POLYNOMIAL PHASE SIGNAL PROCESSING VIA WARPED HIGH-ORDER AMBIGUITY FUNCTION

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ABSTRACT

The high-order ambiguity function (HAF) was introduced for the estimation of polynomial-phase signals (PPS). Currently the HAF suffers from noise-masking effects and from the appearance of undesired cross terms in the presence of multi-components PPS. The multi-lag product HAF (PHAF) concept was then proposed as a way to improve the performances of the HAF. Nevertheless, the "optimal" choice of lag sets implies many tries, undesirable in an automatically signal processing context. On the other hand, multiplying many mlHAFs might lead to abnormal results.

In this paper we propose a warped-based algorithm in order to accurately estimate the coefficients of the polynomial phase. We compute the HAF for different lag values. Knowing the variation law of the frequency with respect to these values, we can construct a warping function leading to a linear dependence between the HAF maxima coordinates and the lag set.

1. INTRODUCTION

Signals encountered in engineering applications, such as communications, radar or sonar often involve amplitude (AM) and/or frequency modulation (FM). It is practically observed that the phase function of a large class of AM-FM processes can be modeled by a polynomial function of *t*. Consequently, these processes are called Polynomial-Phase Signals (PPSs).

This class of signals provides a good model in a variety of applications, such as synthetic aperture radar (SAR) imaging and mobile communications.

Single-component PPSs have been extensively investigated in recent years using *the high-order ambiguity function* (HAF) introduced by Peleg and Porat [1]. HAF has proved to be an efficient tool for parameter estimation of single-component PPS.

Signals rising from real life applications have often multiple components and their parameter estimation poses a great challenge. When HAF is applied to multi-component PPS there are introduced a large number of cross-terms which are themselves PPS. Consequently, some techniques have been proposed, in order to reduce the influence of these cross-terms [1], [2]. Generally, these approaches perform very well for a signal-to-noise ratio (SNR) superior to 10 dB. In order to enhance the noise robustness, a possible solution is to multiply the HAFs obtained for some lag sets (Product HAF - PHAF). This method improves also the performances related to noise robustness and cross-terms [3].

In this paper, we propose a warped-based algorithm to accurately estimate the polynomial phase coefficients for a multi-component PPS for SNR inferior to 10 dB. In section 2, we introduce the HAF and illustrate the cross-terms apparition in the multi-component PPS case. Furthermore, we present the multi-lag PHAF concept. In section 3, we present briefly the unitary transformation principle. Its application in polynomial coefficients estimation is illustrated in section 4. The section 5 - "Conclusion" will close this discussion.

2. PRODUCT HIGH-ORDER AMBIGUITY FUNCTION

As it was illustrated in [1], [2], the classical HAF algorithm presents some limitations, related to the *noise robustness* and the *cross-terms presence*. In order to solve these aspects, the concept of *multi-lag HAF* (mlHAF) has been initially proposed in [2]. In fact, the mlHAF is based on the generalization of the high-order instantaneous moment HIM [2] :

$$HIM_K\left[s(t);\boldsymbol{\tau}_{K-1}\right]=HIM_{K-1}\left[s(t+\tau_{K-1});\boldsymbol{\tau}_{K-2}\right]HIM^*_{K-1}\left[s(t-\tau_{K-1});\boldsymbol{\tau}_{K-2}\right] \quad (1)$$

where $\boldsymbol{\tau}_\mathbf{i}=(\tau_1,\tau_2,...,\tau_i)$ is the lag set. Applying the Fourier transform to (1), we obtain the ml-HAF of the signal $s(t)$:

$$mlHAF_K\left[s;\alpha,\boldsymbol{\tau}\right]=\int_{-\infty}^{\infty}HIM_K\left[s(t);\boldsymbol{\tau}\right]e^{-j\alpha t}dt \quad (2)$$

Assuming a PPS model for the analyzed signal, i.e.

$$s(t)=A\exp j\phi(t)=A\exp\left[j\sum_{k=0}^{K}a_kt^k\right] \quad (3)$$

the main property of HIM is that, the K^{th} order HIM is reduced to a harmonic with amplitude $A^{2^{k-2}}$, frequency $\widetilde{\omega}$ and phase $\widetilde{\phi}$:

$$HIM_k\left[s(t);\boldsymbol{\tau}\right]=A^{2^{k-1}}\exp j\left(\tilde{\omega}_k\cdot t+\tilde{\phi}_k\right) \quad (4)$$

where $\widetilde{\omega}_k=k!\tau^{K-1}a_k$ (5).

Based on these results, Porat [2] has proposed an algorithm which estimates sequentially the coefficients $\{a_k\}$. At each step, using a spectral analysis method, we estimate the spectral peak and, using the HAF, we compute an estimation value ($\hat{a}_k$) of a_k. With this value, the effect of the phase term of the higher order is removed:

$$s^{(k-1)}(t) = s^{(k)}(t) \cdot e^{-j\hat{a}_k t^k} \quad (6)$$

Using the ml-HIM concept (relation (1)), Barbarossa and *al* [3] introduced the *Product* HAF: the mlHAFs computed, via relation (2), for different lag sets

$$\mathbf{T} = \left\{ \boldsymbol{\tau}_{K-l}^{(l)} \right\}_{l=\overline{1,L}} ; \boldsymbol{\tau}_{K-l}^{(l)} = \left\{ \tau_i \right\}_{i=\overline{1,K-1}} \quad (7)$$

are multiplied, obtaining also a more robust method and a cross-terms free representation :

$$PHAF(\alpha;\mathbf{T}) = \prod_{l=1}^{L} mlHAF_K \left[s; \frac{\prod_{i=1}^{K-1} \tau_i^{(l)}}{\prod_{i=1}^{K-1} \tau_i^{(1)}} \alpha, \boldsymbol{\tau}_{K-1}^{(l)} \right] \quad (8)$$

The simulation results given in [1] prove that the PHAF solves both noise robustness and ambiguity problems, providing as well a correct IFL estimation. Nevertheless, these results are obtained for a particular lag set, generated by empirical trials. Thence, the use of PHAF is conditioned by the selection of good lags. This task constitutes a considerable limitation in practical applications.

Otherwise, multiplying many mlHAFs could lead to abnormal situation if one of the mlHAFs involved in (8) has a low value. The results will be closed to zero as shown in the figure 1.

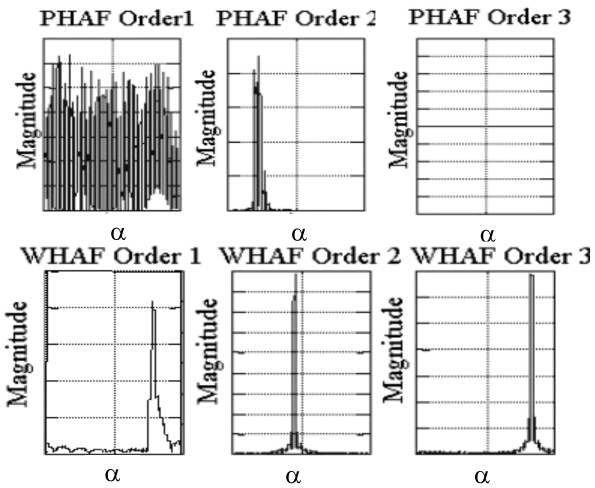

Figure 1: Example of an abnormal result related to product (8).

Supposing a 3rd order PPS given by :

$$s(t) = \exp\left[j2\pi \left(0.25t - 4.55 \cdot 10^{-4} t^2 + 1.78 \cdot 10^{-6} t^3 \right) \right] \quad (9)$$

we remark the effect of the multiplication by a mlHAF close to 0 : since the 3rd order PHAF must normally peak to a frequency related, via (5), to the 3rd order coefficient, the result is very small. It means that there is a lag set which leads to a very small mlHAF value. Furthermore, the product (8) leads to a wrong polynomial coefficient estimation.

As proposed in the section 3, this effect could be avoided by a particular set of the lag. We will see that the new method – called Warped HAF (WHAF) solves the problem caused by this effect. As depicted in figure 1, this method provides a correct estimation of signal given in (9). It gives also a general way for the lag set selection.

3. UNITARY TRANSFORM PRINCIPLE

Unitary similarity transformations furnish a simple powerful tool for generating new classes of joint distributions based on concepts different from time, frequency and scale [6]. Actually, it is possible to construct (via unitary transformation) distributions to match almost any one-to-one group delay or instantaneous frequency characteristics. One of the most used unitary transforms is the *axis transformation* [6] (or axis warping), defined for a signal $s(t)$ as an operator $\mathbf{U}$ on $L^2(\Re)$, whose effect is given by

$$(\mathbf{U}s)(x) = |w'(x)|^{1/2} s[w(x)] \quad (10)$$

where w is a smooth, one-to-one function, comprising a large subclass of unitary transformations ([5]). The functions $w(x) = e^x$ and $w(x) = |x|^k sgn(x), k \neq 0$ are examples of useful warpings. Since the resulting signal $\mathbf{U}s$ has a linear time-frequency behavior, we may say that the effect of this transform is the *signal "stationarization"* ([6]). In practical applications, the choice of the $w(t)$ might be critical if the analyzed signal has an unknown nature. This can be solved by assuming a correct model for the signal. In the next section, we choose as signal model the *polynomial* one. Its generality provides, via elementary warping operator, an attractive flexibility and an accurate signal representation in the time-frequency plane.

4. WARPED HIGH-ORDER AMBIGUITY FUNCTION

By definition [4], a *warping operator* applied to a structure which depends non-linearly on a parameter leads to a linear dependence of this parameter. Knowing the dependence law between α and τ : $\alpha_k(\tau) = k!\tau^{k-1} a_k$ and according to the warping operator principle [6], we propose to use the following set of lags :

$$\tau_w = \tau^{\frac{1}{k-1}} \quad (11)$$

In this condition, the mlHAF evaluated for this lag set will peak to an angular frequency α_w depending *linearly* of τ. The original expression $\alpha_k(\tau) = k!\tau^{k-1}a_k$, the warping law, and the new peak location in the warped frequency domain are depicted in figure 2.

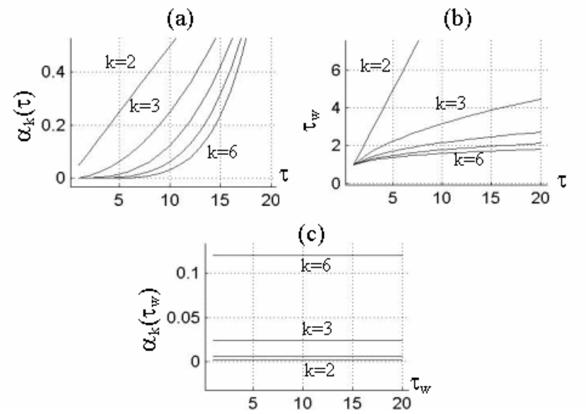

Figure 2: (a) The original frequency locations of the HAF maxima; (b) The time warping laws; (c) The new peak locations in the warped frequency domain.

The effect of the warping function (11) in the frequency-lag plane consists of disposing the mlHAFs peaks on parallel lines with the lag axis (figure 2.c). According to polynomial orders, the original peak values of the mlHAFs are placed on different curves. But, in the warped plane, these peaks correspond to the *same* frequency location. Therefore, an idea to exploit this property is to sum the mlHAFs obtained for the warped lag set. We generate also the WHAF as :

$$WHAF_k[s;\alpha] = \sum mlHAF_k[s;\alpha,\tau_w] \qquad (12)$$

which peaks at the locations $\alpha_k = k!a_k$ (12). The term $mlHAF_k[s;\alpha,\tau_w]$ represents the multi-lag HAF obtained using the warping operator given in (10).

The WHAF block diagram is given in figure 3.

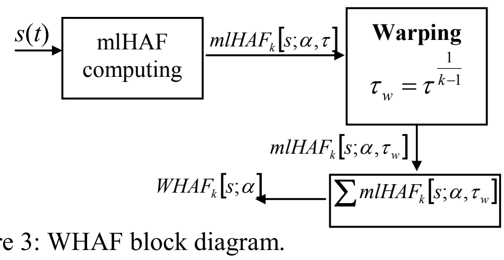

Figure 3: WHAF block diagram.

For the signal given by (9) the WHAF is plotted in the next figure. We analyze also the noised version of this signal (SNR=5 dB).

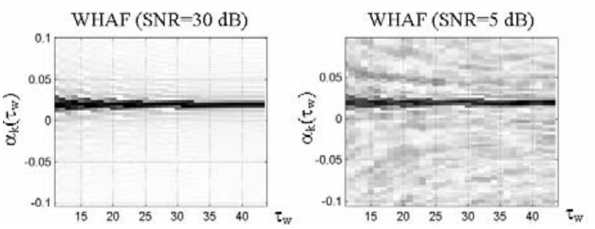

Figure 4: Warping High-Order Ambiguity Function.

We remark the peaks of mlHAF computed for an arbitrary lag set. Their arrangement around $k!a_k$ line facilities the estimation of k^{th} order polynomial coefficient. This is illustrated in the next figure where the frequency marginals of WHAF values, for both SNRs, are plotted.

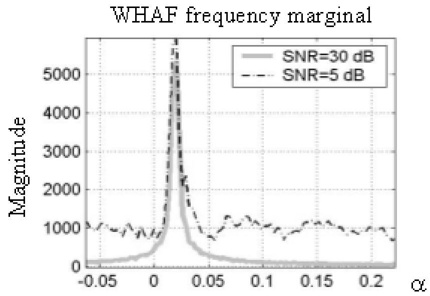

Figure 5: Localization of WHAF maximas.

As shown on this figure, in spite of noise presence, the frequency coordinate associated to 3[rd] order polynomial coefficient is the same as in the noise reduced case (SNR=30 dB). To compare the performances, we present in the next figure the variance of the estimation of a 3[rd] order polynomial coefficient for the signal given in (9).

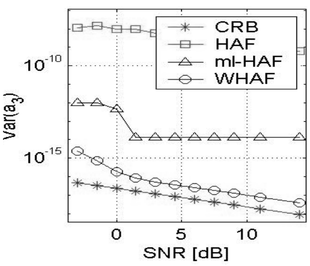

Figure 6: Variance of the estimation vs. SNR.

We observe that the WHAF- based estimation method provides good results compared to the mlHAF procedure. The values of the variance are closed to the Cramer-Rao Bound - CRB (computed in [1]) for a SNR around 0 dB.

On the other hand, from the operational point of view, this method is more advantageous than the PHAF. Since the lag sets are object of warping operation, their choice does not influence the result. Practically, we compute the HIM and, via (2), the mlHAF for an arbitrary lag set. Then, using the warping operator defined in (11), we transform the coordinate of mlHAFs as indicated in figure 2. The summation (12) ensures the absence of abnormal results such the ones illustrated in figure 1.

The next example illustrates the capability of the WHAF approach to deal with noisy multi-component signals. We consider a two component 3 order PPSs given by :

$$x(t)=\exp\left[j2\pi\left(0.25t+9.7656\cdot 10^{-4}t^2+4.2915\cdot 10^{-6}t^3\right)\right]+ \\ +\exp\left[j2\pi\left(0.15t+4.8828\cdot 10^{-4}t^2+1.9073\cdot 10^{-6}t^3\right)\right]+ \\ +w(t) \qquad (13)$$

where w is a white Gaussian noise (SNR=8 dB).

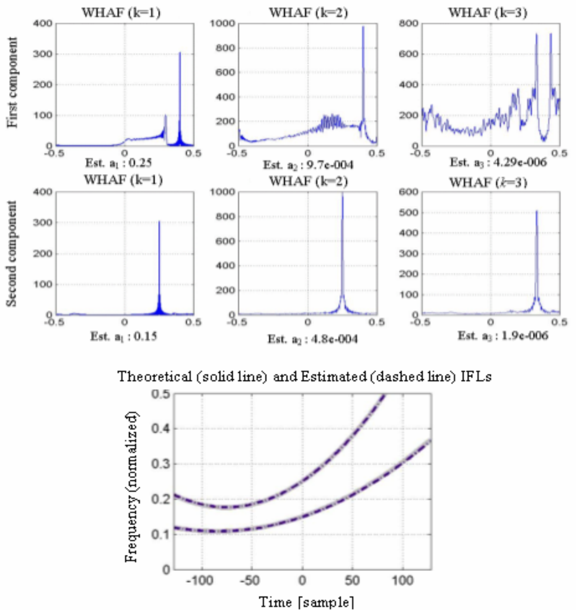

Figure 7: WHAF approach for a two component PPSs.

To manage the multi-component character of the analyzed signal, we use an adapted version of the Matching Pursuit principle proposed in [3]. For each iteration, we use the WHAF method to estimate the polynomial coefficients of components. The estimated PPS is eliminated from the analyzed signal and the residual is analyzed in the same manner. We note that the polynomial coefficients are correctly estimated. This is also reflected by a correct IFL estimation.

Finally, to illustrate the capability of WHAF to deal with real-life signals, we present the results obtained for an emission of a Blue Whale from Atlantic ocean [7].

The IFLs presented in figure 8 have been obtained by WHAF analysis of several time-frequency regions of interest, detected by the approach proposed in [8]. As the spectrogram, the WHAF provides a *visual* time-frequency information for the analyzed signal (figure 8). In addition, the WHAF furnishes a complementary *parametric* information concerning the estimated IFLs. For the signal considered in figure 8, the three IFLs have the following polynomial description :

$$\xi_1(t)=0.42-3\cdot 10^{-4}t^2+5.1\cdot 10^{-7}-2.03\cdot 10^{-8}t^3 \\ \xi_2(t)=0.42-4.1\cdot 10^{-4}t^2+5.32\cdot 10^{-7}-3.03\cdot 10^{-8}t^3 \\ \xi_3(t)=0.39-2.25\cdot 10^{-4}t^2+3.02\cdot 10^{-9}-4.02\cdot 10^{-10}t^3 \qquad (14)$$

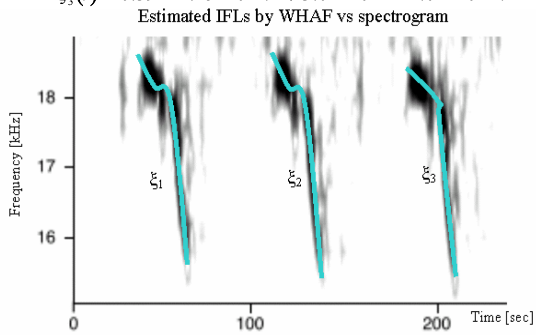

Figure 8: IFLs estimation, via 4th order WHAF, for a blue whale vocalization.

5. CONCLUSION

A new method for the polynomial signal processing has been presented in this paper. This method can be viewed as a new version of the HAF that improves the noise robustness performances of the classical high-order methods. We have also presented the HAF based method as a basic procedure for the polynomial coefficient estimation. We have seen that there are many limitations, and, consequently, based on the warped operator, we have proposed a new method to solve the noise robustness problem. The presented results highlight the performances of the WHAF in a noisy environment.

As perspectives, we intend to use this method for speech and radar signal characterization purposes.

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