OPTIMAL MONOPULSE TRACKING OF SIGNAL SOURCE OF UNKNOWN AMPLITUDE

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ABSTRACT

Estimation of the direction of arrival of a signal source by means of a monopulse antenna is one of the oldest and most widely used high resolution techniques [1]. Although the statistical performance of this estimation technique has been extensively investigated for decades, recent work [2] based on an analysis of the problem from the point of view of optimal detection applied to a two-sensors system, has shown that the common solution (detector/estimator) restricts the accessible performance. Indeed, changing the detector is necessary to optimize the overall performance. First derived in the particular case of Rayleigh-type signal source, this approach can be extended to the case of a signal source of unknown amplitude (including the non fluctuating case). The present paper establishes analytical performance of both new and common solutions in that case.

1. NOTATION

$\overrightarrow{X}, \overrightarrow{x}$ denote vectors (complex or real)

$\mathbf{C}, \widehat{\mathbf{R}}, \mathbf{m}$ denote matrices (complex or real)

$\Sigma_i, \beta_i, g_\Sigma, n_{\Sigma_i}, r, \ldots$ denote scalar values (complex or real)

$P()$ denotes a probability

$f()$ denotes a probability density function (pdf)

$F()$ denotes a cumulative distribution function (cdf)

$\widehat{r}$ denotes an estimator of r

$\mathbf{C}_{\overrightarrow{X}}$ denotes the covariance matrix of random vector $\overrightarrow{X}$

$\mathbf{Id}_I$ denotes Identity matrix with dimensions (I,I)

$$f_{\chi_2^I}(t, \mu^2, \sigma^2) = \frac{e^{-\frac{\left(t+\mu^2\right)}{\sigma^2}}}{\sigma^2} I_{I-1}\left(\frac{2\mu\sqrt{t}}{\sigma^2}\right)\left(\frac{\sqrt{t}}{\mu}\right)^{(I-1)}$$

$$e_N(T) = \sum_{n=0}^{N} \frac{T^n}{n!}, \quad I_I(t) = \frac{1}{2\pi}\int_0^{2\pi} e^{t\cos(\theta)}\cos(I\theta)\, d\theta$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt$$

2. INTRODUCTION

A monopulse antenna (radar or telecom) determines the angular location of a signal source (radar target or telecom transmitter) by comparing the returns from difference (Δ) and sum (Σ) antenna pattern [1][3]. It is a particular solution of the more general problem of finding the direction of arrival (DOA) of a signal source. As mentioned lately in [2], the usual approach related to this particular solution [3][4][5](ref.[3],[12],[20],[27]) limits the contribution of the difference channel to the estimation part of the problem, and therefore reveals an "historical" separate analysis of detection and estimation. A improved (detector, estimator) couple has been derived in [2] in the case of Rayleigh-type signal source by applying optimal detection theory to the monopulse antenna. Nevertheless, if the Rayleigh case is of fundamental interest for some applications (mainly radar applications), most of signal sources (telecom for example) have different amplitude fluctuation laws. If it seems unrealistic to try to solve the problem of optimal detection/estimation using the statistics averaged over all possible observations, for every amplitude fluctuation law of interest, a suboptimal but very general approach consists in solving the problem for each observation, regarding the amplitude fluctuation law as unknown (the most likely hypothesis when measuring an unknown actual signal source). Additionally, this characterization may also be useful to support tracking performance analysis where monopulse measurement is a preliminary step of nonstationary process as the Kalman Filter.

In order to allow straightforward comparison with results derived in [2], the present paper keeps the same analysis breakdown of the optimal detection problem [6], including equations numbering. Thus, we first formulate the optimal detector – the Neyman-Pearson criterion – applied to the monopulse antenna, and its associated composite hypotheses testing problem, as certain parameters are unknown. To solve the composite hypotheses testing problem, we apply the GLRT method and establish the analytical expressions of the detector and associated estimators, in particular that of the monopulse ratio. Lastly, we develop two approximations of the (detector, monopulse ratio estimator) pair. The first is based on the common "historical" approach [4][7]. The second, which we characterized analytically, proposes an appreciable improvement of the performances of the composite hypotheses testing problem. While retaining a comparable estimation Root Mean Square Error (RMSE), it helps achieve better on average detection performance characteristics, an improvement which is illustrated by an example.

3. PROBLEM FORMULATION

A common model for the receiver signal vector is:

$$\overrightarrow{v}(t) = \begin{pmatrix} \Sigma(t) \\ \Delta(t) \end{pmatrix} = \alpha(t)\,\overrightarrow{g}(\theta_0) + \overrightarrow{n}(t) \qquad (1)$$

where $\overrightarrow{g}(\theta) = (g_\Sigma(\theta), g_\Delta(\theta))^T$ is the array response vector (steering vector). It represents the array complex response to a narrowband point source situated at an angle θ. The complex envelope of the source is denoted by $\alpha(t)$, and $\overrightarrow{n}(t)$ is an 2x1 additive noise vector. Consider the following detection problem:

$$\begin{array}{l} H_0 : \overrightarrow{v}(t) = \overrightarrow{n}(t) \\ H_1 : \overrightarrow{v}(t) = \alpha(t)\,\overrightarrow{g}(\theta_0) + \overrightarrow{n}(t) \end{array} \qquad (2)$$

Based on an observation consisting of I independent array snapshots $\overrightarrow{v}(t_1),...,\overrightarrow{v}(t_I)$, we want to decide whether to accept the null hypothesis (noise only) H_0, or to accept the alternate hypothesis (signal plus noise) H_1.

3.1 Optimal Detector: LRT

If the pdf of the measurement is known under both hypotheses, the optimal detector - in the Neyman-Pearson sense [6] -

is the Likelihood Ratio Test (LRT). In the problem at hand, the additive noise $\overrightarrow{n}(t)$ is a circular, zero mean, white (both temporally and spatially), complex Gaussian random vector process with variance σ_n^2. The signal $\alpha(t)$ represents the complex envelope of the source (including power budget equation, signal processing gains) at time t and its a priory fluctuation law is unknown. The signal source does not alter its relative position with respect to the array during the I snapshots (static situation: θ_0 is constant). Denote by $\overrightarrow{V} = \left(\overrightarrow{\Sigma}^T, \overrightarrow{\Delta}^T\right)^T$, where $\overrightarrow{\Sigma} = (\Sigma_1, .., \Sigma_I)^T$ and $\overrightarrow{\Delta} = (\Delta_1, .., \Delta_I)^T$, the $2I$ dimensional observation vector related to the I snapshots, then:

$$f\left(\overrightarrow{V} \mid H_0\right) = \frac{e^{-\frac{I}{\sigma_n^2} Tr\left(\widehat{\mathbf{C}}_{H_0}\right)}}{\left(\pi^2\sigma_n^2\right)^I}$$

$$f\left(\overrightarrow{V} \mid H_1\right) = \frac{e^{-\frac{I}{\sigma_n^2} Tr\left(\widehat{\mathbf{C}}_{H_1}\right)}}{\left(\pi^2\sigma_n^2\right)^I}$$

$$\widehat{\mathbf{C}}_{H_0} = \frac{1}{I}\sum_{i=1}^{I} \overrightarrow{v}(t_i)\,\overrightarrow{v}(t_i)^H$$

$$\widehat{\mathbf{C}}_{H_1} = \frac{1}{I}\sum_{i=1}^{I} \left[\overrightarrow{v}(t_i) - \alpha(t_i)\,\overrightarrow{g}(\theta_0)\right]\left[\overrightarrow{v}(t_i) - \alpha(t_i)\,\overrightarrow{g}(\theta_0)\right]^H$$

Under these assumptions the LRT takes the form of:

$$LRT = \frac{f\left(\overrightarrow{V} \mid H_1\right)}{f\left(\overrightarrow{V} \mid H_0\right)} = e^{-\frac{I}{\sigma_n^2} Tr\left(\widehat{\mathbf{C}}_{H_1} - \widehat{\mathbf{C}}_{H_0}\right)} \underset{H_0}{\overset{H_1}{\gtrless}} T' \tag{3}$$

and can be reduced to:

$$\frac{\operatorname{Re}\left\{\overrightarrow{g}(\theta_0)^H \left[\sum_{i=1}^{I} \alpha(t_i)^H\,\overrightarrow{v}(t_i)\right]\right\}}{\sqrt{\frac{\sigma_n^2}{2}\left[\left\|\overrightarrow{g}(\theta_0)\right\|^2 \left(\sum_{i=1}^{I} \left|\alpha(t_i)\right|^2\right)\right]}} \underset{H_0}{\overset{H_1}{\gtrless}} T$$

Denote by D the event of a threshold detection. Then, Probability of False Alarm - $P_{FA} = P(D \mid H_0)$ - and Probability of Detection - $P_D = P(D \mid H_1)$ - are given by (real Gaussian law):

$$P_{FA} = \frac{1 - \operatorname{erf}(T)}{2} \tag{4a}$$

$$P_D = \frac{1 - \operatorname{erf}\left(T - \sqrt{2I\sigma_\alpha^2 \left\|\overrightarrow{g}(\theta_0)\right\|^2}\right)}{2} \tag{4b}$$

where: $\sigma_\alpha^2 = \frac{\overrightarrow{\alpha}^H \overrightarrow{\alpha}}{I} = \frac{1}{I}\left(\sum_{i=1}^{I} |\alpha_i|^2\right)$

3.2 GLRT

For cases in which some of parameters are unknown, the detection problem in (2) becomes a composite hypotheses testing problem (CHTP) [6]. Although not necessarily optimal, the GLRT (Generalized LRT [6]) is widely used in such problem. Let's denote by $\overrightarrow{\varphi}_j$ the unknown parameters vector under hypothesis j, the GLRT for deciding whether to accept H_0 or to accept H_1 is given by:

$$GLRT = \frac{\max_{\overrightarrow{\varphi}_1} f\left(\overrightarrow{V} \mid \overrightarrow{\varphi}_1\right)}{\max_{\overrightarrow{\varphi}_0} f\left(\overrightarrow{V} \mid \overrightarrow{\varphi}_0\right)} = \frac{f\left(\overrightarrow{V} \mid \widehat{\overrightarrow{\varphi}}_1\right)}{f\left(\overrightarrow{V} \mid \widehat{\overrightarrow{\varphi}}_0\right)} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{5}$$

where $\widehat{\overrightarrow{\varphi}}_j$ stands for the Maximum Likelihood Estimates (MLE) [6] of the unknown parameters under hypothesis j. In the problem at hand, the observation equation (1) may be rewritten according to an equivalent form:

$$\overrightarrow{v}(t_i) = \beta(t_i)\,\overrightarrow{x} + \overrightarrow{n}(t_i) \tag{6}$$

where: $\beta(t_i) = \alpha(t_i)\,g_\Sigma(\theta_0)$, $\overrightarrow{x} = (1, r(\theta_0))^T$, $r(\theta) = \frac{g_\Delta(\theta)}{g_\Sigma(\theta)}$

This is the "Monopulse Ratio" reformulation of the observation equation. Under this formulation, the possible unknown parameters are $\left\{\sigma_n^2, r, \overrightarrow{\beta}\right\}$, $\overrightarrow{\beta} = (\beta_1, .., \beta_I)^T$, and the final form of (5) depends on whether the noise power (σ_n^2) is an unknown parameter (7) or not (8) [8]:

$$GLRT \Longleftrightarrow \frac{Tr\left(\widehat{\mathbf{R}}\right) + \sqrt{Tr\left(\widehat{\mathbf{R}}\right)^2 - 4\left|\widehat{\mathbf{R}}\right|}}{2\widehat{\sigma}_n^2} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{7}$$

$$GLRT \Longleftrightarrow \frac{Tr\left(\widehat{\mathbf{R}}\right) + \sqrt{Tr\left(\widehat{\mathbf{R}}\right)^2 - 4\left|\widehat{\mathbf{R}}\right|}}{2\sigma_n^2} \underset{H_0}{\overset{H_1}{\gtrless}} T \tag{8}$$

where: $\widehat{\mathbf{R}} = \frac{1}{I}\sum_{i=1}^{I} \overrightarrow{v}(t_i)\,\overrightarrow{v}(t_i)^H$

In both cases:

$$\widehat{r} = \frac{Tr\left(\widehat{\mathbf{R}}\right) + \sqrt{Tr\left(\widehat{\mathbf{R}}\right)^2 - 4\left|\widehat{\mathbf{R}}\right|} - 2\left\|\overrightarrow{\Sigma}\right\|^2}{2\overrightarrow{\Delta}^H\overrightarrow{\Sigma}} \tag{9}$$

Form (7) of GLRT is a constant false alarm rate (CFAR) detector which assesses the noise power (σ_n^2) under H_0 and H_1 [8] using the smallest eighenvalue of $\widehat{\mathbf{R}}$:

$$\widehat{\sigma}_n^2 = \frac{Tr\left(\widehat{\mathbf{R}}\right) - \sqrt{Tr\left(\widehat{\mathbf{R}}\right)^2 - 4\left|\widehat{\mathbf{R}}\right|}}{2}$$

As most of CFAR process, its performance (P_D vs. P_{FA}) is poor for small number of snapshots. This is the reason why σ_n^2 estimation is always performed at a different stage of the processing, generally at the output of the Matched Filter, where a large amount of samples is available. Therefore, hereinafter, it is assumed that σ_n^2 can be estimated precisely enough to be a known parameter of observation model (6) leading to form (8) of GLRT. It is worth noticing that in case of Rayleigh signal source (8) and (9) have the same form [2].

For completeness, let's mention that in the particular case where $\beta_1 = \beta_2 = .. = \beta_I$ (non fluctuating case), expressions (7), (8), (9) and $\widehat{\sigma}_n^2$ are different [8]. From an operational point of view, it is preferable not to use expressions derived in that case because of their specificity and the dubious validity of the hypothesis of non fluctuating signal source in most cases.

3.3 Practical GLRT approximations

Except for case $I = 1$, where:

$$GLRT \Longleftrightarrow |\Delta|^2 + |\Sigma|^2 \underset{H_0}{\overset{H_1}{\gtrless}} T,\ \widehat{r} = \frac{\Delta}{\Sigma} \tag{10}$$

the exact solution of the CHTP, forms (8) of the GLRT and (9) of the MLE of r, is unpractical for establishing analytical results. Although the computing power of today's computers

allows a precise study of its performance through a Monte-Carlo type simulation with a large number of draws, it is always interesting to be able to establish analytical results based on approximated solutions which may be used as calibration tools for this type of simulation (number of draws necessary for a representative measurement).
The usual "historical" approximation consists in restricting the use of the difference channel Δ to computation of MLE of r only, where detection is achieved using the sum channel Σ only. Under this assumption, the samples which pass the detection test and participate in the estimation process mostly belong to the sum beam width (see figure 1) and verify $\left\|\overrightarrow{\Sigma}\right\|^2 > \left\|\overrightarrow{\Delta}\right\|^2$. In this case [2]:

$$GLRT \Longleftrightarrow \left\|\overrightarrow{\Sigma}\right\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} T,\ \widehat{r} \approx \frac{\overrightarrow{\Sigma}^H \overrightarrow{\Delta}}{\left\|\overrightarrow{\Sigma}\right\|^2} \tag{11}$$

This approximated form of r was introduced by Mosca [3] as the solution of "the problem of estimation of angle of arrival in amplitude comparison monopulse radars", but with no reference to the associated detection test (see introduction).
A more global approach is the theoretical approach disclosed above. It leads to a symmetrical form (relative to $\overrightarrow{\Delta}$ and $\overrightarrow{\Sigma}$) of the GLRT (8) and therefore suggests an approximation based on a symmetrical criterion, such as the correlation of the 2 channels under H_1 with $\left\|\overrightarrow{\beta}\right\|$ large. In this case $\frac{\left|\overrightarrow{\Sigma}^H\overrightarrow{\Delta}\right|^2}{\left\|\overrightarrow{\Sigma}\right\|^2\left\|\overrightarrow{\Delta}\right\|^2} \approx 1$ and [2]:

$$GLRT \Longleftrightarrow Tr\left(\widehat{\mathbf{R}}\right) \underset{H_0}{\overset{H_1}{\gtrless}} T,\ \widehat{r} \approx \frac{\left\|\overrightarrow{\Delta}\right\|^4 + \left|\overrightarrow{\Sigma}^H\overrightarrow{\Delta}\right|^2}{\overrightarrow{\Delta}^H\overrightarrow{\Sigma}\left(\left\|\overrightarrow{\Sigma}\right\|^2 + \left\|\overrightarrow{\Delta}\right\|^2\right)} \tag{12}$$

Under this form, the GLRT becomes a simple quadratic detector based on the use of the energy available on the 2 reception channels. The detection performance (P_D vs. P_{FA}) of this type of detector are well known (2I order non central Chi-Square laws). However, the form of $\widehat{r}$ obtained is not a great deal simpler than (9). It is simplified when $\left\|\overrightarrow{\Sigma}\right\|^2 > \left\|\overrightarrow{\Delta}\right\|^2$. We then have again the form (11) of $\widehat{r}$ and (12) becomes:

$$GLRT \Longleftrightarrow \left\|\overrightarrow{\Sigma}\right\|^2 + \left\|\overrightarrow{\Delta}\right\|^2 \underset{H_0}{\overset{H_1}{\gtrless}} T,\ \widehat{r} \approx \frac{\overrightarrow{\Sigma}^H \overrightarrow{\Delta}}{\left\|\overrightarrow{\Sigma}\right\|^2} \tag{13}$$

A large number of Monte-Carlo simulations have shown [8] that solution (13) offers better performances than solution (12) over the complete main lobe of channel Σ: same P_D but lower RMSE (see figure 2 for an example). Solution (13) is therefore a better solution of the CHTP for which an analytical formulation of the performances has been derived. We shall designate hereinafter the various solutions (8-9) (11) (12) and (13) of the CHTP as "exact glrt", "mosca sum", "power glrt", "mosca power", respectively.

4. STATISTICAL PREDICTION

Assessing the statistical performances of the CHTP requires a joint analysis of the performance of the detector (GLRT) and the MLEs of the unknown parameters. It is indeed the expressions of the unknown parameters estimators which determine the form of the GLRT, which in turns selects (conditions) the observations participating in the estimation. Thus, in strict logic, studying the performances (mean, variance) of the MLEs should make use of conditional expectation, as the estimation is conditioned by the detection test. This aspect is seldom covered in the open literature, including reference works [6] (and others) where detection performance and estimation performance are covered as separable problems. The main reason is probably the fact that the formulation and assessment complexity increases significantly in the general case. Further, this approximation is fully justified when the detection probability is close to 1, i.e. for SNRs high "enough" - per the detection test.
$E(\mathrm{Re}\{\widehat{r}\} \mid D)$ and $Var(\mathrm{Re}\{\widehat{r}\} \mid D)$ can be computed from $E(\widehat{r} \mid D)$, $E(|\widehat{r}|^2 \mid D)$, $E(\widehat{r}^2 \mid D)$ using the following identities:

$$\begin{aligned} \mathrm{Re}\{\widehat{r}\}^2 &= \frac{1}{2}\left[|\widehat{r}|^2 + \mathrm{Re}\{\widehat{r}^2\}\right] \\ Var(\mathrm{Re}\{\widehat{r}\}) &= E\left(\mathrm{Re}\{\widehat{r}\}^2 \mid D\right) - \mathrm{Re}\{E(\widehat{r} \mid D)\}^2 \end{aligned}$$

which also enables to assess statistical prediction of $\mathrm{Im}\{\widehat{r}\}$ (see [5](ref.[12],[27]) for applications). For sake of simplicity in formulas, we assume that the noise has been normalized ($\sigma_n^2 = 1$). Characterization of solution "mosca sum" in the general case of colored noise has been covered in [4] ($I = 1$) and [7] ($I \geq 1$). In the case of white noise, it comes:

$$D = \left\{\overrightarrow{V} \mid \left\|\overrightarrow{\Sigma}\right\|^2 \geq T\right\}$$

$$P_{FA} = e^{-T} e_{I-1}(T), \quad P_D = \int_{t \geq T} f_{\chi_2^I}\left(t, I\sigma_\beta^2, 1\right) dt$$

$$E(\widehat{r} \mid D) = \mu \int_{t \geq T} \frac{f_{\chi_2^{I+1}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t} dt$$

$$E\left(\widehat{r}^2 \mid D\right) = \mu^2 \int_{t \geq T} \frac{f_{\chi_2^{I+2}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t^2} dt$$

$$\begin{aligned} E\left(|\widehat{r}|^2 \mid D\right) = &\int_{t \geq T} \frac{f_{\chi_2^I}\left(t, I\sigma_\beta^2, 1\right)}{P_D t} dt \\ &+ \lambda \int_{t \geq T} \frac{f_{\chi_2^{I+1}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t^2} dt \\ &+ |\mu|^2 \int_{t \geq T} \frac{f_{\chi_2^{I+2}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t^2} dt \end{aligned}$$

where: $\mu = rI\sigma_\beta^2$, $\quad \lambda = |r|^2 I\sigma_\beta^2$, $\quad \sigma_\beta^2 = \frac{1}{I}\left(\sum_{i=1}^{I} |\beta_i|^2\right)$.

In the case of solution "mosca power" [8]:

$$D = \left\{\overrightarrow{V} \mid \left\|\overrightarrow{\Sigma}\right\|^2 + \left\|\overrightarrow{\Delta}\right\|^2 \geq T\right\}$$

$$P_{FA} = e^{-T} e_{2I-1}(T)$$

$$P_D = \iint_{x+t \geq T} f_{\chi_2^I}\left(x, I\sigma_\beta^2 |r|^2, 1\right) f_{\chi_2^I}\left(t, I\sigma_\beta^2, 1\right) dx dt$$

$$E(\widehat{r} \mid D) = \mu \iint_{x+t \geq T} f_{\chi_2^{I+1}}\left(x, I\sigma_\beta^2 |r|^2, 1\right) \frac{f_{\chi_2^{I+1}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t} dx dt$$

$$E\left(\widehat{r}^2 \mid D\right) = \mu^2 \iint_{x+t \geq T} f_{\chi_2^{I+2}}\left(x, I\sigma_\beta^2 |r|^2, 1\right) \frac{f_{\chi_2^{I+2}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t^2} dx dt$$

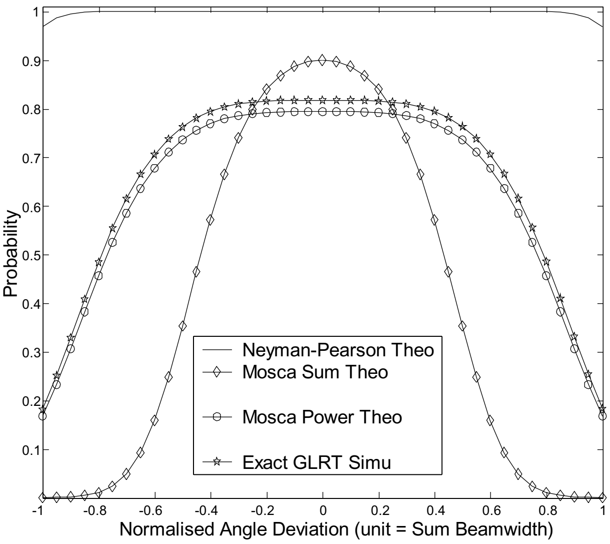

Figure 1: Probability of Detection, $P_{FA} = 10^{-4}$

$$
\begin{aligned}
E\left(|\widehat{r}|^2 \mid D\right) = & \iint\limits_{x+t\geq T} f_{\chi_2^{I+1}}\left(x, I\sigma_\beta^2 |r|^2, 1\right) \frac{f_{\chi_2^{I}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t} dxdt \\
& +\lambda \iint\limits_{x+t\geq T} f_{\chi_2^{I+2}}\left(x, I\sigma_\beta^2 |r|^2, 1\right) \frac{f_{\chi_2^{I+1}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t^2} dxdt \\
& +|\mu|^2 \iint\limits_{x+t\geq T} f_{\chi_2^{I+2}}\left(x, I\sigma_\beta^2 |r|^2, 1\right) \frac{f_{\chi_2^{I+2}}\left(t, I\sigma_\beta^2, 1\right)}{P_D t^2} dxdt
\end{aligned}
$$

The above expressions ("mosca power") are simple to compute [8]. When $I \geq 2$, they can all be reduced to simple convergent integrals of bounded functions and assessed using numerical integration. The only difficulty arises when $I = 1$ for computing $E\left(|\widehat{r}|^2 \mid D\right)$ which requires the evaluation of a simple integral on domain $[0, T]$ of a unbounded function.

5. PERFORMANCE COMPARISON

As an example of performance comparison, we consider the multifunction Radar case. Due to time budget constraint, the maximum number of observations available per target is generally 2 ($I = 2$). A likely probability of false alarm is $P_{FA} = 10^{-4}$. The Signal to Noise Ratio (SNR) is adapted to obtain $P_D = 0.9$ when signal source is on boresight and detected on Σ channel only. The monopulse antenna model corresponds to a rectangular surface sum antenna (1° beamwidth) with a plane surface uniform current distribution associated with an appropriate difference beam. Figure (1) and (2) depicts respectively the variation of P_D and RMSE within Σ channel main lobe, according to (detector,estimator) solution pair of the CHTP. In figures (1) and (2) "Theo" and "Simu" stands for Theoretical (assessed using analytical formula) and Simulation (assessed using Monte-Carlo runs). All P_{FA} measurements has been performed on 10^9 independent trials. All P_D and RMSE measurements has been performed on 10^6 independent trials. The two figures illustrates the on average superiority of "mosca power" solution over "mosca sum" solution (almost equal RMSE and improved on average P_D), and additionally

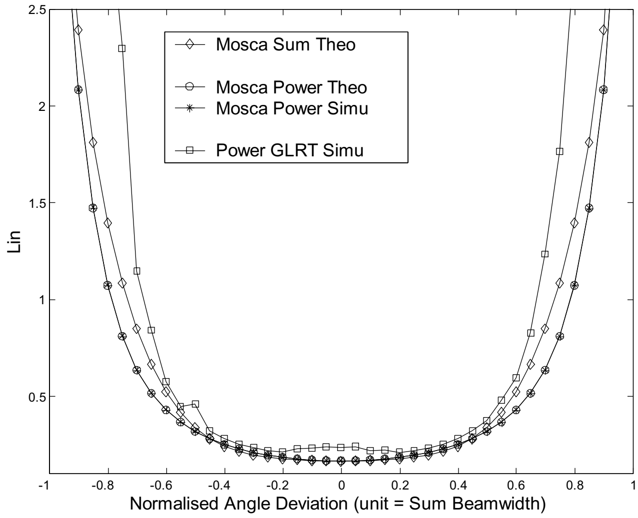

Figure 2: Conditional RMSE, $P_{FA} = 10^{-4}$

demonstrate the perfect adequacy between simulations and theoretical formulas derived for "mosca power" solution.

6. CONCLUSION

This paper, generalizing results derived in [2], emphasizes the existence of a better (detector, estimator) solution pair of the monopulse antenna CHTP, whatever the amplitude fluctuation law, and sets forth its analytical characterization. In addition to the expected impact on the future implementation of monopulse antennas, it contributes to illustrate the often unacknowledged or underestimated interaction between the components of the (detector, estimator) solution pairs of the CHTP. This is particularly true in real systems (radar, telecoms, sonar) where the (contractual) operating area of interest seldom corresponds to $P_D \approx 1$, which is the only case where detection and estimation are disconnected problems.

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