New Challenges in Non-Linear Signal and Image Processing

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ABSTRACT

This paper will trace the origins of non-linear signal and image processing techniques from their separate sources based in set theory and logic to the more unified field that we see today. It will outline the technical difficulties resulting from the abandonment of well-known linear design methods and the subsequent advantages in terms of improved results. Some new design methods such as statistical estimation will be summarised. Finally, new challenges, such as the extension to multivariate data and applications in security, control and bioscience will be introduced. These will topics will be covered in more detail by subsequent speakers.

1. INTRODUCTION

In recent years the area of non-linear signal and image processing (NSIP) has emerged as a distinct research field in its own right. It has formed from a fusion of techniques which, whilst coming from very different sources, share many characteristics. In essence, these techniques have appeared because in many applications the classical linear techniques have not worked.

Whilst linear techniques have many advantages, they mainly arise in a mathematical framework based on orthogonal spaces. For example, in Fourier analysis the modelling of both signals and systems by a summation of complex sinusoids leads to an elegant analysis with all cross terms conveniently vanishing from the calculation. Filters can in effect be designed one frequency at a time. Such powerful superposition properties result in neat closed-form solutions of optimal filters. In many branches of engineering, from circuit analysis to audio processing, these techniques have worked well. In transient and discrete systems, Laplace and z transforms have proved to be similarly invaluable. However, solutions are limited to those based on superposition of sinusoids. Relaxation of this constraint is the basis of non-linear signal and image processing. It can lead to better solutions to problems at the price of increased mathematical complexity, particularly in operator design.

2. ORIGINS OF NON LINEAR SIGNAL AND IMAGE PROCESSING

2.1 Mathematical Morphology

Mathematical Morphology was conceived by Matheron and Serra at the École des Mines de Paris in Fontainebleau in 1964 and has continued to grow ever since. Their motivation was to investigate the relationship between the geometry of porous materials and their permeability. Their data was in the form of 2D images rather than 1D signals and their approach was structural. They introduced the concept of structuring elements. These are sub-images which interact with the original image to modify and extract information. The work was initially developed with 2D binary images and a whole set of notions emerged, such as the hit-or-miss transform, openings, closings, top-hat, skeletons, watersheds and Boolean models [1].

Since its early days the theory has been extended, first to greyscale signals and images and then to complete lattices [2]. The underlying theme was motivated by a need to understand shape and structure which could not be solved by methods based in linear concepts. In particular, tools and concepts such as granulometries [3] allow images and signals to be analysed across a range of scales. Granulometries are useful in texture analysis and in the decomposition of signals and shapes into features which are ordered on the basis of size.

One way to view morphology is to say that it focuses on a pyramid of four operator classes, each containing the one below it: (1) translation-invariant operators [$\Psi = \Psi_{x,y}$]; (2) increasing [$f \leq g \Rightarrow \Psi(f) \leq \Psi(g)$], translation-invariant operators; (3) idempotent [$\Psi\Psi = \Psi$], increasing, translation-invariant operators]; (4) anti-extensive [$\Psi(f) \leq f$] (extensive [$\Psi(f) \geq f$] , idempotent, increasing, translation-invariant operators]. Operators satisfying all four conditions are called τ-opening (τ-closings). If one is interested in a purely lattice-theoretic approach, translation invariance can be omitted, and operators satisfying the three remaining conditions are called openings (closings). For instance, for a binary image containing discrete grains of different sizes, the process of removing all grains smaller than a given number of pixels is a τ-opening.

Various representation theorems apply to different classes of operators. Focusing only on binary images, we have the following representations: (a) every translation-invariant operator can be represented as a union of hit-or-miss transforms [4]; (b) every increasing, translation-invariant operator can be represented as union of erosions [5]; and every τ-opening can be represented as a union of elementary openings by structuring elements [5].

Whilst morphology was developed in terms of set theory, all of the concepts have equivalent statements in terms of Boolean logic. For example, an increasing, translation-invariant set operator represented by a union of erosions corresponds to a positive Boolean function represented as a sum of products void of complements. This equivalent interpretation has the advantage of bringing morphology quickly to other communities such as electronics engineers, and it also clarifies how the operations may be implemented in digital hardware.

2.2 Stack filters

Stack filters were introduced by Wendt and Coyle in the mid 1980s [6]. The key to their analysis is based on the concept of threshold decomposition, which allows grey scale operations to be broken down into a series of binary operations. The resultant simplification means that the analysis and design of such filters may be carried out by studying only binary signals. These are then reconstructed (or 'stacked') to give the final grey scale output signal. It is a small step to go from these binary signals to implementations in digital electronic hardware. The design of such filters reduces in this case to the selection of appropriate Boolean functions for processing the binary signals.

Only a limited set of binary functions preserve the stacking property and these are the positive Boolean functions. As noted above, these are functions whose logical representation may be expressed without complementation of the variables. Consequently, stack filters form a special class of morphological filters.

With a stack filter, the task is to estimate the value of a signal Y from an input window of sample values $\mathbf{x} = \{X_0, X_1, \ldots, X_{n-1}\}$. The samples within $\mathbf{x}$ may be thresholded at every level t to produce a set of n binary samples, $\mathbf{x}^t$ which are then processed with a Boolean function ψ' to produce a stack of filtered binary signals, $Y^t = \psi'(\mathbf{x}^t)$. When this stack of binary signals is summed it yields the greyscale output signal. This is identical to the one which would be produced by filtering the original greyscale signal, $\mathbf{x}$ with the equivalent greyscale filter $\psi(\mathbf{x})$. The process is summarised in the equation

$$Y = \sum_t Y^t = \sum_t \psi^t([\mathbf{x}]^t) = \psi(\mathbf{x}) \qquad (1)$$

where $[\ ]^t$ is the threshold operator, $[]^t = 1$ if $[] \geq t$ and $[]^t = 0$ if $[] < t$, and $[\mathbf{x}]^t = \{X^t_0, X^t_1, \ldots, X^t_{n-1}\}$.

2.3 Statistical Estimation

Whereas the origin of both mathematical morphology and stack filters is fairly recent, statistical estimation techniques have been in existence for over a century. In particular, calculations based on the rank ordering of samples have been used to estimate signals in noisy environments. The estimation of the signal Y also takes the form of a weighted sum of the rank ordered values of the observations in the input window $\mathbf{x} = \{X_0, X_1, \ldots, X_{n-1}\}$:

$$Y(k) = \sum_i a_i r(i) \qquad (2)$$

where $r(i)$ is the i th largest value in the set $\mathbf{x} = \{X_0, X_1, \ldots, X_{n-1}\}$.

A subset of these filters is composed of the **rank-order filters** in which $a_i = 1$ for $i = p$ and $a_i = 0$ for all $i \neq p$. These are filters for which only a single input sample, p, is passed to the output. They compose a special class of stack filters. The most common rank-order filter is the median filter for which $p = n/2$, i.e. the centre value when n is odd. Other examples of rank-order filters are the minimum, $p = 0$, and the maximum, $p = n - 1$. A further variation is the **weighted rank-order filter** in which the input samples are entered into the set **x** a variable number of times depending on their position within the input window.

3. CURRENT APPLICATIONS OF NSIP

A vast array of processing methods has been developed for applications such as restoration, target detection, segmentation, and texture analysis. These applications have driven the development of a large number of processing tools and methods. Amongst these tools are hit-or- miss transforms, stack and order-statistic filters, watershed algorithms, skeletonization, granulometries and aperture filters.

3.1 NSIP Design Techniques

The most challenging aspects of applying morphological algorithms to real world problems is in designing optimum operators for given tasks. This usually involves statistical estimation techniques which require knowledge of both the prior probabilities of the inputs and the conditional output probabilities. Usually these are estimated from a large number of observations. A delicate balance exists between the complexity of the operators to be designed and the associated training set. This balance is at the intersection between non-linear operator design and the theory of pattern recognition [7]. A summary of optimization for nonlinear filters is beyond this paper but may be found in [8]. Other interesting design techniques include those based on genetic algorithms [9].

At the heart of filter design is the issue of constraints. Non linear operators may have their functionality restricted by a number of constraints as part of the design process and this leads to two consequences, one bad and one good. In the first case the restriction usually results in a filtering operation which is suboptimal as compared to the unconstrained operator. Hence the accuracy of the solution is reduced. The error introduced is known as the *constraint error*. However the constraint also results in the reduction in the size of the overall solution space. This leads to faster convergence using a smaller sized training set.

In cases where the amount of training data is limited, it may be insufficient for the design of an unconstrained operator and an error will result from undertraining. This type of error, known as *precision error*, can be very severe compared to constraint error. It is frequently better to introduce a constraint and accept a small increase in constraint error rather than a large increase in precision error. The ideal constraint is one which reduces the solution space to provide precise training whilst retaining sufficient flexibility to provide an accurate solution.

There are many areas in which NSIP has provided solutions to real imaging problems in industrial situations. Several books provide over views of applications, a few of which will be mentioned here. As well as providing a review of basic principles, Soille [10] decribes many real world examples and case studies of the use of morphological techniques in image processing. Applications range from blob analysis and counting to skeletonization, filtering, segmentation and classification. Case studies are drawn from geoscience, material science, medical and bioscience, security and image coding.

Loce and Dougherty [11] demonstrate how non-linear and logical image processing techniques may be applied to a range of problems in document processing. As well as restoration they include many operations relevant to the print industry such as anti-aliasing operations and techniques for changing both the spatial and grey level resolution of images.

Dougherty and Lotufo [12] present techniques for developing solutions to a range of imaging problems in their hands-on book.

4. NEW CHALLENGES IN NSIP

As non-linear signal processing matures it is being applied to many new applications and in new contexts. This session considers just four of these exciting fields.

A key area is in the analysis of vector data. This is particular important in the processing of colour data. It is well known that colour is a three-dimensional quantity and most of us wish to view colour images. The filtering problem is that there is no natural ordering to colour data for the calculation of such features as the median. Mathematical rigour is not sufficient in itself to solve the problem. The human visual system has a highly non-linear response to colour information and operators with the minimum MAE or MSE may give terrible pictures. It requires the inclusion of both psycho-visual properties and rigorous experimentation to achieve acceptable results in this area.

The treatment and diagnosis of cancer and other complex genetic diseases is one of the most important challenges facing mankind today. New gene-expression microarray technology allows the behaviour of thousands of genes to be monitored simultaneously over time; however modelling, prediction and eventual intervention require highly complex non-linear signal processing tools. These tools are amongst the first techniques beginning to bear fruit, in this field but require extensive study owing to the even more complex design processes required for modelling than those confronted in standard signal processing tasks. Dougherty, *et al.* provide a review of the salient issues as they pertain to non-linear signal processing [13].

Security and copyright of data are important issues considering the proliferation of digital media and internet file sharing mechanisms. Subsequent legal disputes regarding the authorship of material will depend on the robustness of embedded features known as 'watermarks' for authentication. Digital watermarking has been an active topic of research for several years. Watermarks must be both invisible to the viewer and resistant to attack.

Non linear image processing techniques may be used to generate piecewise- linear Markov maps for both additive and multiplicative watermark embedding. A system using chaotic watermark generators will be presented in this session.

The area of active noise control has been the subject of much research in recent years. Whilst most of the techniques are based on linear models emerging work, described in this session, has employed non-linear techniques based on polynomial filters.

5. CONCLUSIONS

A brief review on non linear signal and image processing methods has been presented. More detailed papers on a number of new areas will be given in the remainder of the session.

REFERENCES

[1] J. Serra, "Image Analysis and Mathematical Morphology", Vol 1, Academic Press, London, 1982.

[2] J. Serra, "Image Analysis and Mathematical Morphology", Vol 2, Academic Press, London, 1988.

[3] E. R. Dougherty, and Y. Chen, "Optimal and Adaptive Design of Logical Granulometric Filters", *Advances in Imaging and Electron Physics, Vol. 117*, ed. P. Hawkes, Academic Press, 1-71, New York, 2001.

[4] G. J. F. Banon, and J. Barrera, "Minimal Representation for Translation-Invariant Set Mappings by Mathematical Morphology," *SIAM J. Applied Mathematics*, **51**, 1991.

[5] G. Matheron, *Random Sets and Integral Geometry*, John Wiley, New York, 1975.

[6] P. Wendt, and E. Coyle, N. Gallagher, "Stack Filters", IEEE Trans. ASAP, vol 34, 898-911, August 1986.

[7] E. R. Dougherty, and J. Barrera, "Pattern Recognition Theory in Nonlinear Signal Processing," *Mathematical Imaging and Vision*, **16**, 181-197, 2002.

[8] E. R. Dougherty, and J. Astola, eds., *Nonlinear Filtering for Image Processing*, SPIE and IEEE Presses, Belingham, 1999.

[9] M S Hamid, N Harvey, S Marshall, "GA optimisation of multidimensional grey-scale soft morphological filters with applications in archive film restoration", *IEEE Trans. Circuits and Systems for Video technology*, Vol 13, No 5, 406-416, May 2003. See also Vol 13, No 7, 726, July 2003.

[10] P. Soille, *Morphological Image Analysis: Principles and Applications*, 2nd Edition, Springer, Berlin, 2003.

[11] R. P. Loce, and E. R. Dougherty, *Enhancement and Restoration of Digital Documents: Statistical Design of Nonlinear Algorithms*, SPIE Press, 1997.

[12] E. R. Dougherty, and R. Lotufo, *Hands-on Morphological Image Processing*, SPIE Press, 2003.

[13] Dougherty, E. R., Shmulevich, I., and M. L. Bittner, "Genomic Signal Processing: The Salient Issues," *Applied Signal Processing*, **4**, 2004.