REDUCING THE BIAS OF A BEARINGS-ONLY TLS TARGET LOCATION ESTIMATOR THROUGH GEOMETRY TRANSLATIONS

Kutluyıl Doğançay

School of Electrical and Information Engineering
University of South Australia
Mawson Lakes, SA 5095, Australia
Email: Kutluyil.Dogancay@unisa.edu.au

ABSTRACT

Despite its simplicity, the pseudolinear estimator for bearings-only target localization is known to suffer from severe estimation bias. To overcome this problem, the method of total least squares (TLS) can be employed which attempts to correct errors not only in the data vector but also in the measurement matrix. The paper examines the relationship between the target localization geometry and the TLS estimation bias for the target location. The paper shows that translations of the target localization geometry through simple shift and/or rotation operations do affect the bias of the TLS estimator and can be exploited to reduce the TLS estimation bias significantly.

1. INTRODUCTION

Passive target localization has a long history. The pioneering work in this area is that of Stansfield [1]. The Stansfield estimator is a weighted least squares (WLS) estimator that can be viewed as a small bearing noise approximation of the maximum likelihood (ML) estimator for independent Gaussian bearing noise and no observer position error [2]. It also assumes the prior knowledge of the target range from the observer positions. This strong assumption can be dispensed with by using the pseudolinear estimator [3]. The passive target localization problem can be recast as a nonlinear LS problem by using the ML solution.

Despite having low complexity and no convergence problems, a major drawback of the pseudolinear estimator is the large estimation bias that does not vanish with the increasing number of measurements. The bias is due to the correlation between the measurement matrix and the bearing noise, and its severity depends on the target localization geometry as well as the noise statistics. The bias of the pseudolinear and Stansfield estimators has been studied in the target tracking and localization literature (see e.g. [4, 3, 2, 5]). To overcome the bias of the pseudolinear estimator, various fixes have been proposed based on batch iterative and closed-form instrumental variables [3, 6, 7], and total least squares (TLS) [8, 9]. The iterative techniques have the disadvantage of being sensitive to initialization and the stepsize [10].

In this paper we examine the effect of target localization geometry on the bias of the TLS target location estimator. The TLS estimator attempts to correct the errors in both the measurement matrix and the data vector unlike the pseudolinear estimator. The bias performance of the TLS algorithm will be shown to depend on the absolute localization geometry. The analysis will obtain best target localization geometries that can be obtained from a given arbitrary geometry by simple shift and/or rotation operations.

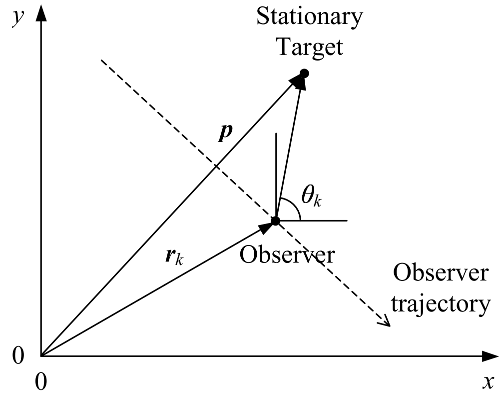

Figure 1: Two-dimensional bearings-only target localization geometry.

The paper is organized as follows. Section 2 introduces the target localization problem. In Section 3 an overview of the pseudolinear estimator is provided. Section 4 develops the TLS estimator. The dependence of the TLS estimation bias on target localization geometry and its equivalent translations is explained in Section 5. Section 6 presents simulation studies. The conclusions are drawn in Section 7.

2. PASSIVE BEARINGS-ONLY TARGET LOCALIZATION AND ASSUMPTIONS

The two-dimensional passive target localization problem using bearing measurements is depicted in Fig. 1 where p is the location of a stationary target, and θ_k and r_k are the bearing angle and observer position, respectively, at time instant k. The relationship between the bearing angle, observer position and target location is given by the nonlinear equation:

$$\theta_k = \tan^{-1} \frac{\Delta y_k}{\Delta x_k}, \quad k = 1, \ldots, N \tag{1}$$

where $\Delta y_k = p_y - r_{y,k}$, $\Delta x_k = p_x - r_{x,k}$, $p = [p_x, p_y]^T$ and $r_k = [r_{x,k}, r_{y,k}]^T$.

The objective of target localization is to estimate the target location p from a sequence of bearing measurements over the interval $1 \leq k \leq N$. In practice, the bearing measurements are corrupted by additive noise, i.e.,

$$\tilde{\theta}_k = \theta_k + n_k \tag{2}$$

where the $\tilde{\theta}_k$, $k = 1, \ldots, N$, are the bearing measurements and n_k is the bearing noise. We assume that n_k is white Gaussian with zero mean and variance $\sigma_{n_k}^2$. The bearing noise variance can vary with k. We also assume that the target is observable from the available observer positions and bearing measurements. Regarding the observer trajectory, we assume that the observer moves along a linear path and collects bearing measurements at equally spaced positions.

3. OVERVIEW OF THE PSEUDOLINEAR ESTIMATORS

The pseudolinear estimator can be derived from the orthogonal vector sum relationship

$$p = r_k + \tilde{s}_k + e_k \tag{3}$$

where $\tilde{s}_k$ is the noisy bearing vector and $e_k^T \tilde{s}_k = 0$. The orthogonal error vector e_k is defined by

$$e_k = d_k \sin n_k a_k, \quad a_k = \begin{bmatrix} \sin\tilde{\theta}_k \\ -\cos\tilde{\theta}_k \end{bmatrix}. \tag{4}$$

Here $d_k = \|s_k\|_2$ is the target range from the observer position r_k and a_k is a unit vector orthogonal to $\tilde{s}_k$. To eliminate $\tilde{s}_k$, pre-multiply both sides of (3) with a_k^T, resulting in

$$a_k^T p = a_k^T r_k + \eta_k \tag{5}$$

where $\eta_k = d_k \sin n_k$ is a nonlinearly transformed version of the bearing noise that has zero mean and variance $d_k^2 E\{\sin^2 n_k\}$. For small bearing noise, i.e., $\sin\eta_k \approx \eta_k$, the variance of η_k can be approximated by $d_k^2\sigma_{n_k}^2$. Concatenating (5) for $k = 1,\ldots,N$, we get

$$Ap = b + \eta \tag{6}$$

where

$$A = \begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_N^T \end{bmatrix}, \quad b = \begin{bmatrix} a_1^T r_1 \\ a_2^T r_2 \\ \vdots \\ a_N^T r_N \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{bmatrix}. \tag{7}$$

An LS solution to (6) is given by

$$\hat{p}_{\text{LS}} = \arg\min_{p\in\mathbb{R}^2} \|Ap - b\|_2^2 \tag{8a}$$

$$= (A^T A)^{-1} A^T b \tag{8b}$$

which is referred to as the pseudolinear estimator. Equation (8b) assumes that $A^T A$ is nonsingular in accordance with the observability assumption.

4. THE TLS ESTIMATOR

The LS problem assumes that all errors occur in the vector b. However, this is not the case with the bearings-only target location problem since in (6) both the "measurement" matrix A and the "data" vector b are subject to error due to the bearing measurement noise. The LS estimators such as the pseudolinear estimator and the Stansfield estimator exhibit large bias due to the correlation between the entries of A and η, which can be asymptotically expressed as [5]

$$E\{\hat{p}_{\text{LS}}\} - p = -E\{A^T A\} E\{A^T \eta\}. \tag{9}$$

The estimation bias can be reduced significantly by employing the TLS estimation algorithm. Central to TLS is the concept of perturbing both A and b in a minimal fashion, rather than b only as in the case of LS estimation, to achieve a consistent equation that relates the perturbed A to the perturbed b. TLS aims to solve the following constrained minimization problem [11, 12]

$$\min_{b+\delta\in\text{Range}(A+\Delta)} \|L[\Delta,\delta]T\|_F \tag{10}$$

where L and T are nonsingular diagonal weighting matrices

$$L = \text{diag}(l_1, l_2, \ldots, l_N)$$
$$T = \text{diag}(t_1, t_2, t_3)$$

and $\|\cdot\|_F$ denotes the Frobenius norm. The TLS solution is given by the vector $\hat{p}_{\text{TLS}}$ that satisfies

$$(A + \hat{\Delta})\hat{p}_{\text{TLS}} = b + \hat{\delta} \tag{11}$$

where $\hat{\Delta}$ and $\hat{\delta}$ are the perturbations that minimize (10).

The TLS estimate can be easily obtained by using the singular value decomposition (SVD) of the weighted augmented matrix

$$L[A,b]T = U\Sigma V^T = \sum_{i=1}^{3} \sigma_i u_i v_i^T \tag{12}$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$ are the singular values, and $U = [u_1, u_2, u_3]$ and $V = [v_1, v_2, v_3]$ are orthogonal matrices, i.e., $U^T U = I$ and $V^T V = I$. The perturbations minimizing (10) are obtained from a reduced rank approximation of $L[A,b]T$:

$$L[A,b]T + L[\hat{\Delta},\hat{\delta}]T = \sum_{i=1}^{2} \sigma_i u_i v_i^T \tag{13}$$

where $\|L[\hat{\Delta},\hat{\delta}]T\|_F = \sigma_3$. Using (13), (11) can be rewritten as

$$[A + \hat{\Delta}, b + \hat{\delta}] \begin{bmatrix} \hat{p}_{\text{TLS}} \\ -1 \end{bmatrix} = 0 \tag{14a}$$

$$L^{-1} \left(\sum_{i=1}^{2} \sigma_i u_i v_i^T \right) T^{-1} \begin{bmatrix} \hat{p}_{\text{TLS}} \\ -1 \end{bmatrix} = 0. \tag{14b}$$

Noting that $v_i^T v_j = 0$ if $i \neq j$, we obtain

$$\begin{bmatrix} \hat{p}_{\text{TLS}} \\ -1 \end{bmatrix} = -\frac{1}{t_3 v_{33}} T v_3 \tag{15a}$$

$$\hat{p}_{\text{TLS}} = -\frac{1}{t_3 v_{33}} \begin{bmatrix} t_1 v_{13} \\ t_2 v_{23} \end{bmatrix} \tag{15b}$$

where $v_3 = [v_{13}, v_{23}, v_{33}]^T$.

For the bearings-only target localization problem, the TLS weighting matrices are usually set to $L = I$ and $T = I$ because of the intractability of optimal weights.

5. GEOMETRY TRANSLATION FOR BIAS REDUCTION

The TLS performance does not remain the same for different equivalent translations of a given target localization geometry. Since we have the freedom to translate a given geometry

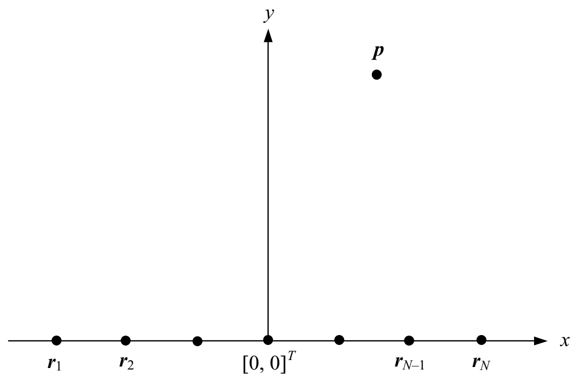

Figure 2: Normalized geometry obtained from an arbitrary localization geometry by shifting and rotation where the centre of mass of the r_k is at the origin.

by means of rotation and shifts, it is advantageous to seek a translation that will minimize the TLS bias and mean-square error (MSE).

Th TLS bias can be minimized by reducing the bearing noise variance, or by shifting and/or rotating the target location geometry as we shall see. In a given target localization problem, we cannot change the bearing noise variance. However, shifting and/or rotation of the geometry can be done without changing the location problem. A given target location geometry can be shifted to change the r_k, and can be rotated to change both the r_k and the θ_k. If the shifting and rotation is done in such a way as to minimize the norm of the TLS bias, this will lead to the best TLS target localization geometry. It is interesting to note that the bias of the pseudolinear estimator is invariant to the translation of the localization geometry through shifts and rotations. This is readily seen from the bias expression for the pseudolinear estimator [5]

$$E\{A^T A/N\}^{-1} E\{\sin^2 n_k\}(c-p) \quad (16)$$

where the bearing noise is assumed to be i.i.d., and c is the centre of mass of observer positions r_k. Thus, for the pseudolinear estimator it is not possible to reduce the bias by means of geometry translation.

To shift the target location geometry by a displacement vector s, we perform $r_k + s$, $k = 1, \ldots, N$, and $p + s$, which amounts to shifting the observer positions and the target location by s, respectively. The bearing angles are not affected by a shift. To rotate the target geometry by ϕ, we use the rotation matrix

$$R = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}.$$

The rotated bearing angles, bearing measurements, observer positions and target location are simply given by

$$\theta_k + \phi, \quad k = 1, \ldots, N$$
$$\tilde{\theta}_k + \phi, \quad k = 1, \ldots, N$$
$$Rr_k, \quad k = 1, \ldots, N$$
$$Rp.$$

By an appropriate selection of s and ϕ, any target localization geometry with a linear observer trajectory can be translated to the *normalized* geometry shown in Fig. 2 in which the observer trajectory is placed on the x-axis (i.e., $r_{y,k} = 0$, $k = 1, \ldots, N$) and the center of mass of the observer positions is at the origin, i.e., $\sum_{k=1}^{N} r_k = 0$. For bias comparison purposes we will use the normalized geometry for a given arbitrary target localization geometry and determine the necessary shift that needs to be applied to the normalized geometry to yield the minimum TLS bias.

In target localization applications, only observer positions r_k and bearing measurements $\tilde{\theta}_k$ are available. The translation of the geometry is carried out by shifting and/or rotating the r_k and $\tilde{\theta}_k$. In this case, the true target location is also implicitly translated. In fact, the translation process is equivalent to rotating and/or shifting Cartesian coordinates. After the target location estimate is obtained from the translated r_k and $\tilde{\theta}_k$, the target location estimate in the original geometry (before translation) can be recovered by simply undoing the translation on the location estimate.

6. SIMULATION STUDIES

As alluded to in Section 5, the TLS bias is dependent on the absolute location of the observer positions. The key question is "how does the absolute location affect the TLS bias?" There does not appear to be a simple answer to this question. The purpose of this section is to illustrate by simulation the dependence of the TLS estimation bias on the shift applied to the normalized geometry of a target localization scenario.

The simulated target localization geometry is shown in Fig. 3 where the true target location is $p = [47.97, 98.60]^T$ and $N = 40$ bearing measurements are taken at regular intervals along a linear trajectory between $r_1 = [11.2061, 26.8404]^T$ and $r_N = [48.7939, 13.1596]^T$. The bearing noise standard deviation is 5°. Fig. 3 also shows the mean pseudolinear and TLS location estimates and their 1-σ error ellipses obtained from 10,000 Monte Carlo simulation runs. Both the pseudolinear and TLS estimates exhibit large estimation biases even though the TLS bias is smaller.

To demonstrate the effect of translation, the geometry of Fig. 3 was first translated to the normalized geometry as in Fig. 2 by applying an appropriate shift and rotation. Using the normalized geometry, the effect of the shift $s = [s_x, s_y]^T$ was simulated by estimating the TLS bias for each shift in the range $-30 \leq s_x \leq 40$ and $-20 \leq s_y \leq 10$. For simulation purposes, only integer-valued shifts were used, and the TLS bias was estimated using 10,000 Monte Carlo simulation runs. A smoothed version of the resulting TLS bias norm estimates is plotted in Fig. 4. The figure demonstrates the dependence of the TLS estimation bias on the shift vector s. The bias appears to be reduced on an elliptical trajectory on the s_x-s_y plane. However the minimum bias was obtained at shift $s_{\min} = [0, 4]^T$. The geometry resulting from this shift is shown in Fig. 5 along with the pseudolinear and TLS estimation results. A comparison of Fig. 5 with Fig. 3 confirms that the TLS estimation bias is indeed significantly reduced if the normalized geometry is shifted by $s_{\min}$. Table 1 summarizes the simulated performance of the pseudolinear and TLS estimators.

7. CONCLUSION

For a given target localization geometry, we have shown that there exists an equivalent translated geometry obtained

Estimator	Geometry	Bias Norm	MSE
Pseudolinear	Fig. 3	21.01	463.35
TLS	Fig. 3	6.55	90.51
Pseudolinear	Fig. 5	20.94	461.06
TLS	Fig. 5	0.07	63.95

Table 1: Bias and MSE performance.

through shift and rotation operations that minimizes the TLS estimation bias. The resulting bias reduction was demonstrated to be quite significant. The existence of favourable geometries for the TLS estimator can be exploited to determine an optimal geometry translation using offline simulations based on an initial target location estimate obtained from the initial localization geometry.

REFERENCES

[1] R. G. Stansfield, "Statistical theory of DF fixing," *Journal of IEE*, vol. 94, no. 15, pp. 762–770, December 1947.

[2] M. Gavish and A. J. Weiss, "Performance analysis of bearing-only target location algorithms," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 28, no. 3, pp. 817–828, 1992.

[3] S. C. Nardone, A. G. Lindgren, and K. F. Gong, "Fundamental properties and performance of conventional bearings-only target motion analysis," *IEEE Trans. on Automatic Control*, vol. 29, no. 9, pp. 775–787, September 1984.

[4] V. J. Aidala and S. C. Nardone, "Biased estimation properties of the pseudo-linear tracking filter," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 18, pp. 432–441, July 1982.

[5] K. Doğançay, "On the bias of linear least squares algorithms for passive target localization," *Signal Processing*, in press.

[6] Y. T. Chan and S. W. Rudnicki, "Bearings-only and Doppler-bearing tracking using instrumental variables," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 28, no. 4, pp. 1076–1083, October 1992.

[7] K. Doğançay, "Passive emitter localization using weighted instrumental variables," *Signal Processing*, in press.

[8] ——, "Bearings-only target localization using total least squares," *Signal Processing*, under review.

[9] K. D. Rao and D. C. Reddy, "A new method for finding electromagnetic emitter location," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 30, no. 4, pp. 1081–1085, 1994.

[10] J. P. Le Cadre and C. Jauffret, "On the convergence of iterative methods for bearings-only tracking," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 35, no. 3, pp. 801–818, July 1999.

[11] G. H. Golub and C. F. Van Loan, "An analysis of the total least squares problem," *SIAM Journal on Numer. Anal.*, vol. 17, no. 6, pp. 883–893, December 1980.

[12] ——, *Matrix Computations*, 2nd ed. Baltimore, MD: Johns Hopkins Univ. Press, 1989.

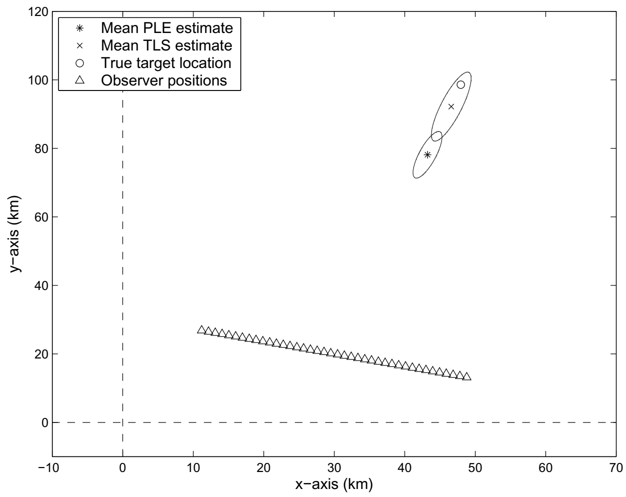

Figure 3: Simulated geometry with error ellipses for the pseudolinear (PLE) and TLS estimates.

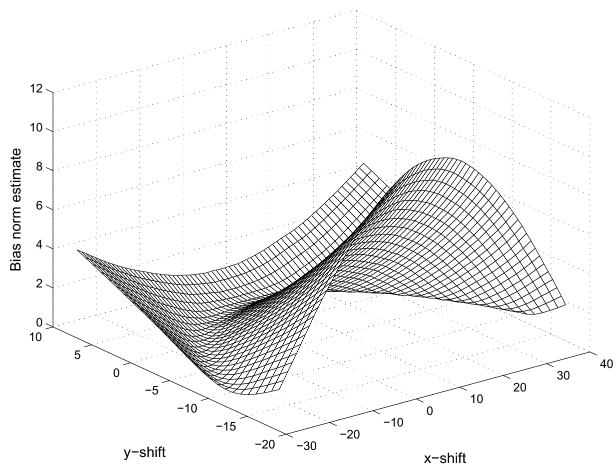

Figure 4: TLS estimation bias as a function of shift from the normalized geometry.

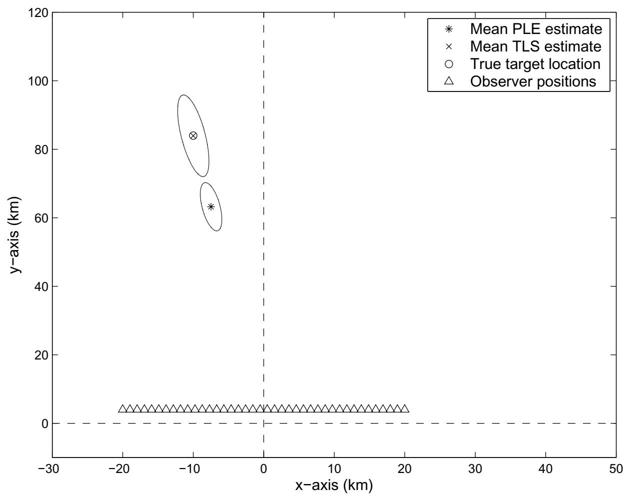

Figure 5: Best (minimum bias) geometry and error ellipses for the PLE and TLS estimates.