \quad (15)$$

Supposing the perturbations to be independent, we introduce the perturbation vector which contains the errors induced by each type of nuisance:

$$errf = \left[\Delta r_{xbias\_ext} \quad \Delta r_{xbias\_el} \quad \Delta r_{xbias\_n}\right] \quad (16)$$

Notice that the squared norm of this vector is our minimizing functional in (15). However, we need to add a supplementary term to the vector in (16). Indeed, from equations (1) and (4):

$$F_x = \left(I - FF^t\right)\frac{h_x}{\|h\|};\; h_x = \frac{\partial h}{\partial r_x}$$

Considering equation (9), the influence of external sources can be rewritten as:

$$\Delta r_{xbias\_ext} = \left(F_x^{\,t} F_x\right)^{-1} \frac{h_x^t}{\|h\|}\left(F_{ext} - F\left\langle F, F_{ext}\right\rangle\right) y_{ext\_x}$$

One obvious minima will occur if $F_{ext} = F$. This is exactly what we want to avoid, so we need to force our minimization towards the second minima, which implies $F_{ext} \perp h_x$ and $F_{ext} \perp F$. This way, the signal from the nuisance sources will be orthogonal to the sensitivity function and to the relevant signal, thus easy to discard. We reinforce the later orthogonality through the supplementary term:

$$\Delta_{orth} = \frac{1}{\|F_x\|}\left\langle F, F_{ext}\right\rangle y_{ext\_x}$$

Finally, the sensor position are given by:

$$\vec{r}_y = \arg\min\left(Err\Phi\right) = \arg\min\left(\|errf\|^2\right)$$
$$errf = \left[\Delta r_{xbias\_ext} \quad \Delta r_{orth} \quad \Delta r_{xbias\_el} \quad \Delta r_{xbias\_n}\right] \quad (17)$$

Given the nature of our problem, we used a hybrid Simulated Annealing ([6]) with Gradient Descent algorithm for minimization. The algorithm is briefly described below:

- Move one electrode until the configuration is accepted according to SA probability law $p = \exp\left(-\frac{\Delta Err\Phi}{k \cdot Err\Phi_{\min}}\right)$

- Apply gradient descent
- If $Err\Phi_{new} < Err\Phi_{min}$ replace $Err\Phi_{min} = Err\Phi_{new}$ and save the configuration
- Loop.

8. RESULTS AND DISCUSSION

Using analytical expansions into Legendre polynomials for the computation of the scalp potentials on a unit radius spherical 4-layer isotropic head model ([7], [8]) we simulated a relevant source producing the upper side of the potential shown in Figure 1. We used as perturbations additive Gaussian noise with a PSNR of 10 dB, electrode misplacements of standard deviation 5° (spherical coordinates), and an external source of equivalent power producing the lower side of the potential in Figure 1.We considered 4 electrodes and optimized their placement for different combinations of perturbations, according to (17). Figure 1 shows an optimal sensor placement for robustness against the external source (left) and the optimal sensor placement when considering jointly all perturbations (right).

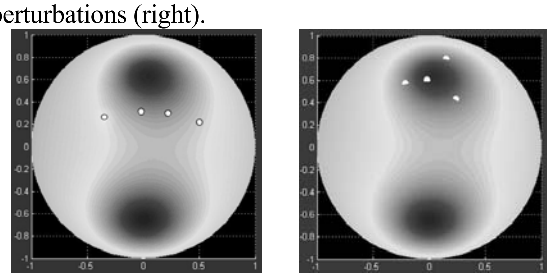

Figure 1. Optimal sensor configurations

The configuration optimized for external sources is dictated by the orthogonality conditions imposed, while the necessity of robustness against noise pushes the electrodes in the zone of maximum source intensity.

In order to test the obtained configurations, we have computed the signal produced by the relevant source on the specific sensor configurations, perturbed it and then used a correlation detector to recover the position of the source. The localization errors are presented in Table 1. Each column represents a type of perturbation and each row a sensor configuration, optimized for the specified perturbations. The row named HUG corresponds to a standard 123 electrode cap from the Hopital Universitaire de Geneve, courtesy of R.Grave de Peralta and S. Gonzalez. The last row shows average results for a set of 1000 random configurations, tests performed with reduced perturbations (standard deviation divided by 100).

	ext (equiv)	el (5°)	noise (10dB)	ext+el+noise
ext	0.011	0.138	0.160	0.161
ext+el	0.065	0.105	0.127	0.134
ext+noise	0.065	0.105	0.109	0.127
el+noise	0.084	0.104	0.111	0.129
ext+el+noise	0.072	0.106	0.110	0.126
HUG	0.094	0.068	0.057	0.098
rand, std/100	0.067	0.031	0.096	0.100

Table 1. Localization errors of selected configurations. *el* is short for electrode misplacement, *ext* for external source

The optimized configurations have performed much better than random configurations, yielding the same magnitude of errors for much higher perturbations (40 dB difference). Moreover their performance is equivalent to that of the 123 electrode system. One may also notice how the first configuration (Figure 1, left) outperforms all other configurations when tested for robustness against external sources (including the HUG 123 electrodes configuration), and that optimization for robustness against noise seems to imply robustness against electrode misplacement also. This is quite natural, as the random movements of electrodes are equivalent to random variations of the signal's amplitude.

9. CONCLUSION

We have studied in this paper the localization properties inherent to an EEG sensor system in the context of a Brain Computer Interface. We have used our results to derive principles for the optimal design of such a system. We have proved that systems based on such principles can perform as well as normal systems with a much higher number of electrodes. More importantly, we have proved that it is possible to impose near-orthogonality of signals from different sources through optimal sensor placement, allowing for straightforward source identification.

We intend to pursue this approach and design a sensor system which would be composed out of optimal subsystems, each dedicated to one specific brain region, thus allowing for robust source identification and reconstruction.

10. REFERENCES

[1] R. Grave de Peralta-Menendez, S. L. Gonzalez-Andino, G. Lantz, C. M. Michel and T. Landis, " Non-invasive localization of electromagnetic epileptic activity I. Method Descriptions and Simulations", Brain Topogr., Vol.14, pp. 131-137, 2001.

[2] R .D. Pascual-Marqui, C. M. Michel, and D. Lehmann, "Low resolution electromagnetic tomography: a new method to localize electrical activity in the brain", Int. J. Psychophysiol. 18, pp. 49–65, 1994.

[3] A. K. Liu, A. M. Dale, and J. W. Belliveau, "Monte Carlo Simulation Studies of EEG and MEG Localization Accuracy", Human Brain Mapping 16, pp. 47–62, 2002.

[4] R. Grave de Peralta-Menendez, S. L. Gonzalez-Andino, "A Critical Analysis of Linear Inverse Solutions to the Neuroelectromagnetic Inverse Problem", IEEE Transactions On Biomedical Engineering, Vol. 45., No. 4, pp. 440-448, April 1998.

[5] W. W. Piegorsch, "Notes On Minimum Variance Point Estimation For A Course In The Theory Of Statistical Inference", Statistics Technical Report No. 195, University of South Carolina, August 2002.

[6] S. Rajasekaran, "Randomization in Discrete Optimization: Annealing Algorithms", in *Encyclopedia of Optimization*, edited by P.M. Pardalos, Oxford University Press, 2001

[7] Z. Zhang, "A fast method to compute surface potentials generated by dipoles within multilayer anisotropic spheres", Phys. Med. Biol 40, pp. 335-349, 1995.

[8] Z. Shuang-Ren, J. Grotendorst and H. Halling, "Calculation of the Potential Distribution for a Three-Layer Spherical Volume Conductor", The Maple Technical Newsletter, Volume 2, Number 1, pp. 59-66, April 1995.