3D - tangential Diffusion

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Abstract

This article proposes a novel 3D diffusion approach to filter oriented patterns. This approach was first developed to denoise and enhance seismic 3D data composed of a stack of layers disturbed by noise and broken by faults.

The proposed filter is an extension of a 2D filter based on a classical diffusion equation including also additional orientation information. Considering 3D structures containing oriented planes, we search at each pixel the orientation of the tangential plane. The mean orientation information is included in the classical diffusion equation; the diffusion is steered at each pixel according to the directional tendency. Through some results on synthesized 3D blocks we will show that our method is able to eliminate noise while keeping edges and transitions.

1. Introduction

Initially proposed by Perona and Malik, the non-linear diffusion filters have been widely used in the last decade in edge preserving filtering. The basic concept of the initial approach is the adaptive smoothing of a noisy input image by defining a diffusion force depending on the local gradient [8]:

$$\frac{\partial U}{\partial t} = div(g(\|\nabla U\|)\nabla U) \quad (1)$$

Where g() is a non-increasing function of the local gradient ∇U called diffusivity function. This function governs the behavior of the diffusion process. A typical choice for g is [8]:

$$g(\|\nabla U\|) = \frac{1}{1 + \left(\frac{\|\nabla U\|}{K}\right)^2} \quad (2)$$

with *K,* a positive constant acting as a gradient threshold: the process leads to an enhancement (or backward diffusion) along the gradient's direction when $\|\nabla U\| \geq K$ whereas a diffusion is obtained otherwise. Practical implementations of the P-M filter are giving impressive results, noise is eliminated and edges are kept or even enhanced provided that their gradient value is greater than *K*.

Several drawbacks of the P-M diffusion filter were mentioned in the literature [3],[6]. In [3] Catte et al. showed that the P-M filter is ill-posed and leads sometimes to a noise enhancement when the noise level is high. The authors proposed to use a Gaussian smoothing in the computation of the diffusivity. They prove the existence, uniqueness and regularity of the solution provided by their improved filter. The results obtained are similar to those of Perona and Malik for small sizes of the Gaussian kernel, however, for larger sizes, the filter tends to eliminate small scale details. Another undesired effect- the pinhole effect- was recently mentioned in [6]. Typically when a group of pixels of intermediate values is placed near a high transition zone the diffusion does not stop and extra-region smoothing is performed. The PM filter can also develop terraces for slow varying edges. This is called the staircasing effet.

In [11] Weickert and Benhamouda, by using central differences in approximating the gradient, show that due to the implicit diffusion induced by such a discrete scheme, the corresponding discrete PM filter is well posed.

In [1], Alvarez et al. proposed a mean curvature motion approach for the non-linear diffusion filter:

$$\frac{\partial U}{\partial t} = g(|G_\sigma * \nabla U|)[1 - h(|\nabla U|)\Delta U + h(|\nabla U|)|\nabla U| div(\frac{\nabla U}{|\nabla U|})] \quad (3)$$

g(.) is a decreasing function of the smoothed gradient controlling the diffusion, *h(.)* is a smooth non-decreasing function. For small gradient values (lower than a given threshold), the process is isotropic. For high values, the diffusion is anisotropic: and acts along the direction of the structures.

Recently, Kornprobst et al. proposed in [4] a combined diffusion-reaction-coupling model. The filter uses a diffusion term according to (4), a reaction term based on the theory of shock filters developed by Osher and Rudin [7] and Alvarez and Mazorra [2], and a coupling term that keeps the solution close to the original image.

A global, coherence based method, was proposed by Weickert [10]. Starting from a smoothed version of the gradient at a local scale σ, by convolving the components of the gradient at a global scale ρ, a structure tensor is constructed.

The author builds up a tensor driven diffusion:

$$\frac{\partial U}{\partial t} = div[D(J_\rho(\nabla U_\sigma))\nabla U], \quad (4)$$

imposing the way to diffuse along the smallest contrast direction and the orthogonal one, weighting the diffusion according to a coherence measure. Such a filter is capable of closing interrupted line-like structures but, at junctions, corner rounding is also observed and the filter can develop false, anisotropic structures.

Finally, let us note that the different diffusion processes presented above can be viewed as:

$$\frac{\partial U}{\partial t} = c_\xi U_{\xi\xi} + c_\eta U_{\eta\eta} \quad (5)$$

where η represents the direction of the gradient and ξ is locally tangential to the structures.

The approaches described above work as well on 2D and 3D images, or they can be easily extended to the 3D case. Specific works were also proposed in the recent literature for 3D applications. For instance, we can cite the level set approach of Preusser and Rumpf for anisotropic diffusion of 3D medical images [9].

We propose an original approach dedicated to the filtering and the enhancing of strongly oriented 3D structures. The considered data can be viewed as a stack of surfaces. First, the process consists in computing the orientation in the neighbourhood of each pixel and then to force the diffusion along the directions of the structures. Considering the local approximation of the oriented plane, the proposed numeric scheme is simply derived from the link between finite differences and interpolation techniques and is based on a grey level interpolation. We devote Section 2 in presenting our filter. Some experimental results are given in Section 3 and finally some conclusions and future work related aspects are presented in Section 4.

2. 3D directional diffusion

In this section we will briefly present the 2D version of the directional diffusion filter. We then extend the filter to the 3D case.

2.1. 2D Directional diffusion filter

We consider a particular case of the general equation (5) with no diffusion term along the direction of the gradient:

$$\begin{cases} \frac{\partial U}{\partial t} = c_\xi U_{\xi\xi} \\ \vec{\xi} = \vec{i}\cos\theta + \vec{j}\sin\theta, |\xi| = 1 \\ c_\xi \in [-1,1] \end{cases} \quad (6)$$

Where the vector ξ is oriented along the structure.

● Computation of the structure orientation

Our first aim is to obtain a robust estimation of the orientation of the surfaces.

The use of a classical gradient provides us with a vector orthogonal to the local surface.

Then we consider a more regional orientation by computing a Principal Component Analysis. If $V = \{V_i\}_{1 \le i \le n}$ is the field of n gradient vectors in the neighborhood of a given pixel, the regional orientation of the field is obtained as the principal axis of the moment tensor [11]:

$$M = \frac{1}{n}\sum_{i=1}^{n} V_i V_i^T \quad (7)$$

This can be seen as a Principal Component Analysis of the autocorrelation matrix of the considered vectors. So, the orientation of the eigenvector corresponding to the highest eigenvalue gives us the normal direction of the surface.

● Diffusivity function

The diffusion is locally determined by θ and by the diffusivity function c_ξ. The approaches described in section 1 use generally a diffusivity that is function of the gradient norm. We propose to use a function of the absolute value of first derivative in the direction of the structure $c_\xi = f(|U_\xi|)$ more precisely, we propose :

$$\frac{\partial U}{\partial t} = [\underbrace{g(|U_\xi|) + g'(|U_\xi|)U_\xi}_{c_\xi}]U_{\xi\xi} \quad (8)$$

(8) is equivalent to:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial \xi} g(|U_\xi|)U_\xi \quad (9)$$

where g() is a decreasing function similar to those proposed by Perona-Malik (eq.1).

● Numerical scheme

We propose the following numerical scheme:

$$\frac{\partial U}{\partial t} = \{g[D_\xi^+(U)]D_\xi^+(U) + g[D_\xi^-(U)]D_\xi^-(U) \quad (10)$$

with:

$$D_\xi^+(U) = \frac{U(\xi + d\xi) - U(\xi)}{d\xi}; D_\xi^-(U) = \frac{U(\xi - d\xi) - U(\xi)}{d\xi}$$

In each pixel, the tangent to the surface is determined by the use of a PCA (subsection 2.3). As the figure 1 shows, for any direction $\vec{\xi}$, the estimation of the values $U(\xi + d\xi), U(\xi - d\xi)$ corresponding respectively to a positive and negative moving $d\xi$ needs a sub-pixel precision.

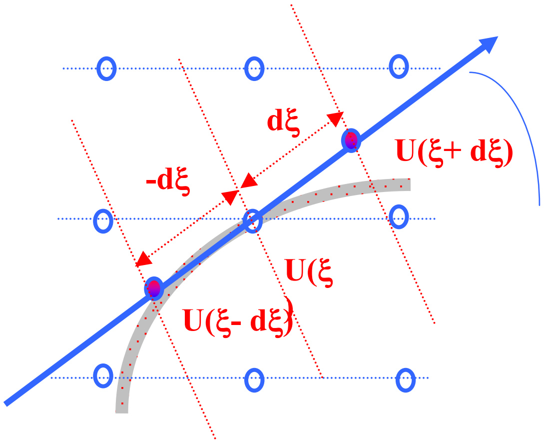

Figure 1: Interpolation on the tangent of the surface

As the values of the luminance are known for the nodes of the rectangular grid, the problem is a classical interpolation problem.
There are many interpolation techniques described in the literature. In order to limit the time consumption in the iterative process, we propose to use a classical 3 order polynomial interpolation.

2.2. 3D Extension

The 3D extension of the directional diffusion is immediate.
The first eigenvector of a PCA provides us with the direction of the normal of the surface. Then the diffusion will be operated in the plane **P** corresponding to the second and third eigenvectors.
As the direction of these eigenvectors on the tangential plane are arbitrary, the proposed numerical scheme take into account several moves $d\xi$: we will consider a set of pairs of points (M_i^+ ; M_i^-) shared out on a circle **C** with radius $d\xi$ around the pixel (figure 2).
The set of points selected on **C** can be considered as a sample chosen on the plane at a distance $d\xi$.
Let us denote *n*, the number of pairs. Then, the points are located on **C** and spaced out with an angle π / n . *n* is a parameter of the method. The results presented in section 3 were obtained using 4 pairs (M_i^+ ; M_i^-).
As in the 2D version, the method needs a sub-pixel precision. The values $U(M_i^+);U(M_i^-)$ are obtained using a two step quadratic interpolation [5]. This choice was motivated by the need to reduce the computation time for 3D applications. This stage will be developed in the full paper.

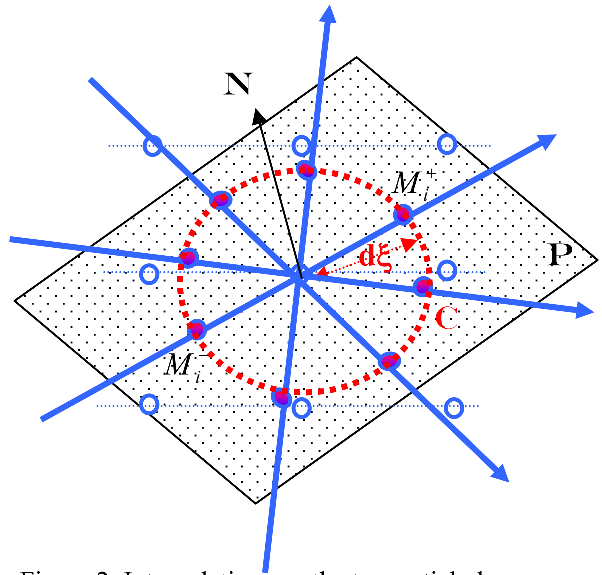

Figure 2: Interpolations on the tangential plane

Finally the proposed approach consists in computing:

$$\frac{\partial U}{\partial t} = \sum_i \left[g[D_i^+(U)]D_i^+(U) + g[D_i^-(U)]D_i^-(U) \right] \quad (11)$$

where :

$$D_i^+(U) = \frac{U(M_i^+) - U(\xi)}{d\xi} ; D_i^-(U) = \frac{U(M_i^-) - U(\xi)}{d\xi}$$

3. Results

Considering 3D seismic data, the aim of the geologist is to focuse on the faults. Then our approach is used as a pre processing leading to a reduction of the noise while preserving the faults, so as to improve the detection of them. The efficiency of our filter will then be proved if the faults detector produces a more accurate result. Thus we will use the results obtained by a faults detector developed by our partner TOTAL to evaluate the quality of the filtering.
As the seismic data produced by TOTAL are confidential, we will show here results obtained with synthetic 3D images, composed of a stack of planes, broken by two faults, a right one and a curved one. Fig 3a shows a front view of the data block. A gaussian white noise was added to this block leading to an SNR of 5dB (fig3c). Finally the restored block using our approach is given in Fig. 2e. The right part of the figure 3 presents a top view of the faults blocks respectively computed on the original block (3b), the noisy block (3d) and the restored one (3f).
As can be seen, the smoothing has been done along the layers; however the faults have been enhanced. This allows the faults detector to produce better results, while noisy points, resulting of a misdetection of a fault due to the noise become rare.

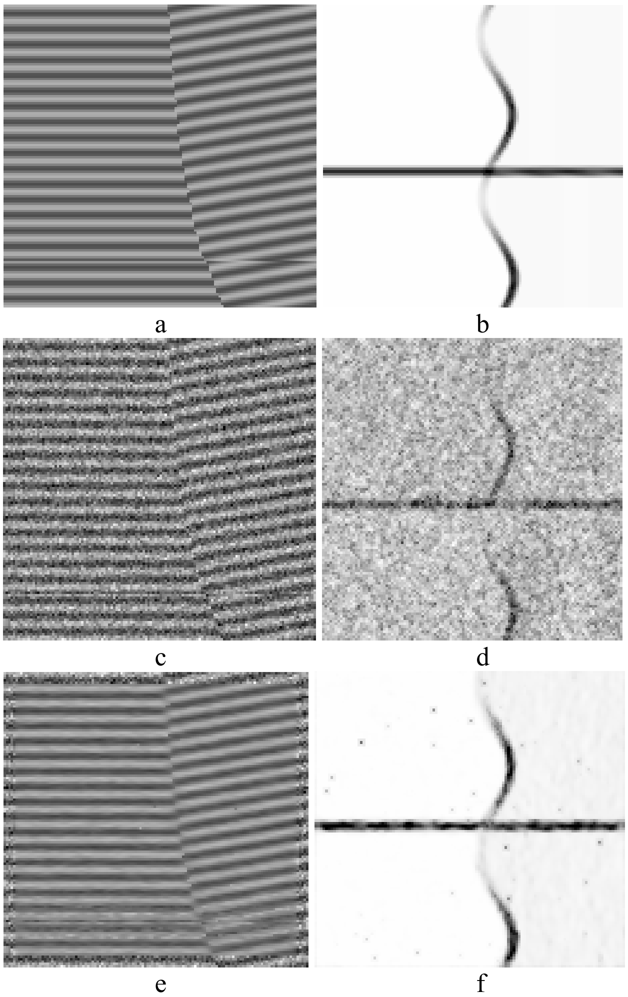

Figure 3. Comparative results of faults detection on the original bloc (top), the noisy one (middle) and the restored one (bottom).

4. Conclusion

This paper presents a method of 3-D data filtering, which is able to smooth along a surface. This method is used as a preprocessing for seismic images. The process of filtering itself follows a step of computation of a regional orientation, which indicates for each pixel the dip of the local plane. Then the diffusion is operated by using interpolated values corresponding to a set of points belonging to the plane.

As an extension of the method, and in order to reduce the computation time, future works will be devoted to the extension of the neighborhood, and to the study of the impact that this extension can have on the quality of the results.

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