IF ESTIMATION OF MULTICOMPONENT CHIRP SIGNAL IN IMPULSIVE α-STABLE NOISE ENVIRONMENT USING PARAMETRIC AND NON-PARAMETRIC APPROACHES

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ABSTRACT

In this paper we address the problem of instantaneous frequency (IF) estimation of multicomponent chirp signals in impulsive α-stable noise environment. Three new parametric techniques are introduced first based on robust versions of the MUSIC algorithm using truncated data (TRUNC-MUSIC), robust covariance estimate (ROCOV-MUSIC) and generalized covariation coefficients which are a fractional lower order statistics (FLOS-MUSIC), respectively. These methods are compared with a new proposed method based on "robust" time-frequency signal analysis.

I. INTRODUCTION

This paper is concerned with the analysis of multicomponent linear frequency-modulated (FM) signals, corrupted by heavy-tailed α-stable noise. This kind of signal, also known as chirp signals, arises in many engineering applications such as radar, sonar, telecommunications, automotive signal analysis and biomedical engineering. A concept intimately related to mono-component FM signals is that of instantaneous frequency (IF) [2]. In many situations, the IF characterizes important physical parameters of the signal. Diverse IF estimation methods have been developed for mono or multi components signals embedded in additive noise [7]. A well-known class of non-parametric methods, for IF estimation, is based on the time-frequency distributions of the signal [2], [3], [4]. On the other hand, parametric methods exploit a polynomial phase representation of the FM signals to achieve IF estimation through the estimation of the polynomial phase parameters [7]. Analysis of non-stationary signals affected by additive Gaussian noise has been addressed in details in several places [3]. There are many situations wherein the Gaussian assumption does not hold. Here, we consider noise processes characterized by infrequent but high amplitude events. This kind of noise, also known as *impulsive noise*, can be encountered in many engineering applications [6]. Modeling *spikes or impulsive* events by Gaussian models will lead to poor estimation and detection performances. A class of distributions adopted by the signal processing community to model the statistical behavior of impulsive processes is the heavy-tailed distributions class. Examples of heavy-tailed distributions include Laplace, Cauchy and α-stable distributions with $\alpha < 2$. Modeling of impulsive random processes by the α-stable statistics has been shown to be an effective tool in modern statistical signal processing [6]. Theoretical justifications for using the stable distribution as a basic statistical modeling tool come from the *Generalized Central Limit Theorem* (GCLT) [6]. Another defining feature of the stable distribution is the so-called *stability property*, which says that the sum of independent stable random variables with the same characteristic exponent is again stable and has the same characteristic exponent.

Unfortunately, standard techniques for parameter estimation such as the maximum likelihood (ML) or time-frequency based methods are not easily implemented or present poor performances in the α-stable case. Consequently, it is important to have suitable IF estimation methods dedicated to the impulsive noise context. The objective of this paper is to study this particular problem by parametric and non parametric time-frequency-based methods.

II. PROBLEM STATEMENT

Consider a multicomponent chirp signal given by:

$$x(t) = \sum_{i=1}^{I} s_i(t) + z_0(t) = \sum_{i=1}^{I} a_i(t)\, cos\{\phi_i(t)\} + z_0(t) \quad (1)$$

where $t = 0, \ldots, N-1$, $\phi_i(t) = 2\pi(f_i t + \delta_i t^2) + \theta_i$ is the phase of the ith chirp component. $f_i, \delta_i, i = 1, \ldots, I$ are *unknown* real coefficients. $\{\theta_i, i = 1, \cdots, I\}$ are realizations of random variables, distributed uniformly and independently over $[0, 2\pi)$. N is the sample size and I is the number of components of the observed signal. The amplitudes $a_i(t)$ are assumed α-stable, independent from the noise term $z_0(t)$ and with location parameters $a_i \neq 0$ and dispersions γ_i. The random noise $z_0(t)$ is modeled as a symmetric with zero location parameter α-stable process ($S\alpha S$). The $S\alpha S$ do not have closed form probability density function (pdf) except for the cases $\alpha = 1$ (Cauchy distribution) and $\alpha = 2$ (Gaussian distribution). The $S\alpha S$ pdf is defined by means of its characteristic function $\psi(t) = \exp\{jat - \gamma|t|^{\alpha}\}$, where α $(0 < \alpha \leq 2)$ is the characteristic exponent, controlling the heaviness of the pdf tail, $\gamma(\gamma > 0)$ is the dispersion, which plays an analogous role to the variance, and a is the location parameter, the symmetry axis of the pdf. Due to their heavy tail, stable distributions do not have finite second or

higher-order moments, except for the case of $\alpha = 2$.
Our primary interest is to estimate the instantaneous frequency IF_i of each signal component s_i.
By decomposing $a_i(t) = \gamma_i^{\frac{1}{\alpha}} a_{i,0}(t) + a_i$ where $a_{i,0}(t)$ being a standard (zero location parameter and unit dispersion) α-stable process, we can re-write the signal expression as

$$\begin{aligned} x(t) &= \sum_{i=1}^{I} a_i cos\{\phi_i(t)\} + \underbrace{\sum_{i=1}^{I} \gamma_i^{\frac{1}{\alpha}} a_{i,0}(t) cos\{\phi_i(t)\} + z_0(t)}_{z(t)} \\ &= \sum_{i=1}^{I} a_i cos\{\phi_i(t)\} + z(t) \end{aligned}$$

where $z(t)$ is an α-stable process according to the stability property [6]. Thus, the problem of estimation $(IF_i)_{1\le i\le I}$ of the multicomponent chirp signal affected by multiplicative and additive α-stable noise is reduced to that of estimating $(IF_i)_{1\le i\le I}$ of a constant amplitude chirp signals, i.e. having the same IF laws as the original signals, but affected by the additive noise only.

III. PARAMETRIC IF ESTIMATION

This section introduces three parametric methods for IF estimation robust to impulsive noise.

III-1. IF estimation of linear FM signals

Consider the quadratic phase estimation of the signal $x(t)$ in Eq.(1). The first step consists in transforming the quadratic phase on linear phase using the polynomial transform [7]:

$$\begin{aligned} y(t) &= x(t+\tau)x(t) & (2) \\ &= \sum_{i=1}^{I} \frac{|a_i|^2}{2} \cos\{2\pi(2\tau\delta_i t) + \varphi_i\} + z^1(t) & (3) \end{aligned}$$

where τ is the delay parameter (to choose preferably in $[\frac{N}{2}, \frac{2N}{3}]$), $\varphi_i = 2\pi(\tau f_i + \tau^2\delta_i)$ and $z^1(t)$ is the term of noise plus interferences.[1] Now we apply one of the proposed algorithms in section III.2 to $y(t)$ to estimate the parameters $\delta_i, i = 1, \ldots, I_1$. [2] In order to estimate the parameters $f_i, i = 1, \ldots, I$, we consider the demodulation of the signal as follows: For $i = 1, \ldots, I_1$, we compute

$$\begin{aligned} x^{(i)}(t) &= x^a(t)\exp(-j2\pi\hat{\delta}_i t^2) \\ &\approx \sum_{k\in J_i} \exp\{2j\pi(f_k t) + \theta_i\} + w(t) \end{aligned}$$

where J_i is the set of component indices with the same coefficient δ_i, $\hat{\delta}_i$ is the estimate of δ_i, $x^a(t)$ is the analytic signal of $x(t)$, and $w(t)$ represents noise plus interference. For each demodulated signal, we estimate the frequencies $\{f_k, \quad k \in J_i\}$ using one of the proposed algorithms (see section III.2) applied to the real part of the demodulated signal $\Re\{x^{(i)}(t)\}$. Note that it is not necessary to use a high resolution method in the case where J_i contains one single signal index.

[1] Note that $z^1(t)$ is an impulsive noise but not necessarily $S\alpha S$.
[2] We might have $I_1 < I$ in the case where certain chirp components of the signal have the same phase coefficients δ_i but different coefficients f_i.

III-2. IF estimation of sinusoidal signals

In this section, we address the frequency estimation problem of multicomponent sinusoidal signals observed in impulsive noise environment given by equation (1) with $\phi_i(t) = 2\pi f_i t + \theta_i$. We propose to apply the high resolution subspace algorithm MUSIC (Multiple Signal Classification) [1] for the frequency estimation. As the performance of the standard MUSIC algorithm based on the sample covariance matrix degrades if the underlying noise is impulsive, we propose to apply MUSIC in the following three ways: *(i)* In the first one, we apply MUSIC to the truncated harmonic signal, *(ii)* in the second one, we apply MUSIC to the robust covariance estimate of the harmonic signal and *(iii)* in the third one, we apply MUSIC to the generalized covariation function of the signal.

III-2-1. TRUNC-MUSIC

In α-stable environment, the use of sample covariance is no longer appropriate for frequency estimation due to the infinite variance of the noise. To avoid this difficulty, we propose to truncate in amplitude the 'large-valued' observations that represent "large" impulsive noise realizations and apply MUSIC to the finite covariance matrix of the truncated process. TRUNC-MUSIC (TRUNC stands for truncation) can be summarized as follows:

- *Choice of a truncation constant* K: We propose to compute the histogram of observations and choose K such that $[-K, K]$ contains 90 % of the data.
- *Pre-Processing*: We truncate the signal according to:

$$\tilde{x}(t) = \begin{cases} x(t) & \text{if} \quad |x(t)| \le K \\ sign[x(t)]K & \text{if} \quad |x(t)| > K \end{cases}$$

- *Frequency estimation*: Apply MUSIC algorithm to the covariance matrix of the truncated signal $\tilde{x}(t)$.

III-2-2. ROCOV-MUSIC

Huber considered the parameter estimation problem in the presence of outliers or impulsive noise and proposed the concept of M-estimation [5]. In this subsection, we consider M-estimates for the signal auto-covariance function $\gamma(k) \triangleq E[x(t+k)x(t)]$. Note that the robust autocovariance estimation is equivalent to the robust variance estimation according to $E(XY) = \frac{1}{4}[Var(X+Y) - Var(X-Y)]$ where Var is the variance. For an α-stable distribution we have infinite variance, for that we propose to first truncate the observations using a large valued constant $K \gg 1$. The M-estimator of the variance σ^2 is a solution of the following equation [5]

$$\frac{1}{N}\sum_{i=0}^{N-1} u(d_i^2)\frac{x^2(i)}{\sigma^2} - u(d_i^2) = 0 \qquad (4)$$

where $d_i^2 = \frac{x^2(i)}{\sigma^2}$ is the Mahalanobis quadratic distance and u is a weighting function defined in $\mathbb{R}^+$. The existence and uniqueness of the solution of Eq.(4) was shown by Huber [5] under mild assumptions about the weighting function such as boundedness and continuity. This function are typically chosen such that observations coming from the tails of the assumed contaminated distribution are down-weighted. Here, we use the *robust non-descending* weighting function which is based on Huber's minimax function [5]. We can compute the M-estimate of the variance as a solution of the latter equation [5]. Thus, the algorithm ROCOV-MUSIC proceeds as follows:

- Compute the M-estimates $\hat{\gamma}(k)$, $k = 0, \ldots, L-1$ using the M-estimator of the variance of $[x(t+k)+x(t)]$ and $[x(t+k)-x(t)]$ through the following so called ROCOV algorithm:
  1. Initialize the ROCOV algorithm by the standard variance estimator $\sigma_0^2 = \frac{1}{N}\sum_{i=0}^{N-1} x^2(i)$,
  2. At the $(j+1)$th iteration compute
  $$\sigma_{j+1}^2 = \frac{\sum_{i=0}^{N-1} \omega_{i,j}^2 x^2(i)}{\sum_{i=0}^{N-1} \omega_{i,j}^2 - 1};$$
  $\omega_{i,j} = u(d_{i,j}) = \omega(d_{i,j})/d_{i,j}$, $d_{i,j}^2 = \frac{x^2(i)}{\sigma_j^2}$, ω is the Huber function given by $\omega(d) = \min(d, k)$ and k is a suitable constant.
  3. Stop the ROCOV algorithm when the error is smaller than a given threshold ϵ.
- Apply MUSIC to the robust covariance matrix estimate $\hat{\Gamma}_x = Toeplitz[\hat{\gamma}(k)_{0\le k\le L-1}]$ for the frequency estimation.

III-2-3. FLOS-MUSIC

In this section we propose to use the fractional lower order statistics (FLOS) of the signal for the frequency estimation. We consider an $L \times L$ generalized covariation coefficient (GCC) matrix Γ, whose (n,l)th entry is given by [1]:

$$\Gamma_{n,l} = \frac{[x(n), x(l)]_\alpha}{[x(l), x(l)]_\alpha} = \frac{E[x(n)x(l)^{<p-1>}]}{E[\mid x(l)\mid^p]}, \quad 1 \le p < \alpha \quad (5)$$

where $x^{<p-1>} = |x|^{p-1} sign(x)$. It has been shown in [1] that

$$\Gamma_{n,l} = \sum_{i=1}^{I} \eta_i \cos\{2\pi f_i(n-l)\} + P_z \delta_n \quad (6)$$

where $\{\eta_i, i = 1, \cdots, I\}$ are positive real constants depending on α and a_i, P_z is a real constant depending on noise pdf and δ_n is the Kronecker coefficient. In practice, an estimate of $\Gamma_{n,l}$ (for $p = 1$) is given by

$$\hat{\Gamma}_{n,l} = \frac{\sum_{i=1}^{N-M+1} x(n+i-1) sign(x(l+i-1))}{\sum_{i=1}^{N-M+1} \mid x(l+i-1)\mid} \quad (7)$$

Equation (6) shows that we can obtain the frequency estimates by applying MUSIC algorithm to the GCC-matrix Γ. This algorithm is referred to as FLOS-MUSIC [1].

IV. TIME-FREQUENCY IF ESTIMATION

Signal time-frequency analysis has proved to be a powerful tool in the analysis of non-stationary FM signals. In the time-frequency representation, the noise energy is spread over all time-frequency domain while the component energies are well localized around their respective IFs leading to high energy peaks for the latter. However, in the heavy-tailed noise case, to get a good performances, we need a pre-processing of the signal to attenuate the impulsive noise effect. In [3], a robust TFD-based technique has been proposed for IF estimation of mono-component FM signal. Here, we propose to generalize this approach to the multicomponent case according to the following steps:

Pre-Processing: The first step consists in reducing the impulsive noise amplitudes in order to improve the SNR. To do so, two solution might be suggested.

- *Compressing technique*: We propose here to pass the noisy signal through a nonlinear device that compresses the large amplitudes (i.e., reduces the dynamic range of the noisy signal) before further analysis [2]. The output of the nonlinear device, is expressed as
  $$\tilde{x}(t) = |x(t)|^\beta sign[x(t)]$$
  where $0 < \beta \le 1$ is a real coefficient that controls the amount of compression applied to the input noisy signal $x(t)$.
- *Truncating technique*: In this case, we apply the truncating technique which is presented in TRUNC-MUSIC algorithm (section III-2-1).

Time-Frequency distribution: The choice of a TFD depends on the specific application at hand and the representation properties that are desirable for this application. In order to separate the signal components and estimate their IFs, we need to have a "clean" TFD. That is, we need a distribution that can reveal the features of the multicomponent signal as clearly as possible (with reduced cross-terms energy). In this work, we have used the modified B-distribution [4] given by:

$$S(t,f) = \int\int_{-\infty}^{+\infty} G_{MB}^\sigma(t')[z(t-t'+\frac{\tau}{2})z^*(t-t'-\frac{\tau}{2})]e^{-j2\pi f\tau}dt'd\tau$$

where $G_{MB}^\sigma(t') = \frac{k_\sigma}{\cosh(t')^{2\sigma}}$, $0 \le \sigma \le 1$ is a real parameter that controls the tradeoff between component's resolution and cross-terms suppression and $k_\sigma = \Gamma(2\sigma)/(2^{2\sigma-1})\Gamma^2(\sigma)$ is the normalizing factor ($\Gamma(.)$ stands for the gamma function). The choice of the MB-distribution, stems from the fact that it presents a good performance in terms of resolution and cross-terms suppression [4].

Component separation & IF estimation: In [3] an algorithm that separates the signal components and estimate their respective IF laws from the signal TFD, has been presented. Here, we propose to apply this algorithm to the TFD of the pre-processed signal $\tilde{x}(t)$. In order to compare the estimation performance with the previously presented parametric methods, we use a simple polynomial fit (in the least squares sense) to extract the phase parameters $(\hat{f}_i, \hat{\delta}_i)$ from the ith component IF estimate.

V. PERFORMANCE EVALUATION & COMPARISON

In this section, a comparative study of the previous IF estimation methods of multicomponent chirp signal is addressed. For this purpose, we consider a mixture of two chirp components of the same amplitude $a_1 = a_2 = 1$, with $f_1 = 0.05$, $f_2 = 0.3$, $\delta_1 = 0.0001$ and $\delta_2 = 0.0003$ embedded in impulsive α-stable noise with parameter exponent $\alpha = 1$. The estimation performance is measured by the normalized MSE defined by

$$NMSE = \frac{1}{N_r} \sum_{r=1}^{N_r} \frac{\|\hat{\theta}_r - \theta\|^2}{\|\theta\|^2}$$

where θ is the considered parameter, $\hat{\theta}_r$ is the estimate of θ at the rth experiment and N_r is the number of Monte-Carlo runs chosen here equal to 500. For the non-parametric TFD-based method, we propose the compressing technique with parameter $\beta = 0.1$ (we chose $\sigma = 0.01$ for the MB-distribution kernel).
Figures 1 and 2 represent the NMSE of the phase parameters versus the sample size and the noise dispersion, respectively. In this simulation context, the best results are obtained by the non-parametric method followed by the parametric method based on robust covariance estimation (ROCOV-MUSIC).

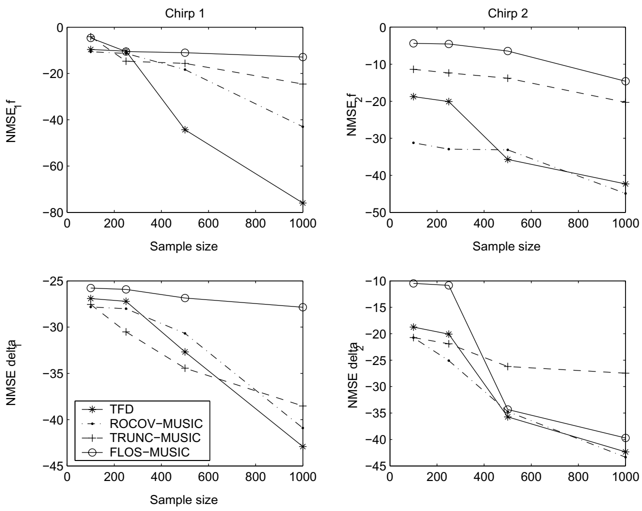

Figure 1: Normalized MSE of the various phase parameters versus sample size, $\gamma = 0.1$.

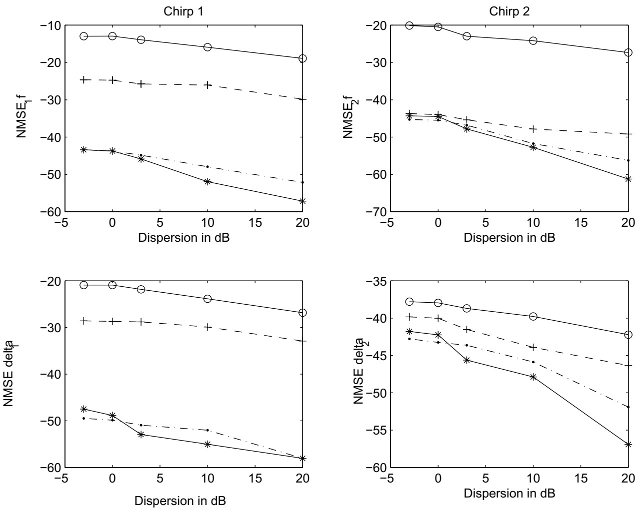

Figure 2: Normalized MSE of the various phase parameters versus noise dispersion in dB, N=1000.

VI. CONCLUSION

In this paper, several methods for IF estimation in heavy-tailed noise are introduced. New non-parametric method based on a pre-processing stage and on the signal TFD was proposed and compared with the proposed parametric methods based, respectively, on signal truncation, robust covariance estimation and generalized covariation coefficients. Simulations results are presented to validate our IF estimation methods. In the considered simulation context, the comparative study shows the superiority of the non-parametric (TFD-based) method and the parametric method using the robust covariance estimation technique (ROCOV-MUSIC).

VII. REFERENCES

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