A NOVEL APPROACH TO CONVOLUTIVE BLIND SEPARATION OF CLOSE-TO-ORTHOGONAL PULSE SOURCES USING SECOND-ORDER STATISTICS

Aleš Holobar and Damjan Zazula

Faculty of Electrical Engineering and Computer Science, University of Maribor
Smetanova 17, 2000 Maribor, Slovenia (Europe)
phone: +386 2 220 7485, fax: +386 2 251 1178, email: ales.holobar@uni-mb.si
web: storm.uni-mb.si/staff/holobar.html

ABSTRACT

We introduce a novel approach suitable for blind separation of convolutive Multiple-Input-Multiple-Output mixtures of orthogonal pulse sources. The approach is based on a joint-diagonalization of the algebraic combinations of whitened correlation matrices at non-zero lags. It supposes constant and finite system impulse responses and more measurements than sources. Preliminary tests on the synthetic signals prove 100% accuracy in detection of source pulses even if the sources are not completely orthogonal and when the measurements are noisy.

1. INTRODUCTION

Blind source separation (BSS) is a computational technique for identifying mutually independent sources out of their instantaneous or convolutive mixtures provided no further information is given either on the mixing process or on the nature of the sources. Many BSS approaches to separation of instantaneous mixtures were developed in the past. They usually exploit at least one of the following three properties: the non-Gaussianty of the independent and identically distributed (i.i.d.) sources, temporal correlations of the sources, and possible non-stationarity of the sources.

The separation of convolutive mixtures proved to be much more difficult problem and was less frequently addressed. The developed approaches are typically based on the Bussgang methods [2], higher-order statistics [8], second-order statistics [6], and on linear independent component analysis [7]. Very efficient method for separation of general non-stationary sources was proposed in [1]. The approach exploits the differences of energy locations of sources in time-frequency domain and enables a robust reconstruction of sources, but only up to a filtering effect. The approach was upgraded in [6] where general sources were substituted with the mutually independent pulse sources which are commonplace in communications and biomedical signal processing.

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The approach in this paper follows the work in [6] by operating upon the convolutive mixtures of close-to-orthogonal pulse sources with finite unit sample responses. Both approaches even share the first whitening step, but herein much more robust and, hence, preferable identification of the missing unitary mixing matrix is presented. Section 2 outlines the assumed data model while Section 3 introduces the blind deconvolution method. Simulation results are outlined in Section 4. Section 5 concludes the paper.

2. DATA MODEL

Assume the following linear and time invariant Multiple-Input-Multiple-Output (MIMO) convolutive mixture of pulse sources:

$$x_i(t) = \sum_{j=1}^{n}\sum_{l=0}^{L-1} h_{ij}(l) s_j(t-l) + n_i(t)\,; \quad i=1,...,M, \tag{1}$$

where, $\mathbf{x}(t) = [x_1(t),....,x_M(t)]^T$ stands for the transposed vector of M measurements (channels), $\mathbf{s}(t) = [s_1(t),...., s_N(t)]^T$ denotes the vector of N trains of pulses (sources), and $\mathbf{n}(t) = [n_1(t),....,n_M(t)]^T$ is the noise vector. $h_{ij}(l)$ stands for the unit sample response of the i-th source as detected by the j-th channel. For the simplicity reasons we will (without any loss of generality) suppose the length of all impulse responses equal to L. We further suppose the number of measurements greater than the number of sources $M>N$.

The additive noise $n_i(t)$ is modelled as stationary, temporally and spatially white zero-mean Gaussian random process, being independent from the sources

$$E[\mathbf{n}(t+\tau)\mathbf{n}^*(t)] = \sigma^2\delta(\tau)\mathbf{I}\,, \tag{2}$$

where *E[·]* stands for mathematical expectation, $\delta(.)$ for the Dirac impulse (delta function), σ^2 for the noise variance, and **I** denotes the identity matrix.

Our goal in blind decomposition is to reconstruct the source pulse trains $\mathbf{s}(t) = [s_1(t),....,s_N(t)]^T$ given only the vector of measurements $\mathbf{x}(t) = [x_1(t),....,x_M(t)]^T$.

To extend relation-ship (1) to convolutive MIMO vector form, the vector $\mathbf{x}(t)$ has to be augmented by K delayed repetitions of each measurement:

$$\overline{\mathbf{x}}(t) = [x_1(t),....,x_1(t-K+1),....,x_M(t),....,x_M(t-K+1)]^T, \quad (3)$$

where K is an arbitrary large integer which satisfies

$$KM > N(L+K) \quad (4)$$

Extending the noise vector in the same manner, (1) can be rewritten in a vector form [1]:

$$\overline{\mathbf{x}}(t) = \mathbf{A}\overline{\mathbf{s}}(t) + \overline{\mathbf{n}}(t). \quad (5)$$

$\mathbf{A}$ in (5) stands for the so called mixing matrix of size $KM \times N(L+K)$ which contains the unit sample responses $h_{ij}(l)$:

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1N} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{M1} & \cdots & \mathbf{H}_{MN} \end{bmatrix} \quad (6)$$

with

$$\mathbf{H}_{ij} = \begin{bmatrix} h_{ij}(0) & \cdots & h_{ij}(L) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & h_{ij}(0) & \cdots & h_{ij}(L) \end{bmatrix}, \quad (7)$$

while the extended vector of sources $\overline{\mathbf{s}}(t)$ takes the following form:

$$\overline{\mathbf{s}}(t) = [s_1(t),....,s_1(t-L-K+1),....,s_N(t),....,s_N(t-L-K+1)]^T. \quad (8)$$

Following the above assumptions the correlation matrix of extended measurements can be expressed as:

$$\mathbf{R}_{\overline{\mathbf{x}}}(\tau) = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \overline{\mathbf{x}}(t)\overline{\mathbf{x}}^*(t+\tau) = \mathbf{A}\mathbf{R}_{\overline{\mathbf{s}}}(\tau)\mathbf{A}^T + \delta(\tau)\sigma^2\mathbf{I}, \quad (9)$$

where $\mathbf{R}_{\overline{\mathbf{s}}}(\tau)$ denotes the correlation matrix of sources and $\overline{\mathbf{x}}^*(t)$ stands for the conjugate transpose of $\overline{\mathbf{x}}(t)$. Without any loss of generality we can suppose the variance of all sources equal to 1 ($r_{ii} = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \overline{s}_i(t)\overline{s}_i^*(t) = 1$). Hence, the correlation matrix of the extended sources at zero lag $\tau = 0$ can be set equal to the identity matrix:

$$\mathbf{R}_{\overline{\mathbf{s}}}(0) = \mathbf{I}. \quad (10)$$

3. DECOMPOSITION METHOD

Following the commonly used blind source separation route the novel approach utilises a two-step procedure. Firstly, the mixing matrix $\hat{\mathbf{A}}$ is estimated. Secondly, using the matrix $\hat{\mathbf{A}}$ the original sources are reconstructed as

$$\overline{\mathbf{s}}(t) = \mathbf{A}^{\#}\overline{\mathbf{x}}(t), \quad (11)$$

where $\mathbf{A}^{\#}$ denotes a pseudo-inverse of the matrix $\mathbf{A}$.

3.1 Estimation of the mixing matrix

As a first step in estimating the mixing matrix the measurements are whitened (second-order decorrelated) by so called whitening matrix $\mathbf{W}$ satisfying

$$\mathbf{W}\mathbf{A}\mathbf{A}^H\mathbf{W}^H = \mathbf{I}. \quad (12)$$

There are several ways of constructing the whitening matrix $\mathbf{W}$ [7]. Due to the clarity reasons only the most obvious is quickly presented in the sequel. According to (4) there are at least $KM - N(L+K)$ eigenvalues of $\mathbf{R}_{\overline{\mathbf{x}}}(0)$ equal to σ^2. Consequently, the noise variance σ^2 can be estimated by averaging the $KM - N(L+K)$ smallest eigenvalues of $\mathbf{R}_{\overline{\mathbf{x}}}(0)$. Subtracting it from the correlation matrix of measurements we obtain:

$$\overline{\mathbf{R}}_{\overline{\mathbf{x}}}(0) = \mathbf{R}_{\overline{\mathbf{x}}}(0) - \hat{\sigma}^2\mathbf{I} = \mathbf{A}\mathbf{R}_{\overline{\mathbf{s}}}(0)\mathbf{A}^T = \mathbf{A}\mathbf{A}^T. \quad (13)$$

The whitening matrix $\mathbf{W}$ can thus be obtained as an inverse square root of the observation correlation matrix $\overline{\mathbf{R}}_{\overline{\mathbf{x}}}(0)$ [3].

According to (12) the whitening matrix $\mathbf{W}$ transforms the mixing matrix $\mathbf{A}$ to yet unknown $N(L+K)\times N(L+K)$ unitary matrix $\mathbf{U}$:

$$\mathbf{W}\mathbf{A} = \mathbf{U}. \quad (14)$$

Hence, in the second step the matrix $\mathbf{U}$ must be identified. One of the possible ways was already introduced in [6] by joint-diagonalizing [4] so called spatial pseudo Wigner-Ville (PWV) distribution matrices of the whitened observations $\mathbf{W}\overline{\mathbf{x}}(t)$. Much more robust way towards identification of unknown matrix $\mathbf{U}$ is to fully exploit the cross-correlations of the augmented sources. Namely, dealing with temporally independent pulse sources the source correlation matrices take the following form:

$$\mathbf{R}_{\overline{\mathbf{s}}}(\tau) = \begin{bmatrix} \mathbf{J}_1(\tau) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2(\tau) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{J}_N(\tau) \end{bmatrix} \quad (15)$$

where $\mathbf{0}$ denotes the $(L+K)\times(L+K)$ matrix with all entries equal to zero, and $\mathbf{J}_i(\tau)$ stands for the $(L+K)\times(L+K)$ matrix with the non-zero entries only on the τ-th diagonal:

$$\mathbf{J}_i(\tau) = r_{ii}\begin{bmatrix} \delta(\tau) & \delta(\tau-1) & \cdots & \delta(\tau-L-K+1) \\ \delta(\tau+1) & \delta(\tau) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \delta(\tau-1) \\ \delta(\tau+L+K-1) & \cdots & \delta(\tau+1) & \delta(\tau) \end{bmatrix} \quad (16)$$

It can easily be verified that

$$\mathbf{C}_{\overline{\mathbf{s}}}(\tau) = \mathbf{R}_{\overline{\mathbf{s}}}(-\tau)\mathbf{R}_{\overline{\mathbf{s}}}(\tau) = \begin{bmatrix} \mathbf{D}_1(\tau) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2(\tau) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{D}_N(\tau) \end{bmatrix} \quad (17)$$

with $(L+K)\times(L+K)$ $\mathbf{D}_i(\tau)$ matrices defined as

$$\mathbf{D}_i(\tau)=r_{ii}^2\begin{bmatrix} \sum_{i=-L-K+1}^{0}\delta(\tau-i) & 0 & \cdots & 0 \\ 0 & \sum_{i=-L-K+2}^{1}\delta(\tau-i) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sum_{i=0}^{L+K-1}\delta(\tau-i) \end{bmatrix} \quad (18)$$

According to (18) $\mathbf{C}_{\bar{\mathbf{s}}}(\tau)$ matrices are diagonal for $-K-L+1\le\tau\le k+L-1$. Some algebra upon (9), (14) and (17) produces:

$$\begin{aligned}\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau) &= \mathbf{W}\mathbf{R}_{\bar{\mathbf{x}}}(-\tau)\mathbf{W}\mathbf{W}^H\mathbf{R}_{\bar{\mathbf{x}}}(\tau)\mathbf{W}^H = \\ &= \mathbf{U}\mathbf{R}_{\bar{\mathbf{s}}}(-\tau)\mathbf{R}_{\bar{\mathbf{s}}}(\tau)\mathbf{U}^H+\delta(\tau)\sigma^2\mathbf{I} = \quad . \quad (19) \\ &= \mathbf{U}(\mathbf{C}_{\bar{\mathbf{s}}}(\tau)+\delta(\tau)\sigma^2\mathbf{I})\mathbf{U}^H\end{aligned}$$

The matrices $\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau)$ effectively suppress the noise. and are all diagonal in the basis of the columns of the matrix $\mathbf{U}$. Hence, the missing matrix $\mathbf{U}$ can be obtained as a unitary diagonalizing matrix of the $\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau)$ matrices. However, to guarantee the uniqueness of the unitary matrix which simultaneously diagonalizes the set of $\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau)$ matrices the following uniqueness condition must be fulfilled [3]:

$$\begin{aligned}&\forall i,j; \quad 1\le i\ne j\le N(L+K) \\ &\exists\tau; \ -K-L+1\le\tau\le K+L-1 \quad d_i(\tau)\ne d_j(\tau),\end{aligned} \quad (20)$$

where $d_i(\tau)$ stands for the *i*-th diagonal element of the matrix $\mathbf{C}_{\bar{\mathbf{s}}}(\tau)$. When diagonalizing the set of $\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau)$ matrices ($-K-L+1\le\tau\le k+L-1$) the uniqueness condition (20) is met only if $r_{ii}\ne r_{jj}$ for $i\ne j$. But, according to (10), all the sources have unit variance $\forall i,\ r_{ii}=1$.

This problem can easily be avoided when separating non-stationary pulse sources with different time-varying firing frequencies. Limiting the calculation of the $\mathbf{R}_{\bar{\mathbf{x}}}(\tau)$ matrices in (19) to a short enough time interval $[T_0,T_1]$:

$$\mathbf{R}_{\bar{\mathbf{x}}}(\tau)=\frac{1}{T_1-T_0}\sum_{t=T_0}^{T_1}\bar{\mathbf{x}}(t)\bar{\mathbf{x}}^*(t+\tau), \qquad T_1-T_0<<T, \quad (21)$$

the global correlation factors r_{ii} in (18) are replaced by local correlations factors $\hat{r}_{ii}=\frac{1}{T_1-T_0}\sum_{t=T_0}^{T_1}\bar{s}_i(t)\bar{s}_i^*(t)$. The local differences in the source firing frequencies cause the $\hat{r}_{ii}$ factors to differ among each other and the condition (20) is fulfilled. In order to prevent the equalization of $\hat{r}_{ii}$ factors the $\mathbf{R}_{\bar{\mathbf{x}}}(0)$ matrix entering the whitening procedure (13) must be calculated over the whole time interval $[0,T]$. To increase the statistical efficiency it is recommended to joint-diagonalize the several sets of $\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau)$ matrices with $\mathbf{R}_{\bar{\mathbf{x}}}(\tau)$ matrices calculated over different short enough time intervals $[T_k,T_{k+1}]$.

Similar solution can be applied to the sources with constant firing frequencies. Before entering the whitening procedure (13), all the signals in the extended vector of measurements (3) should be premultiplied by the positive time-varying scalar function $f(t)$:

$$\forall i,\forall t \quad x_i(t)=f(t)\cdot x_i(t) \quad (22)$$

This introduces non-stationarities to the source amplitudes and, hence, to local $\hat{r}_{ii}$ factors. Afterwards, the procedure for separation of the non-stationary sources with different time-varying firing frequencies should be followed. To preserve the temporal whiteness of the noise, the function $f(t)$ should be periodical and limited in amplitude (a raised sine function with strictly positive values is a good example).

3.2 Source reconstruction

Once the mixing matrix $\hat{\mathbf{A}}$ is reconstructed, the sources can be estimated by (11). The noise may seriously hinder the original pulse trains. However, according to (11) $K+L-1$ delayed estimations of each source are reconstructed. By normalizing, classifying, aligning, and summing them together much more reliable decomposition results are obtained.

4. SIMULATION RESULTS

The proposed source separation approach was applied to the synthetic convolutive mixtures of 5 randomly generated close-to-orthogonal pulse sources with length T=5000 and average inter-pulse interval of 40 samples. To guarantee the temporal independence of sources the lower limit of the inter-pulse interval was set to 15 samples. Unit sample responses of length L=10 where randomly generated (conditional number of the simulated mixing matrix $\mathbf{A}$ yielded 312) and convoluted with pulse trains to produce 10 observed measurements. Factor K in (3) was set to 11 producing 110 extended measurements. According to (22) the extended signals were multiplied by a raised sine function with the strictly positive values varying between $\frac{1}{2}a$ and a, where a is the maximal measurement value. Both the period of sine function (22) and the length of the $[T_0,T_1]$ time interval in (21) were set equal to 1000 samples. The missing unitary matrix $\mathbf{U}$ was reconstructed by diagonalization of a single set of $\mathbf{Q}_{\bar{\mathbf{x}}}(-\tau,\tau)$ matrices ($-K-L+1\le\tau\le k+L-1$) with $\mathbf{R}_{\bar{\mathbf{x}}}(\tau)$ matrices calculated over the $[1,1000]$ sample interval.

Three performance indices, the number of accurately recognized pulses (true positive statistic), the number of misplaced pulses (true negative statistic), and the global interference-to-signal ratio (ISR) defined as

$$ISR=\frac{1}{N(L+K)-1}\frac{\sum_{p=1}^{N(L+K)}\sum_{q=1,q\ne p}^{N(L+K)}I_{pq}}{\sum_{p=1}^{N(L+K)}I_{pp}}, \quad (23)$$

with $I_{pq}=\left|\hat{\mathbf{A}}^{\#}\mathbf{A}\right|_{pq}$, were evaluated in the presence of zero-mean white Gaussian noise. The signal-to-noise ratio (SNR) ranged from 0 dB to 20 dB, at steps of 5 dB, while 20 Monte-Carlo simulation runs per each SNR were conducted. Before comparing to the original sources the 21 estimations of each source were first normalized, classified, aligned according to the pulse triggering times and finally summed together. The values of the selected performance indices are presented in Tables 1 and 2. The representative results of reconstructed pulse sources are depicted in Fig. 1 (SNR=10 dB) and Fig. 2 (SNR=5 dB).

Table 1: Global interference-to-signal ratio (ISR) for different SNR (mean ± standard deviation).

SNR [dB]	**20**	**15**	**10**	**5**	**0**
ISR [%]	3.78 ±0.01	4.32 ±0.02	6.53 ±0.09	16.80 ± 0.32	24.54 ±0.78

Table 2: Normalized number (mean ± standard deviation) of accurately recognized pulses (T+), and of misplaced pulses (T-), in reconstructed sources. The values are averaged over all sources.

SNR [dB]	**20**	**15**	**10**	**5**	**0**
T+ [%]	100.0 ±0.0	100.0 ±0.0	100.0 ±0.0	99.64 ± 0.92	93.72 ±4.61
T - [%]	0.0 ±0.0	0.0 ±0.0	0.0 ±0.0	0.18 ±0.28	1.77 ±2.68

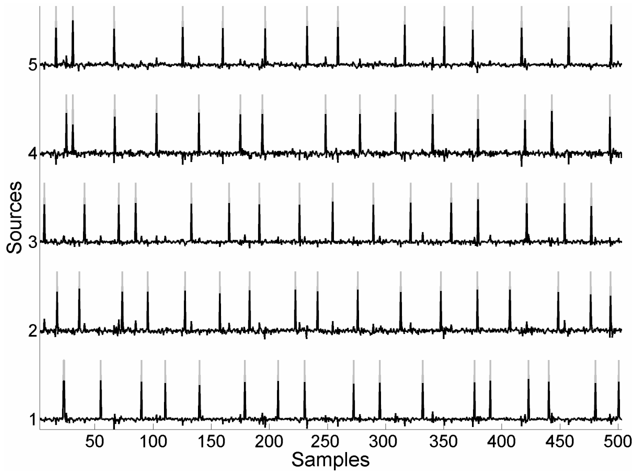

Figure 1: Reconstructed pulse trains (black) for SNR=10 dB in comparison with the original source firing patterns (grey).

5. CONCLUSIONS

As demonstrated by the results in Section 4, the method successfully suppresses the influence of the additive zero-mean white noise. Although closely related to the method presented in [6] the introduced approach does not relay on the identification of special time positions (auto-terms) in which only one source is active, and is, hence, less sensitive to temporal overlappings of the source pulses. Moreover, the *(K+L)* estimations of each source are retrieved up to a scaling factor. Hence, the calculated sources can be further improved by averaging the corresponding estimations what makes the approach even more noise resistant.

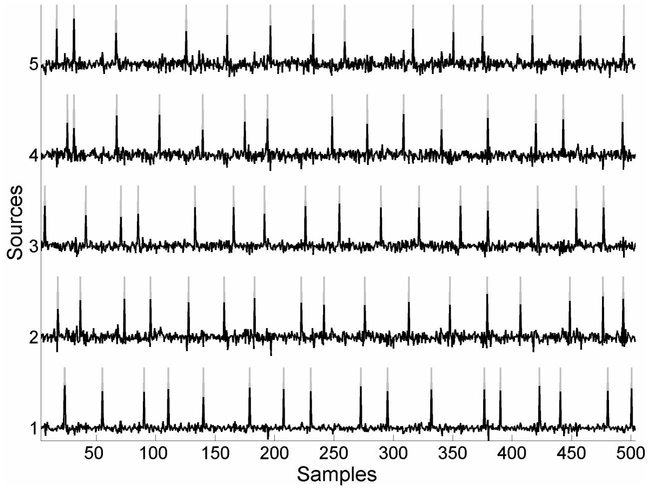

Figure 2: Reconstructed pulse trains (black) for SNR=5 dB in comparison with the original source firing patterns (grey).

Finally, the importance of the differences in firing frequencies was discussed and the method for enhancing the temporal differences among the local source variances $\hat{r}_{ii}$ introduced. When all this fails (when $\forall i \forall j$, $\hat{r}_{ii} = \hat{r}_{jj}$) each reconstructed source corresponds to the linear combination of only *N* (out of *N[K+L]*) original sources which can further be separated by multiplicative BSS methods [7]. The detailed explanation of this problem reaches beyond the scope of this discussion but is available upon the request.

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