BLIND RESTORATION OF BINARY SIGNALS USING A LINE SPECTRUM FITTING APPROACH

Javier Vía, Ignacio Santamaría and Marcelino Lázaro[1]

Dept. Ingeniería de Comunicaciones, Universidad de Cantabria, Spain.
e-mail:{jvia,nacho}@gtas.dicom.unican.es
[1]Dept. Teoría de la Señal y Comunicaciones, Universidad Carlos III de Madrid, Spain.
e-mail:marce@tsc.uc3m.es

ABSTRACT

In this paper we present a new blind equalization algorithm that exploits the parallelism between the probability density function (PDF) of a random variable and a power spectral density (PSD). By using the PDF/PSD analogy, instead of minimizing the distance between the PDF of the input signal and the PDF at the output of the equalizer (an information-theoretic criterion), we solve a line spectrum fitting problem (a second-order statistics criterion) in a transformed domain. For a binary input, we use the fact that the ideal autocorrelation matrix in the transformed domain has rank 2 to develop batch and online projection-based algorithms. Numerical simulations demonstrate the performance of the proposed technique in comparison to batch cumulant-based methods as well as to conventional online blind algorithms such as the constant modulus algorithm (CMA).

1. INTRODUCTION

In many communication systems, digital signals are transmitted through an unknown bandlimited channel with severe intersymbol interference (ISI). When a training sequence is not available, blind equalization techniques must be used to recover the input signal. These techniques exploit the knowledge about the statistical properties of the input signal or the structure of the channel [1].

Benveniste *et al.* [2] showed that a sufficient condition for perfect equalization is that the PDF of the recovered signal be equal to the PDF of the original input signal (a train of impulse functions for an M-ary input constellation). Following this PDF matching approach, several information-theoretic criteria have been proposed for blind equalization and deconvolution [3, 4].

On the other hand, the periodic extension of the PDF of a random variable (normalized to be between $-\pi$ and π) can be viewed as the power spectral density (PSD) of a certain stochastic process. This analogy has been previously exploited to estimate the PDF of a random variable by using PSD estimate methods [5, 6]. Similar ideas have also been applied to blind source separation problems [7], as well as to develop new nonlinear models [8].

In this paper we exploit the PDF/PSD parallelism for blind restoration of binary input signals. Instead of pursuing a PDF matching approach, here we fit the estimated PSD at the output of the equalizer to the target PSD, which corresponds to a line spectrum for a digital input signal. The rationale behind posing blind equalization as a PSD fitting problem is that some properties of the target PSD can be exploited in the algorithm. For instance, for a binary input signal the target autocorrelation has rank 2; this fact is exploited to develop efficient batch and online projection-based algorithms. Through numerical simulations, the proposed method is compared to batch cumulant-based algorithms [9] and to online blind adaptive algorithms [10].

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2. PROBLEM FORMULATION

We consider a baud-rate sampled baseband representation of a digital communications system. Throughout this paper we assume an equiprobable i.i.d. binary signal $s[n] \in \{-1,+1\}$; the method, however, can be easily generalized to M-ary constellations. The binary signal is sent through a linear time-invariant channel with coefficients $h[n]$. Therefore, the channel output is obtained by

$$x[n] = \sum_k h[k]s[n-k] + e[n],$$

where $e[n]$ is a zero-mean white Gaussian noise process.

The objective of a blind linear equalizer is to remove the ISI at its output without using any training sequence. Usually, the equalizer is designed as a FIR filter with L coefficients $\mathbf{w}$; then, its output is given by

$$y[n] = \sum_{k=0}^{L-1} w[k]x[n-k] = \mathbf{w}^T \mathbf{x}_n.$$

Benveniste *et al.* [2] proved that a sufficient condition for perfect equalization is that the PDF of the recovered symbols $y[n]$ be equal to the PDF of the original input signal $s[n]$ (a pair of impulse functions for a binary input). Later, Shalvi and Weinstein [9] proved that perfect recovery of the input signal (in a noiseless situation) can be achieved by maximizing the output kurtosis

$$K_y = E\left[|y[n]|^4\right] - 2\left(E\left[|y[n]|^2\right]\right)^2 - \left|E\left[y[n]^2\right]\right|^2,$$

subject to $E\left[|y[n]|^2\right] = E\left[|s[n]|^2\right]$.

On the other hand, online algorithms typically minimize a nonlinear cost function employing a gradient descent approach. The constant modulus algorithm (CMA) [10], for instance, minimizes the following cost function

$$J(\mathbf{w}) = E\left[\left(|y[n]|^2 - R_2\right)^2\right],$$

where $R_2 = E\left[|s[n]|^4\right] / E\left[|s[n]|^2\right]$. Applying a gradient descent technique, the CMA actualizes the equalizer taps as

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mu_{\text{CMA}}\left(|y[n]|^2 - R_2\right) y[n]\mathbf{x}_n.$$

3. PDF/PSD FITTING

The probability density function (PDF) of a random variable has similar properties to a power spectral density (PSD). This analogy has been previously exploited to estimate the PDF of a random variable by using PSD estimate methods [5, 6].

In particular, let us assume that we are given a set of N observations at the channel output: $(\mathbf{x}_1,\ldots,\mathbf{x}_N)$, where $\mathbf{x}_n = (x[n], \ \ldots, x[n-L+1])^T$. The output of the equalizer, $y[n] = \mathbf{w}^T\mathbf{x}_n$, is a random variable (RV) with PDF $p_{\mathbf{Y}}(y)$. Without loss of generality we can assume that $y[n]$ is constrained to be between $-\pi$ and π: this can be achieved by a proper normalization step. The periodic extension of $p_{\mathbf{Y}}(y)$ can alternatively be viewed as the PSD of a certain wide sense stationary (WSS) stochastic process, whose autocorrelation function is given by

$$r_y[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} p_{\mathbf{Y}}(y)e^{jyk}dy = \frac{1}{2\pi}E(e^{jyk}). \quad (1)$$

Note that, according to this analogy, the output of the equalizer plays here the role of a radian frequency ω. Furthermore, the autocorrelation function (1) can be estimated as

$$\hat{r}_y[k] = \frac{1}{2\pi N}\sum_{n=0}^{N-1} e^{jy[n]k}. \quad (2)$$

In an ideal noiseless situation the linear filter $\mathbf{w}$ attains perfect equalization if

$$y[n] = \mathbf{w}^T\mathbf{x}_n = Ks[n-d],$$

where d is the equalizer's delay and K is an arbitrary scaling factor. On the other hand, in digital communications the input signal $s[n]$ belongs to a finite alphabet. For example, for an M-ary input constellation, the target PSD is a line spectrum composed of M equispaced impulse functions. Correspondingly, its autocorrelation function can be written as

$$r_s[k] = \frac{1}{2\pi M}\sum_{i=1}^{M} e^{j\omega_i k}, \quad (3)$$

where ω_i is a set of known radian frequencies, which depend on the input constellation. Specifically, for a binary input signal the target PSD has two impulse functions at $(-\omega_1, \omega_1)$ and (3) reduces to

$$r_s[k] = \frac{1}{2\pi}\cos(\omega_1 k).$$

The basic idea of the paper consists in finding the coefficients of the equalizer $\mathbf{w}$ with the goal of fitting the estimated autocorrelation function (2) to the target autocorrelation (3), which corresponds to a line spectrum PSD. Actually, we are fitting the estimated PDF at the output of the equalizer to the known PDF of the input constellation: this is a widely used information-theoretic criterion for blind equalization and deconvolution. Several distance measures such as the Kullback-Leibler distance [3], or a quadratic distance [4] can be used for PDF fitting. As an alternative, in this paper we exploit the fact that, after a nonlinear transformation given by (2), the problem reduces to fit the estimated and target PSD's, for which a number of classical techniques may be applied. In the next section we discuss batch an on-line efficient procedures to solve this line spectrum fitting problem.

4. PROPOSED METHOD

4.1 Batch Algorithm

Let us consider a set of N outputs of the equalizer $y[n]$, $n = 1,\ldots,N$. It is important at this point to remember that we assume that $y[n]$ has been normalized to be constrained between $-\pi$ and π: therefore it can be viewed as a radian frequency. In order to attain perfect equalization, we should find an equalizer such as

$$\hat{r}_y[k] = r_s[k], \qquad \forall k. \quad (4)$$

If we fit the estimated and target autocorrelation functions only at lags $k = 0,\ldots,P$; Eq. (4) can be written for a binary input as

$$\underbrace{\begin{bmatrix} 1 & \cdots & 1 \\ e^{jy[1]} & \cdots & e^{jy[N]} \\ e^{j2y[1]} & \cdots & e^{j2y[N]} \\ \vdots & \ddots & \vdots \\ e^{jPy[1]} & \cdots & e^{jPy[N]} \end{bmatrix}}_{(P+1)\times N} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ e^{j\omega_1} & e^{-j\omega_1} \\ e^{j2\omega_1} & e^{-j2\omega_1} \\ \vdots & \vdots \\ e^{jP\omega_1} & e^{-jP\omega_1} \end{bmatrix}}_{(P+1)\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

or in matrix notation

$$\mathbf{A}(y)\mathbf{1}_N = \mathbf{A}(\omega)\mathbf{1}_2,$$

where $\mathbf{1}_N$ denotes an $N\times 1$ vector of ones. The radian frequency ω_1 can be arbitrarily chosen as long as $|\omega_1| < \pi$. However, a good choice is $\omega_1 = \pi/(P+1)$, in this way the columns of the matrix $\mathbf{A}(\omega)$ become orthogonal. Moreover, this choice seems to speed up the algorithm.

The matrix $\mathbf{A}(\omega)$ is full column rank; then, if we take $P+1 \geq 2$, it follows that $rank(\mathbf{A}(\omega)) = 2$. The proposed batch method is an iterative technique that consists in three stages: in the first step the columns of the $(P+1)\times N$ matrix $\mathbf{A}(y)$ are projected into the subspace spanned by the columns of $\mathbf{A}(\omega)$, i.e.,

$$\mathbf{A}_s(y) = \mathbf{P}_s\mathbf{A}(y),$$

where the projection matrix is given by

$$\mathbf{P}_s = \mathbf{A}(\omega)\left(\mathbf{A}(\omega)^H\mathbf{A}(\omega)\right)^{-1}\mathbf{A}(\omega)^H, \quad (5)$$

and $(\cdot)^H$ denotes the conjugate transpose.

In the second step, from each column of the projected matrix $\mathbf{a}_s(y[n]) = (a_1(y[n]),\ldots,a_{P+1}(y[n]))^T$, we estimate the new objective outputs of the equalizer $\hat{y}[n]$ as the averaged phase differences between consecutive elements of $\mathbf{a}_s(y[n])$, i.e.,

$$\hat{y}[n] = \frac{1}{P}\sum_{j=1}^{P}\left[\arg(a_{j+1}(y[n])) - \arg(a_j(y[n]))\right], \quad (6)$$

where $\arg(\cdot)$ denotes the unwrapped phase. The step resembles the weighted phase averager (WPA) method proposed to estimate the frequency of a single complex exponential [11]. By using the new estimated outputs $\hat{\mathbf{y}} = (\hat{y}[1],\ldots,\hat{y}[N])^T$, in the final step the new equalizer is obtained as

$$\mathbf{w} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\hat{\mathbf{y}}, \quad (7)$$

where $\mathbf{X} = [\mathbf{x}_1| \dots |\mathbf{x}_N]^T$.

The equalizer obtained as (7) is used to obtain the new outputs $\mathbf{y}$ and the procedure is repeated. Regarding the convergence of this method, although a formal proof is not included here due to the lack of space, we can point out that the proposed iterative method is in fact a projection onto convex set (POCS) algorithm [12]: note that (5), (6) and (7) can be viewed as projection operators. Therefore, the convergence of the iterative batch algorithm to a point belonging to the intersection of the closed convex sets can be guaranteed. From a practical standpoint the iterations are stopped when $||\mathbf{w}_{k+1} - \mathbf{w}_k||^2 \leq \varepsilon$. Next, we show a summary of the proposed procedure.

Initialize $\mathbf{w}$, choose P and obtain $\mathbf{A}(y)$ ($y[n] \in (-\pi, \pi]$).
Calculate the projection matrix $\mathbf{P_s}$ as (5).
repeat
  Obtain the projected matrix $\mathbf{A}_s(y) = \mathbf{P}_s\mathbf{A}(y)$.
  Estimate the new desired outputs as (6).
  Calculate the new equalizer $\mathbf{w}$ as (7).
  Calculate $\mathbf{y} = \mathbf{X}\mathbf{w}$ and update $\mathbf{A}(y)$.
until convergence

Algorithm 1: Summary of the batch algorithm.

4.2 Online Algorithm

The online algorithm follows the same steps of the batch procedure. With each new data sample, the current equalizer is employed to obtain the output $y[n] = \mathbf{w}^T\mathbf{x}_n$ and then construct the vector $\mathbf{a}(y[n]) = \left(1, e^{jy[n]}, \dots, e^{jPy[n]}\right)^T$. Similarly to the batch procedure this vector is projected into the subspace spanned by the columns of $\mathbf{A}_s(\omega)$ as

$$\mathbf{a}_s(y[n]) = \mathbf{P}_s\mathbf{a}(y[n]). \tag{8}$$

The estimate $\hat{y}[n]$ is again obtained by means of (6). In the final step, we apply an iteration of the least mean square (LMS) algorithm that uses $\hat{y}[n]$ as desired output

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu(\hat{y}[n] - y[n])\mathbf{x}_n, \tag{9}$$

where $\mu > 0$ is the step size of the LMS. A summary of the proposed online algorithm is shown next.

Initialize $\mathbf{w}$, choose P and $\mu > 0$.
Calculate the projection matrix $\mathbf{P_s}$ as (5).
for $n = 1, 2, \dots$ **do**
  $y[n] = \mathbf{w}^T\mathbf{x}_n$.
  $\mathbf{a}(y[n]) = \left(1, e^{jy[n]}, \dots, e^{jPy[n]}\right)^T$.
  Obtain the projected vector $\mathbf{a}_s(y[n]) = \mathbf{P}_s\mathbf{a}(y[n])$.
  Estimate the new objective outputs as (6).
  Update $\mathbf{w}_{n+1} = \mathbf{w}_n + \mu(\hat{y}[n] - y[n])\mathbf{x}_n$.
end for

Algorithm 2: Summary of the online algorithm.

5. SIMULATION RESULTS

All the simulation results have been obtained using a binary input signal and $P = 5$. The equalizers have been initialized following the tap centering strategy, and we have used the ISI as a measure of equalization performance, which is defined as

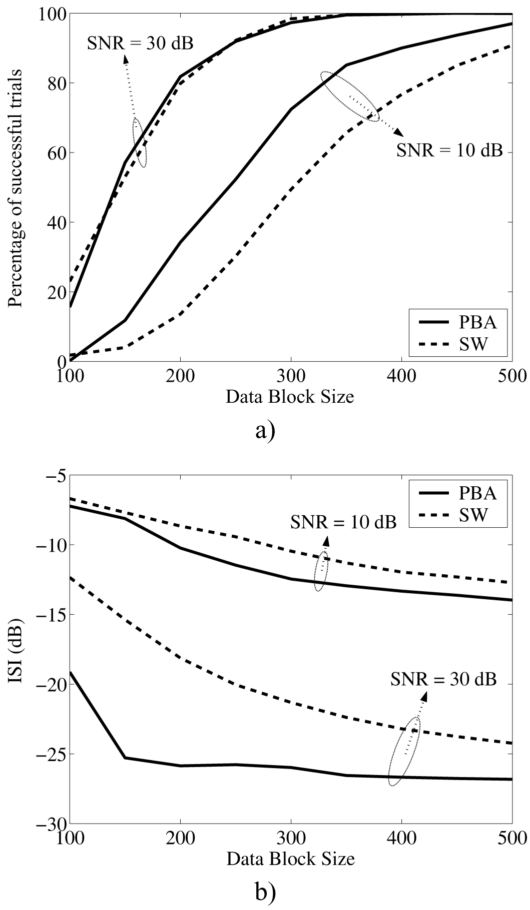

Figure 1: Evaluation of the batch PBA and the SW algorithm for different SNR's and data block sizes. $L = 31$ and channel $H_1(z)$. a) Percentage of successful trials, b) ISI (dB).

$$\text{ISI} = 10\log_{10}\frac{\sum_n |\theta_n|^2 - \max_n |\theta_n|^2}{\max_n |\theta_n|^2}, \tag{10}$$

where $\theta = \mathbf{h} * \mathbf{w}$ is the combined channel-equalizer impulse response.

The batch algorithm was tested, for low (SNR = 30 dB) and moderate (SNR = 10 dB) noise situations, and its performance was compared against the Shalvi and Weinstein algorithm [9] (denoted as SW), which is based on fourth-order cumulants. In the first example a binary signal is sent through the channel $H_1(z) = (0.4 + z^{-1} - 0.7z^{-2} + 0.6z^{-3} + 0.3z^{-4} - 0.4z^{-5} + 0.1z^{-6})$ (used in [9]) and, at the channel output, white Gaussian noise is added.

Fig. 1 compares the projection-based algorithm (denoted as PBA) and the SW algorithms for equalizers of length $L = 31$ using blocks of input data ranging from $N = 100$ to 500 samples. For both methods, if the final ISI after a trial was below -5 dB, we considered that the channel was successfully equalized, since with this level of ISI it is already possible to switch to a decision-directed mode. For each data block size and noise level, both algorithms were tested in 500 Monte-Carlo trials. In particular, Fig. 1.a shows the percent-

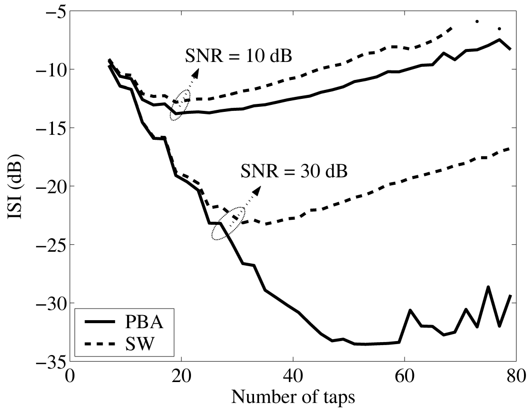

Figure 2: Final ISI for the PBA and the SW algorithm vs. equalizer length. $N = 400$, channel $H_1(z)$.

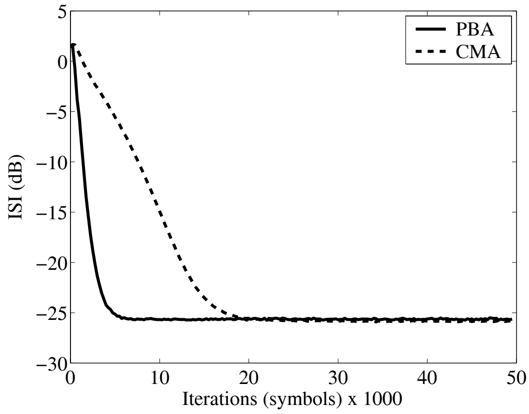

Figure 3: Convergence curves for the online PBA and CMA. $L = 21$, SNR $= 30$ dB and channel $H_2(z)$.

age of trials in which each algorithm successfully converged, and Fig. 1.b shows the mean ISI level after convergence for the successful trials. Fig. 2 compares the performance of both algorithms for different equalizer lengths. Now the data block size is $N = 400$ and the equalizer length ranges from $L = 5$ to 79. From Figs. 1, and 2 we see that the proposed algorithm offers better performance than the SW algorithm, with a similar computational cost.

In our second example we test the online version of the PBA in comparison to the CMA [10]. Again we use a binary input signal distorted by the channel $H_2(z) = (0.2258 + 0.5161z^{-1} + 0.6452z^{-2} + 0.5161z^{-3})$ and corrupted by white Gaussian noise with a SNR of 30 dB. The equalizer length is $L = 21$ and the chosen parameters have been $\mu = 0.01$ for PBA and $\mu = 0.005$ for CMA: these are the largest stepsizes for which both algorithms converged in all trials. The averaged results of 100 independent simulations are compared in Fig. 3, where we can see that PBA converges faster than CMA.

6. CONCLUSIONS

Based on the equivalence between PDF and PSD functions, in this paper we have proposed a new blind equalization technique for binary input signals. By posing blind equalization as a line spectrum fitting problem, some properties of the target PSD can be exploited in the algorithm. Using these properties, batch and adaptive (online) versions of a POCS-like fitting algorithm have been described. A number of simulation examples demonstrate that the batch algorithm shows better performance than cumulant-based techniques. Moreover, the online version of the algorithm seems to be faster than the CMA with a similar computational cost.

Future lines of research include the extension of the proposed algorithms to multilevel and complex modulations as well as the application of similar ideas to blind source separation (BSS) problems.

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