AN EFFICIENT FAST-FADING CHANNEL ESTIMATION AND EQUALIZATION METHOD WITH SELF ICI CANCELLATION

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ABSTRACT

OFDM systems are vulnerable to time-selective fading effect, due to the relatively long symbol duration, compared with single-carrier systems. This results in non-negligible inter-carrier interference (ICI). This paper proposes an effective frequency-domain pilot-symbol-aided channel estimation method for fast-fading channels. By first observing high correlation between ICIs of adjacent subcarriers, we then devise an inter-carrier cancellation scheme. From the ICI-reduced signal, we are able to achieve good channel estimation and equalization accuracy. Simulations show that the new method can effectively combat fast-fading channel conditions. In addition, under the condition of using less pilots, the proposed method still can generate better SER than the current method [1].

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technology for broadband transmission. It has been adopted in state-of-art communication standards. Under the condition of same data rate, the symbol of an OFDM system is much longer than that of a single-carrier (SC) system. Therefore, OFDM systems are less sensitive to inter-symbol interference (ISI) than SC systems, especially with the insertion of a guard interval (GI) in between two consecutive OFDM symbols. If GI is loaded with CP and the channel delay spread is shorter than the CP length, then multi-path channel equalization can be easily implemented with 1-tap division in the frequency domain. Therefore, OFDM system is very robust to frequency-selective fading.

However, OFDM systems with long symbol durations are more vulnerable to time-selective fading than SC systems. This is specially the case in mobile environments and closely related to Doppler spread. Under this condition, the orthogonality between subchannels cannot be maintained and the inter-carrier interference (ICI) will be introduced. ICI will decrease the signal to interference ratio (SIR). Low SIR will introduce an error floor in signal detection.

To mitigate the interference effect, several channel estimation and equalization methods have been proposed for time-variant channels. In [2] and [3], time-domain and frequency-domain compensation techniques, respectively, are proposed to reduce the distortion. However, these two approaches assume flat Rayleigh fading channels. For frequency-selective multipath fading channels, [4] proposes a frequency-domain equalization technique to reduce the Doppler-induced ICI. However, this approach needs a time-domain pilot signal inserted in data stream to get the channel variation information. It is also sensitive to timing error. The frequency-domain pilot-symbol-aided estimation method [1] gives a good modeling of a time-variant channel, and solves channel parameters effectively. However, the estimation is still significantly affected by ICI. Based on this approach, in this work we propose an ICI-reduced channel estimation method for fast-fading channels, by utilizing the high correlation between subcarrier ICIs. Simulations show better performance than the mentioned methods.

2. ESTIMATION OF FAST-FADING CHANNELS

2.1 OFDM system model

The transmitted signal of an OFDM system can be expressed by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} d(k) e^{j2\pi nk/N} \qquad (1)$$

where $d(k)$ is the transmitted data on the k-th subcarier and N is the total number of subcarriers. We assume the discrete time-variant channel impulse response is

$$h(n,m) = \sum_{l=0}^{\nu-1} \alpha_l(n)\delta(m-l) \qquad (2)$$

where $\alpha_l(n)$ is the time-variant gain of the l-th path and ν is the number of paths. If the length of channel impulse response is shorter than the CP, the received time-domain signal is

$$r(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(n,k) \times d(k) \times e^{j\frac{2\pi nk}{N}} + \tilde{n}(n); \ n=0,1,\cdots,N-1 \qquad (3)$$

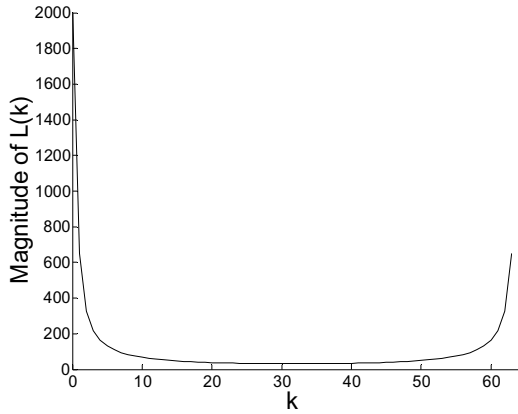where $\tilde{n}(n)$ is the additive white Gaussian noise and $H(n,k)$ is the time-variant channel frequency response:

Figure 1. ICI gain function $L(k)$

$$H(n,k) = \sum_{m=0}^{N-1} h(n,m)e^{-j2\pi mk/N}$$

$$= \sum_{l=0}^{\nu-1} \alpha_l(n)e^{-j2\pi lk/N}; \quad k = 0,1,\cdots,N-1 \qquad (4)$$

Then DFT of the received signal $r(n)$ is

$$Y(m) = DFT\{r(n)\}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} d(k) \times DFT\{H(n,k)\} * \delta(m-k) + DFT\{\tilde{n}(n)\}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} d(k) \times \tilde{H}(m,k) * \delta(m-k) + \tilde{N}(m) \qquad (5)$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} d(k) \times \tilde{H}(m-k,k) + \tilde{N}(m) \quad m = 0,1,\cdots,N-1$$

By this equation, it is obvious that the received signal on the m-th subcarrier not only consists of the desired $d(m)$, but all the $d(k)$'s from other subcarriers. The inter-carrier component in the received signal is

$$ICI(m) = \frac{1}{N}\sum_{k \neq m} d(k) \times \tilde{H}(m-k,k) \qquad (6)$$

If the channel is fixed over an OFDM symbol, then (5) reduces to

$$Y(m) = d(m) \cdot \sum_{l=0}^{\nu-1} \alpha_l e^{-j2\pi lm/N} = d(m) \cdot G(m) \qquad (7)$$

It is verified in [4] and [6] that if $f_d T$ is less than 0.1, the channel variation over an OFDM symbol can be assumed linear, where f_d is the Doppler frequency and T is the OFDM symbol duration. Here, we only consider this relaxed case, because it represents common fading situations. For faster fading conditions, the following demonstration also applies, assuming higher-order non-linear model. With this assumption, the time-variant channel impulse response (2) can be rewritten as

$$h(n,m) = \sum_{l=0}^{\nu-1}(s_l n + a_l)\delta(m-l) \qquad (8)$$

and equation (5) can be rewritten as

$$Y(m) = \frac{1}{N}\sum_{l=0}^{\nu-1} s_l \sum_{k=0}^{N-1} d(k)L(m-k)e^{-j\frac{2\pi lk}{N}}$$

$$+ d(m)\sum_{l=0}^{\nu-1} a_l e^{-j\frac{2\pi lk}{N}} + \tilde{N}(m) \quad m = 0,1,\cdots,N-1 \qquad (9)$$
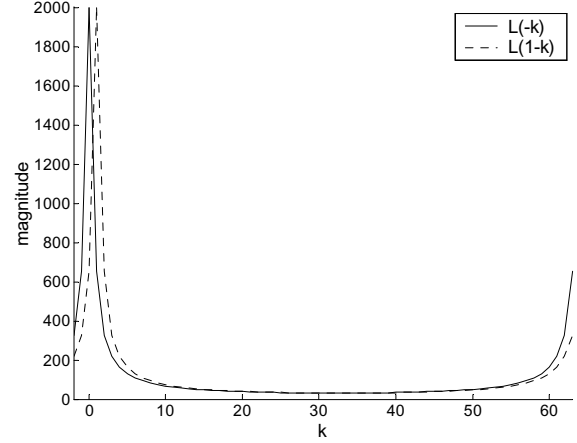

Figure 2. Comparison between ICI gains of adjacent subcarriers

where s_l and a_l are the variation slope and initial value of the path gain, respectively, and $L(k)$ is

$$L(k) = DFT\{n\} = \begin{cases} N(N-1)/2, & k = 0 \\ \dfrac{jN}{2\sin(\frac{\pi k}{N})}e^{j\pi k/N}, & k = 1,\cdots,N\text{-}1 \end{cases} \qquad (10)$$

2.2 Stamoulis' method [1] for channel estimation

Stamoulis' approach is an efficient channel estimation method [1] for fast-fading channels as detailed below. We will later modify and improve its performance. First, we assume that there are P pilot data placed on the subcarriers with indices $p(q)$ $(q = 0,1,\ldots,P\text{-}1)$, then

$$Y(p(q)) = \frac{1}{N}\sum_{l=0}^{\nu-1} s_l \sum_{k \in pilot} d(p(q))L(p(q)-k)e^{-j\frac{2\pi lk}{N}}$$

$$+ \frac{1}{N}\sum_{l=0}^{\nu-1} s_l \sum_{k \notin pilot} d(p(q))L(p(q)-k)e^{-j\frac{2\pi lk}{N}} \qquad (11)$$

$$+ d(p(q))\sum_{l=0}^{\nu-1} a_l e^{-j\frac{2\pi lk}{N}} + \tilde{N}(p(q)) \quad q = 0,1,\cdots,P-1$$

Since the transmitted data except pilots are unknown to the receiver, ICI contributed by non-pilot subcarriers can be collectively treated as a single error term. Therefore, (11) can be rewritten as

$$Y(p(q)) = \frac{1}{N}\sum_{l=0}^{\nu-1} s_l w_s(q,l) + \sum_{l=0}^{\nu-1} a_l w_a(q,l) + Er(q) \qquad (12)$$

where

$$Er(q) = \frac{1}{N}\sum_{l=0}^{\nu-1} s_l \sum_{k \notin pilot} d(p(q))L(p(q)-k)e^{-j\frac{2\pi lk}{N}} + \tilde{N}(p(q))$$

$$w_s(q,l) = \sum_{k \in pilot} d(p(q))L(p(q)-k)e^{-j\frac{2\pi lk}{N}}$$

$$w_a(q,l) = d(p(q))e^{-j\frac{2\pi lk}{N}} \qquad q = 0,1,\cdots,P-1$$

There are 2ν unknown parameters (if neglecting the error term), in the equation. Therefore, if $P \geq 2\nu$, then these parameters can be solved by the following linear system.
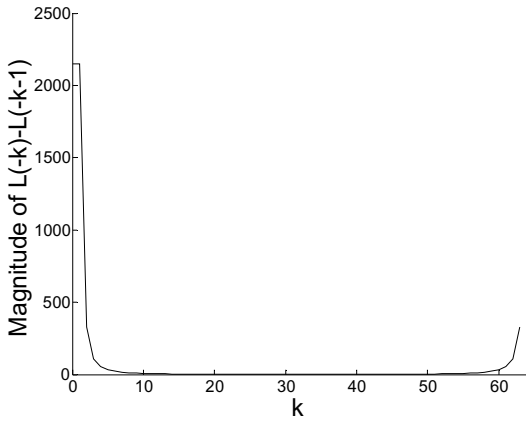
Figure 3. ICI distribution after taking difference between adjacent subcarriers

$$\begin{bmatrix} Y(p(0)) \\ Y(p(1)) \\ \vdots \\ Y(p(P-1)) \end{bmatrix} = \begin{bmatrix} w_s(0,0) & \cdots & w_a(0,2\nu-1) \\ w_s(1,0) & \cdots & w_a(1,2\nu-1) \\ \vdots & \ddots & \vdots \\ w_s(P-1,0) & \cdots & w_a(P-1,2\nu-1) \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ a_{\nu-1} \end{bmatrix} + \begin{bmatrix} Er(0) \\ Er(1) \\ \vdots \\ Er(P-1) \end{bmatrix}$$

$$\Rightarrow \quad \overline{\mathbf{Y}}_p = \overline{\mathbf{WSA}} + \overline{\mathbf{E}} \tag{13}$$

Thus, the least-square (LS) estimate of the channel parameters can be obtained by

$$\overline{\mathbf{SA}} = \overline{\mathbf{W}}^{-1} \overline{\mathbf{Y}}_p \tag{14}$$

where $\overline{\mathbf{W}}^{-1}$ is the pseudo inverse of $\overline{\mathbf{W}}$. We can rewrite (5) in a matrix form as

$$\overline{\mathbf{Y}} = \overline{\mathbf{H}}\overline{\mathbf{D}} + \overline{\mathbf{N}} \tag{15}$$

With the solved parameters, we can use (4), (5) and (8) to determine $\overline{\mathbf{H}}$. Then the transmitted data can be solved by the LS estimate:

$$\overline{\mathbf{D}} = \overline{\mathbf{H}}^{-1} \overline{\mathbf{Y}} \tag{16}$$

In (12), the ICI from non-pilot subcarriers is treated as noise. To reduce the noise, pilot placement is very critical to accurate channel estimation. If the channel variation is not too fast, the ICI of a subcarrier mostly comes from its adjacent subcarriers. Therefore, to reduce the error term, pilots should be placed on subcarriers close to each other. However for time-invariant frequency-selective fading channels, [5] shows that pilot tones should be equispaced and evenly spread to all subcarriers to get the best performance. Under these two considerations, [1] suggests that pilot tones be better split into equispaced groups of subcarriers.

3. THE PROPOSED ESTIMATION SCHEME

Since the term $Er(q)$ of (12) due to non-pilot subcarriers contributes to noise and ICI, it should be minimized as possibly as it can be. Here, by utilizing strong correlation between adjacent ICI subcarriers, we propose an ICI cancellation scheme to reduce the error, for a better channel estimation.

First, from (9), we can find that the ICI components contributed by different subcarriers are mainly determined by the function $L(k)$. Figure 1 shows $L(k)$ magnitude, assuming $N = 64$. It reveals that the power of ICI mostly comes from adjacent subcarriers and ICIs contributed by faraway subcarriers
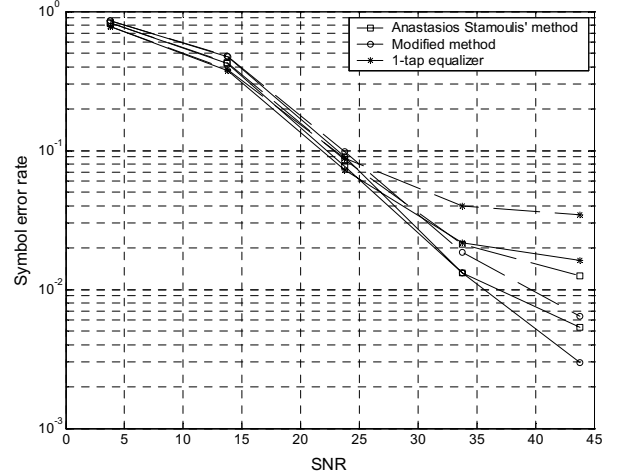

Figure 4. SER performances of Stamoulis's method and the proposed method, 4ν pilots, (solid: $f_d T = 0.04$ dashed: $f_d T = 0.06$)

are almost the same. Based on this observation, we can perform ICI cancellation using adjacent ICI components as discussed below. Let's first rewrite the ICI component of a subcarrier (6) as

$$ICI(m) = \frac{1}{N} \sum_{l=0}^{\nu-1} s_l \sum_{k \neq m} L(m-k) d(k) e^{-j\frac{2\pi lk}{N}} \tag{17}$$

In (17), ICI($m1$) will be very close to ICI($m2$), if $L(m1\text{-}k)$ is close to $L(m2\text{-}k)$. This is particularly the case when $m1$ and $m2$ are close to each other, as depicted in Fig. 2, assuming $m1=0$ and $m2=1$. This figure shows that $L(-k)$ and $L(1\text{-}k)$ are almost the same for $4 \leq k \leq 61$. This implies that for adjacent subcarriers, ICIs contributed by distant subcarriers are almost identical. Under this condition, we can take the difference between the adjacent subcarriers of the received signals, i.e., $Y(m)\text{-}Y(m+1)$, as described by (18), which leads to $L(m\text{-}k)\text{-}L(m+1\text{-}k)$ for the ICI term. As a result, the ICI will be significantly reduced, except those ICI from the subcarriers close to the m-th subcarrier.

$$Y(m) - Y(m+1) = \frac{1}{N} \sum_{l=0}^{\nu-1} s_l \sum_{k=0}^{N-1} d(k) \left[L(m-k) - L(m+1-k) \right] e^{-j\frac{2\pi lk}{N}} \tag{18}$$

$$+ \sum_{l=0}^{\nu-1} a_l \left[d(m) e^{-j\frac{2\pi lm}{N}} - d(m+1) e^{-j\frac{2\pi l(m+1)}{N}} \right] + \tilde{N}(m) - \tilde{N}(m+1)$$

Fig. 3 shows the ICI gain distribution after the differential operation of $L(-k)\text{-}L(1\text{-}k)$. Since $Y(m)\text{-}Y(m+1)$ can be expressed as a linear function of parameters a_l's and s_l's, we can solve these parameters using (18) instead of (13).

We can fully utilize the correlation between all subcarriers and further reduce the ICI term by linearly combining more than two pilot tones as

$$Y'(p(q)) = \sum_{s=0}^{P-1} g_{q,s} Y(p(s))$$

$$= \frac{1}{N} \sum_{l=0}^{\nu-1} s_l \left\{ \sum_{k=0}^{N-1} d(k) \left[\sum_{s=0}^{P-1} g_{q,s} L(p(s)-k) \right] e^{-j\frac{2\pi lk}{N}} \right\} \tag{19}$$

$$+ \sum_{l=0}^{\nu-1} a_l \left[\sum_{s=0}^{P-1} g_{q,s} d(p(s)) e^{-j\frac{2\pi lp(s)}{N}} \right] + \sum_{s=0}^{P-1} g_{q,s} \tilde{N}(p(s))$$
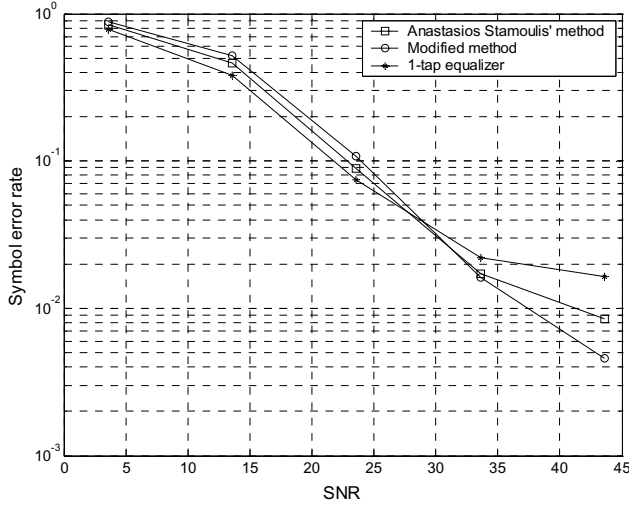
(19) forms a new linear system as shown below:

Figure 5. SER performances of Stamoulis's method and the proposed method, 3ν pilots, $f_d T = 0.04$

$$
\begin{bmatrix} Y'(p(0)) \\ Y'(p(1)) \\ \vdots \\ Y'(p(P-1)) \end{bmatrix} = \begin{bmatrix} w_s'(0,0) & \cdots & w_a'(0,2\nu-1) \\ w_s'(1,0) & \cdots & w_a'(1,2\nu-1) \\ \vdots & \ddots & \vdots \\ w_s'(P-1,0) & \cdots & w_a'(P-1,2\nu-1) \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ a_{\nu-1} \end{bmatrix} + \begin{bmatrix} Er'(0) \\ Er'(1) \\ \vdots \\ Er'(P-1) \end{bmatrix}
$$
(20)

If $g_{q,s}$ is properly chosen, then the ICI can be significantly reduced. Therefore, one can estimate the channel parameters more accurately than from (18). However, the self-ICI cancellation operations also alter the error term $Er(q)$. Hence, the optimization of $g_{q,s}$ should make the influence of error term $Er'(q)$ as small as possible. Since the ICI correlation between pilot tones from different pilot groups is small, the combination set can be narrowed down to the group of pilots close to the object pilot subcarrier. As a result, we can define the following optimization cost function:

$$
\arg \min_{g_{q,s}} \left\{ \sum_{k \notin pilot} \left| \sum_s g_{q,s} L(p(s)-k) \right|^2 \Big/ \sum_{k \in pilot} \left| \sum_s g_{q,s} L(p(s)-k) \right|^2 \right\} \quad (21)
$$

This cost function can be optimized by existing numerical methods. In order to get a better result, the optimization can assume a different set of $g_{q,s}$'s for each pilot subcarrier group. Note that values of $g_{q,s}$'s also affect the second term in (19). In the worst case, this may lead to a very small $a_l \left[\sum_{s=0}^{P-1} g_{q,s} d(p(s)) e^{-j\frac{2\pi l p(s)}{N}} \right]$ term. As such, estimation of a_l's would be noisy. To compensate this effect, one can devise particular $d(m)$'s for pilot subcarriers, so as to produce significant coefficients $\sum_{s=0}^{P-1} g_{q,s} d(p(s)) e^{-j\frac{2\pi l p(s)}{N}}$ of a_l's. This is simpler than re-optimization of $g_{q,s}$'s. Here we find appropriate pilot data $d(m)$'s by exhaustive search.

4. SIMULATIONS AND COMPARISON

In the simulations, we assume 16QAM $d(m)$, $N = 64$, the total bandwidth is 500KHz, the sampling period $T_c = 2\mu s$,

the length of cyclic prefix $T_g = 4T_c$, and the number of delay paths $\nu = 4$. It is also assumed that the path gains follow the exponential-decay power profile, and the last path power is 20dB below the first path.

As for the pilot placement, similar to [1], we assume total 4ν pilot subcarriers with ν groups, which are equispaced in the DFT grid (i.e., the subcarriers with indices {0, 1, 2, 3}, {16, 17, 18, 19}, {32, 33, 34, 35}, and {48, 49, 50, 51} are pilot tones in groups). To evaluate the performance, we assume three different conditions of $f_d T = 0.02$, $f_d T = 0.04$ and $f_d T = 0.06$. For the case of $f_d T = 0.02$, SER performance of our method is almost the same as that of [1], because the ICI effect is very slight in this case. However, for faster time-variant channel conditions of $f_d T = 0.04$ or $f_d T = 0.06$, our method has better performance, especially at high SNR, as shown in Figure 4. In the figure, result due to conventional LMMSE channel equalizer (1-tap) is also included for comparison. Obviously, the conventional equalizer has an error floor due to the ICI noise. Figure 5 shows that the performance comparison for the case of 3ν pilots. In this case, the proposed method has a more significant improvement over the Stamoulis' method, than the condition of pilots.

5. CONCLUSION

This work proposes an efficient fast-fading channel estimation method, which has better performance than the current design. This method utilizes the correlation between subcarriers to reduce ICI in channel. As such, better channel estimates can be achieved than those without ICI cancellation. One can further improve the performance by also taking into account of the noise and channel statistics. This is a challenging and interesting problem which is currently under investigation.

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