LOCALIZATION OF MULTIPLE MOVING SOURCES USING RECURSIVE EM ALGORITHM

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ABSTRACT

This work deals with maximum likelihood (ML) direction of arrival (DOA) estimation of multiple moving sources. Based on the recursive EM algorithm, we develop two recursive procedures to estimate the time-varying DOA parameter. The first procedure requires no prior knowledge about the source movement. The second procedure assumes that the motion of moving sources is described by a linear polynomial model. The proposed recursion updates the polynomial coefficients when a new data arrives. The suggested approaches have two major advantages: simple implementation and easy extension to wideband signals. Numerical experiments show that both procedures provide excellent results in a slowly changing environment. When the DOA parameter changes fast or two source directions cross with each other, the procedure designed for a linear polynomial model has a better performance than the general procedure.

1. INTRODUCTION

The problem of estimating direction of arrival (DOA) of plane waves impinging on a sensor array is of fundamental importance in many applications such as radar, sonar, geophysics and wireless communication. The maximum likelihood (ML) method is known to have excellent statistical performance and is robust against coherent signals and small sample numbers. However, the high computational cost associated with ML method makes it less attractive in practice.

To improve the computational efficiency of the ML approach, numerical methods such as the expectation and maximization (EM) algorithm [6] was suggested in [7]. Recursive procedures based on the recursive EM algorithm for estimating constant DOA parameters were discussed in [5] [9]. Similar procedures for tracking multiple moving sources were studied in [5] [8]. In [8], the authors focused on narrow band sources and assumed known signal waveforms.

The recursive EM algorithm is a stochastic approximation procedure for finding ML estimates. It was first suggested by Titterington [11]. As it was pointed out by Titterington, recursive EM can be seen as a sequential approximation of the EM algorithm. The gain matrix of recursive EM is the inversion of the augmented data information matrix of EM. Since the augmented data usually has a simple structure, the recursive EM algorithm is very easy to implement. For constant parameter, estimates generated by recursive EM are strongly consistent and asymptotically normally distributed. For time-varying parameter, the tracking ability of a stochastic approximation procedure depends mainly on the dynamics of the true parameter, gain matrix and step size [1].

Based on recursive EM, we shall derive two recursive procedures for estimating time-varying DOA. The first procedure does not assume any motion model. The only condition required is that the unknown parameter changes slowly with time. The second procedure assumes that the time-varying DOA parameter $\boldsymbol{\theta}(t)$ is described by a linear polynomial of time. This model is important since a smooth function $\boldsymbol{\theta}(t)$ can be approximated by a local linear polynomial in a short time interval [10]. The procedure reported in [5] employs a decreasing step size to estimate the polynomial coefficients. However, since the DOA parameter $\boldsymbol{\theta}(t)$ and the log-likelihood function change with time, a decreasing step size may not capture the non-stationary feature of the underlying system over a long period. To overcome this problem, we suggest a constant step size to be used in the algorithm. It is noteworthy that both procedures are aimed at maximizing the expected concentrated likelihood function. Introducing a linear polynomial model implies increasing the dimension of the parameter space. With the additional degree of freedom, the procedure designed for a linear polynomial model should perform better than the general one.

This paper is outlined as follows. We describe the signal model and the recursive EM algorithm briefly in section 2 and section 3. Section 4 presents two recursive procedures for localizing moving sources. Simulation results are discussed in section 5. We give concluding remarks in section 6.

2. PROBLEM FORMULATION

Consider an array of N sensors receiving M far field waves from unknown time-varying directions $\boldsymbol{\theta}(t) = [\theta_1(t), \ldots, \theta_M(t)]$. The array output $\boldsymbol{x}(t) \in \mathbb{C}^{N\times 1}$ at time instant t is expressed as

$$\boldsymbol{x}(t) = \boldsymbol{H}(\boldsymbol{\theta}(t))\boldsymbol{s}(t) + \boldsymbol{u}(t), \quad t = 1, 2, \ldots \tag{1}$$

where the steering matrix

$$\boldsymbol{H}(\boldsymbol{\theta}(t)) = [\ \boldsymbol{d}(\theta_1(t))\ \ldots\ \boldsymbol{d}(\theta_M(t))\ ] \quad \in \mathbb{C}^{N\times M} \tag{2}$$

consists of M steering vectors $\boldsymbol{d}(\theta_m(t)) \in \mathbb{C}^{N\times 1}$, $(m = 1, \ldots, M)$. The signal waveform $\boldsymbol{s}(t) = [s_1(t), \ldots, s_M(t)]^T \in \mathbb{C}^{M\times 1}$ is considered as unknown and deterministic. $(\cdot)^T$ denotes vector transpose. The noise process $\boldsymbol{u}(t) \in \mathbb{C}^{N\times 1}$ is independent identically complex normally distributed with zero mean and covariance matrix $\nu\boldsymbol{I}$, where ν represents the unknown noise spectral parameter and $\boldsymbol{I}$ is the identity matrix.

Our central interest is to estimate the time-varying DOA parameter $\boldsymbol{\theta}(t)$ recursively from the observation $\boldsymbol{x}(t)$. We assume that a good initial estimate $\boldsymbol{\theta}^0$ is available at the beginning of the recursion.

3. RECURSIVE PARAMETER ESTIMATION USING INCOMPLETE DATA

The recursive EM algorithm suggested by Titterington is a stochastic approximation procedure for finding maximum likelihood estimates (MLE). As pointed out in [11], there is a strong relationship between this procedure and the EM algorithm [6]. Using Taylor expansion, Titterington showed that approximately, recursive EM maximizes EM's augmented log-likelihood sequentially. The unknown parameter is considered as constant in [11]. In the fixed parameter case, a properly chosen decreasing step size ensures strong consistency and asymptotic normality of the algorithm [4] [11].

Suppose $\boldsymbol{x}(1), \boldsymbol{x}(2), \ldots$ are independent observations, each with underlying probability density function (pdf) $f(\boldsymbol{x};\boldsymbol{\vartheta})$, where $\boldsymbol{\vartheta}$ denotes an unknown constant parameter. The augmented data associated with EM $\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots$ are characterized by the pdf $f(\boldsymbol{y};\boldsymbol{\vartheta})$. Let $\boldsymbol{\vartheta}^t$ denote the estimate after t observations. The following procedure is aimed at finding the true parameter $\boldsymbol{\vartheta}$ which may coincide with the MLE in the asymptotic sense

$$\boldsymbol{\vartheta}^{t+1} = \boldsymbol{\vartheta}^t + \epsilon_t \, \mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)^{-1} \boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t) \tag{3}$$

where ϵ_t is a decreasing step size and

$$\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t) = \mathsf{E}\left[-\boldsymbol{\nabla}_{\boldsymbol{\vartheta}} \boldsymbol{\nabla}_{\boldsymbol{\vartheta}}^T \log f(\boldsymbol{y};\boldsymbol{\vartheta}) \mid \boldsymbol{x}(t), \boldsymbol{\vartheta}\right] |_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t}, \tag{4}$$

$$\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t) = \boldsymbol{\nabla}_{\boldsymbol{\vartheta}} \log f(\boldsymbol{x}(t);\boldsymbol{\vartheta})|_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t} \tag{5}$$

represent the augmented information matrix and gradient vector, respectively. $\boldsymbol{\nabla}_{\boldsymbol{\vartheta}}$ is a column gradient operator with respect to $\boldsymbol{\vartheta}$. Under mild conditions, the estimates generated by (3) are strongly consistent and asymptotic normally distributed. The augmented data $\boldsymbol{y}$ usually has a simpler structure than the observed data $\boldsymbol{x}$. Therefore, the augmented data information matrix $\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)$ is easier to compute and invert than the observed data information matrix $\mathcal{I}(\boldsymbol{\vartheta}^t) = \mathsf{E}\left[-\boldsymbol{\nabla}_{\boldsymbol{\vartheta}} \boldsymbol{\nabla}_{\boldsymbol{\vartheta}}^T \log f(\boldsymbol{x};\boldsymbol{\vartheta}) \mid \boldsymbol{x}(t), \boldsymbol{\vartheta}\right] |_{\boldsymbol{\vartheta}=\boldsymbol{\vartheta}^t}$. Replacing $\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)^{-1}$ with $\mathcal{I}(\boldsymbol{\vartheta}^t)^{-1}$ in (3), we obtain the optimal convergence rate in the Cramér-Rao sense at a much higher computational expense. Using $\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)^{-1}$ as the gain matrix is a trade-off between convergence rate and computational effort.

When the parameter of interest varies with time, a decreasing step size such as $\epsilon_t = t^{-\alpha}, 1/2 < \alpha \leq 1$ can not capture the non-stationary feature of the underlying system. A classical way to overcome this difficulty is to replace ϵ_t with a constant step size ϵ. In general, a large step size reduces the bias and increases the variance of the estimates [1]. A small step size has the opposite effects. Since the time varying parameter $\boldsymbol{\vartheta}(t)$ may follow a complicated dynamics, an exact investigation of the convergence behavior of the algorithm

$$\boldsymbol{\vartheta}^{t+1} = \boldsymbol{\vartheta}^t + \epsilon \, \mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)^{-1} \boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t) \tag{6}$$

is only possible when certain assumptions are made on the parameter model. More discussion about convergence properties of a stochastic approximation procedure in a non-stationary environment can be found in [1].

4. LOCALIZATION OF MOVING SOURCES

The recursive EM with constant step size (6) is applied to estimate the time-varying DOA parameter $\boldsymbol{\theta}(t)$. We start with a general case in which $\boldsymbol{\theta}(t)$ changes slowly with time and then considers a linear polynomial model

$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}_0 + t\,\boldsymbol{\theta}_1, \tag{7}$$

where $\boldsymbol{\theta}_0 = [\theta_{01}, \ldots, \theta_{0M}]^T$ and $\boldsymbol{\theta}_1 = [\theta_{11}, \ldots, \theta_{1M}]^T$. The linear polynomial (7) can be seen as a truncated Taylor expansion which gives a good description for the source motion in a small observation interval [10].

4.1. General Case (REM I)

From the signal model in section 2, we know that the array observation $\boldsymbol{x}(t)$ is complex normally distributed with the the log-likelihood function

$$\log f(\boldsymbol{x}(t);\boldsymbol{\vartheta}) = -\Big[N \log \pi + N \log \nu + \frac{1}{\nu}\big(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\theta}(t))\boldsymbol{s}(t)\big)^H \big(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\theta}(t))\boldsymbol{s}(t)\big)\Big] \tag{8}$$

where $\boldsymbol{\vartheta} = [\boldsymbol{\theta}(t)^T, \boldsymbol{s}(t)^T, \nu]^T$ and $(\cdot)^H$ denotes the Hermitian transpose.

According to (6), all elements in $\boldsymbol{\vartheta}$ should be updated simultaneously. To avoid a complicated gain matrix, the procedure (6) is only applied to $\boldsymbol{\theta}(t)$. The estimate for signal waveform and noise level, denoted by $\boldsymbol{s}^t = [s_1^t, s_2^t, \ldots, s_M^t]^T$ and ν^t respectively, are updated by computing their ML estimates once the current DOA estimate is available. In the following, we omit the dependence of DOA parameter on t and use $\boldsymbol{\theta}$ instead of $\boldsymbol{\theta}(t)$.

Taking the first derivative of $f(\boldsymbol{x}(t);\boldsymbol{\vartheta})$ with respect to θ_m, we obtain the mth element of the gradient vector $\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t)$ [4]

$$\left[\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t)\right]_m = \frac{2}{\nu^t} \mathrm{Re}\left[(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\theta}^t)\boldsymbol{s}^t)^H (\boldsymbol{d}'(\theta_m^t) s_m^t)\right], \tag{9}$$

where $\boldsymbol{d}'(\theta_m) = \partial \boldsymbol{d}(\theta_m)/\partial \theta_m$.

The augmented data $\boldsymbol{y}(t)$ is obtained by decomposing the array output into its signal and noise parts. Formally it is expressed as

$$\boldsymbol{y}(t) = [\,\boldsymbol{y}_1(t)^T, \ldots, \boldsymbol{y}_m(t)^T, \ldots, \boldsymbol{y}_M(t)^T]^T. \tag{10}$$

The augmented data associated with the mth signal

$$\boldsymbol{y}_m(t) = \boldsymbol{d}(\theta_m) s_m(t) + \boldsymbol{u}_m(t) \tag{11}$$

is complex normally distributed with mean $\boldsymbol{d}(\theta_m)s_m(t)$ and covariance matrix $\nu_m \boldsymbol{I}$ with the constraint $\sum_{m=1}^M \nu_m = \nu$. A convenient choice is $\nu_m = \nu/M$. The corresponding log-likelihood is given by

$$\log f(\boldsymbol{y}(t);\boldsymbol{\vartheta}) = -\sum_{m=1}^{M} \Big[N \log \pi + N \log(\nu/M) + \frac{M}{\nu}(\boldsymbol{y}_m(t) - \boldsymbol{d}(\theta_m)s_m(t))^H (\boldsymbol{y}_m(t) - \boldsymbol{d}(\theta_m)s_m(t))\Big]. \tag{12}$$

Since the signals are decoupled through the augmentation scheme (10), $\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)$ is a $M \times M$ diagonal matrix when we only consider the DOA parameters $\boldsymbol{\theta}$. According to (4), the mth diagonal element of $\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)$ is given by

$$[\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)]_{mm} = \frac{2}{\nu^t}\mathrm{Re}\Big[-(\boldsymbol{d}^{''}(\theta_m^t)s_m^t)^H(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\theta}^t)\boldsymbol{s}^t) + M\|\boldsymbol{d}^{'}(\theta_m^t)s_m^t\|^2\Big], \tag{13}$$

where $\boldsymbol{d}^{''}(\theta_m) = \partial^2\boldsymbol{d}(\theta_m)/\partial\theta_m^2$.

Once the estimate $\boldsymbol{\theta}^{t+1}$ is available, the signal and noise parameters are obtained by computing their ML estimates at current $\boldsymbol{\theta}^{t+1}$ and $\boldsymbol{x}(t)$ as follows

$$\boldsymbol{s}^{t+1} = \boldsymbol{H}(\boldsymbol{\theta}^{t+1})^{\#}\boldsymbol{x}(t), \tag{14}$$

$$\nu^{t+1} = \frac{1}{N}\mathrm{tr}\left[\boldsymbol{P}(\boldsymbol{\theta}^{t+1})^{\perp}\widehat{\boldsymbol{C}}_{\boldsymbol{x}}(t)\right], \tag{15}$$

where $\boldsymbol{H}(\boldsymbol{\theta}^{t+1})^{\#}$ is the generalized left inverse of the matrix $\boldsymbol{H}(\boldsymbol{\theta}^{t+1})$, $\boldsymbol{P}(\boldsymbol{\theta}^{t+1})^{\perp} = \boldsymbol{I} - \boldsymbol{P}(\boldsymbol{\theta}^{t+1})$ is the orthogonal complement of the projection matrix $\boldsymbol{P}(\boldsymbol{\theta}^{t+1}) = \boldsymbol{H}(\boldsymbol{\theta}^{t+1})\boldsymbol{H}(\boldsymbol{\theta}^{t+1})^{\#}$ and $\widehat{\boldsymbol{C}}_{\boldsymbol{x}}(t) = \boldsymbol{x}(t)\boldsymbol{x}(t)^H$.

4.2. Linear Polynomial Model (REM II)

We consider moving sources described by the linear polynomial model (7). The recursive EM algorithm is applied to estimate $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_1$. For notational simplicity, we define the extended DOA parameter as $\boldsymbol{\Theta} = [\boldsymbol{\Theta}_1^T, \ldots, \boldsymbol{\Theta}_m^T, \ldots, \boldsymbol{\Theta}_M^T]^T$ where $\boldsymbol{\Theta}_m = [\theta_{0m}, \theta_{1m}]^T$. Similarly to the procedure presented in subsection 4.1, recursive EM is only applied to update the DOA parameter $\boldsymbol{\Theta}$, rather than $\boldsymbol{\vartheta} = [\boldsymbol{\Theta}^T, \boldsymbol{s}(t)^T, \nu]^T$.

Based on this approach, the $2m$th and $(2m+1)$st element of the gradient vector $\boldsymbol{\gamma}(\boldsymbol{x}(t), \boldsymbol{\vartheta}^t)$ are given by [5]

$$\frac{2}{\nu^t}\mathrm{Re}\left[(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}^t)^H(\boldsymbol{d}'(\boldsymbol{\Theta}_m^t)s_m^t)\right] \tag{16}$$

and

$$\frac{2t}{\nu^t}\mathrm{Re}\left[(\boldsymbol{x}(t) - \boldsymbol{H}(\boldsymbol{\Theta}^t)\boldsymbol{s}^t)^H(\boldsymbol{d}'(\boldsymbol{\Theta}_m^t)s_m^t)\right], \tag{17}$$

respectively, where $\boldsymbol{d}'(\cdot) = \partial\boldsymbol{d}(\theta_m)/\partial\theta_m$. Note that the time–varying $\boldsymbol{\theta}(t)$ is calculated at the current $\boldsymbol{\Theta}^t$ according to the linear model (7).

Because each source is described by two unknown parameters, the augmented data information matrix becomes block diagonal. For a higher computational efficiency, rather than using this block diagonal matrix in the recursion directly, we consider an alternative matrix $\tilde{\mathcal{I}}_{EM}(\boldsymbol{\vartheta}^t)$, which is the diagonal part of $\mathcal{I}_{EM}(\boldsymbol{\vartheta}^t)$.

Let $\boldsymbol{d}^{''}(\cdot) = \partial^2\boldsymbol{d}(\theta_m)/\partial\theta_m^2$. According to the augmentation scheme specified above, the $2m$th and $(2m+1)$st diagonal components of $\tilde{\mathcal{I}}_{\mathrm{EM}}(\boldsymbol{\Theta}^t)$ are given by

$$\frac{2}{\nu^t}\mathrm{Re}\Big[(-\boldsymbol{d}^{''}(\boldsymbol{\Theta}_m^t)s_m^t)^H\left(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^t)s_m^t\right)+M\|\boldsymbol{d}^{'}(\boldsymbol{\Theta}_m^t)s_m^t\|^2\Big] \tag{18}$$

and

$$\frac{2t^2}{\nu^t}\mathrm{Re}\Big[(-\boldsymbol{d}^{''}(\boldsymbol{\Theta}_m^t)s_m^t)^H\left(\boldsymbol{x}(t)-\boldsymbol{H}(\boldsymbol{\Theta}^t)s_m^t\right)+M\|\boldsymbol{d}^{'}(\boldsymbol{\Theta}_m^t)s_m^t\|^2\Big], \tag{19}$$

respectively.

Similarly to the general case, the signal and noise parameter are updated by (14) and (15) once the estimate $\boldsymbol{\Theta}^{t+1}$ is available. The parameter $\boldsymbol{\theta}^{t+1}$ in (14) and (15) is replaced by $\boldsymbol{\Theta}^{t+1}$.

5. NUMERICAL EXPERIMENTS

We test the proposed algorithms by numerical experiments. The narrow band signals generated by three sources of equal power are received by a uniformly linear array of 15 sensors with inter-element spacings of half a wavelength. The Signal to Noise Ratio (SNR), defined as $10\log(s_m(t)^2/\nu)$, $m = 1, 2, 3$, is kept at 20 dB. The motion of the moving sources is described by a linear polynomial model (7). Two different parameter sets $\{\boldsymbol{\theta}_0, \boldsymbol{\theta}_1\}$ are assumed in the experiments. Each experiment performs 200 trials.

In the first experiment, we consider fast moving sources. The true parameters are given by $\boldsymbol{\theta}_0 = [10°,\ 60°,\ 66°]$, $\boldsymbol{\theta}_1 = [0.6°,\ -1.0°,\ 0.4°]$ where $\boldsymbol{\theta}_1$ is measured by degree per time unit. The initial estimates for REM II are $\boldsymbol{\theta}_0^0 = [8.5°,\ 58°,\ 68°]$, $\boldsymbol{\theta}_1^0 = [0.4°,\ -0.8°,\ 0.3°]$. The initial estimate for REM I is given by $\boldsymbol{\theta}_0^0$. Both algorithms use a constant step size $\epsilon = 0.8$. Fig. 1 presents the true values of $\boldsymbol{\theta}(t)$ and an example of estimated trajectories obtained by REM I and REM II. Note that two source directions cross with each other at $t = 32$. Obviously, REM I can not follow fast moving sources at all. In contrast, the estimated trajectory obtained by REM II is very close to the true one. Fig. 2 shows the mean square errors (RMSE) of the DOA estimates. Since REM I fails to track the moving sources, the corresponding RMSE grows with increasing time. On the other hand, the RMSE associated with REM II decreases rapidly at the beginning of the recursion and then increases slightly. From time instant $t = 20$ to $t = 50$, the RMSE does not vary much.

The second experiment involves three slowly moving sources. The true parameter values are given by $\boldsymbol{\theta}_0 = [30°,\ 50°,\ 62°]$, $\boldsymbol{\theta}_1 = [0.06°,\ -0.1°,\ 0.05°]$. The initial estimates for REM II are $\boldsymbol{\theta}_0^0 = [29°,\ 49°,\ 61°]$, $\boldsymbol{\theta}_1^0 = [0°,\ 0°,\ 0°]$. The initial estimate for REM I is given by $\boldsymbol{\theta}_0^0$. Both algorithms use a constant step size $\epsilon = 0.6$. Fig. 3 presents the true and estimated trajectories. Note that two source directions cross with each other at $t = 126$. The estimated trajectory by REM I is close to the true one when no crossing happens. Between $t = 100$ and $t = 230$ where two source directions cross with each other, the estimated trajectories associated with the first two sources do not get close to each other. Instead, they just depart in the vicinity of $t = 126$. For the same scenario, REM II provides a more accurate estimate. Fig. 3 shows that the crossing point causes a large deviation of the trajectories. Due to a less accurate estimate for $\boldsymbol{\theta}_1$, REM II leads to a slightly worse estimate than in the first experiment. Fig. 4 shows that the RMSE increases after the crossing point $t = 126$ in fig. 4. Both algorithms perform equally well in this scenario.

From simulation results we conclude that REM I is suitable for tracking slowly time-varying DOA parameters, REM II performs well for both slowly and fast moving sources. In simulation we also observe that both procedures generate accurate estimates when there is no crossing point. When two source directions coincide with each other, the steering matrix $\boldsymbol{H}(\boldsymbol{\theta}(t))$ becomes rank deficient. The signal waveform $\boldsymbol{s}(t)$ can not be determined properly. Consequently the DOA parameter can not be estimated accurately. Since REM II incorporates a linear polynomial model describing the moving sources, it has a better tracking ability than REM I when this critical situation occurs.

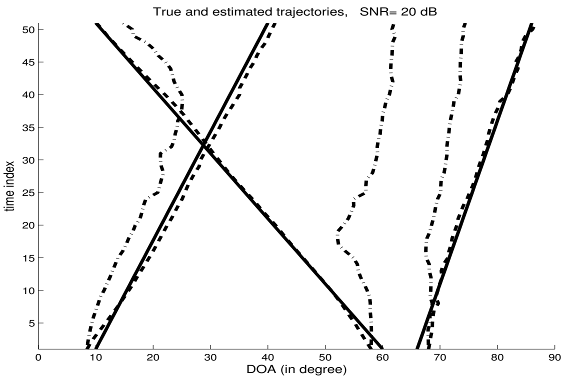

Fig. 1. True trajectory (—) and estimated trajectories by REM I (—.) and REM II (− −).

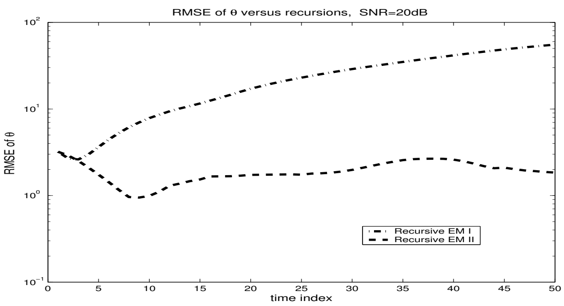

Fig. 2. RMSE vs time. SNR=20 dB.

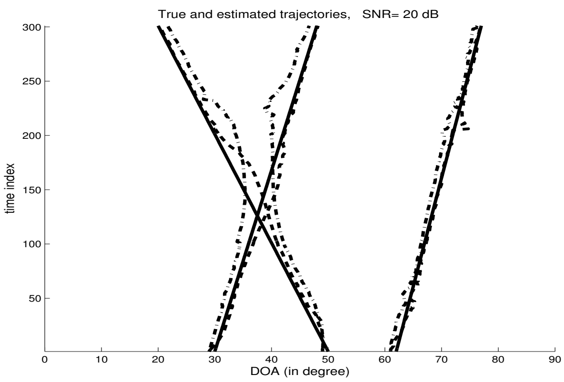

Fig. 3. True trajectory (—) and estimated trajectories by REM I (—.) and REM II (− −).

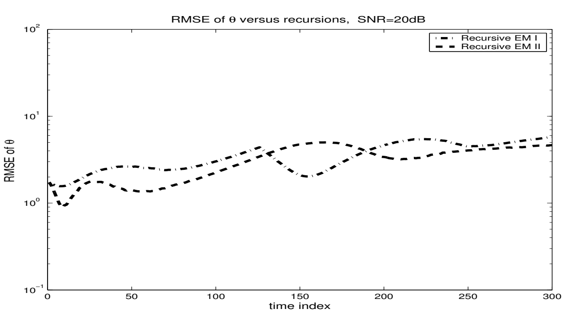

Fig. 4. RMSE vs time. SNR=20 dB.

6. CONCLUSION

We addressed the problem of localizing multiple moving sources. Two recursive procedures are proposed to estimate the time-varying DOA parameter. We applied the recursive EM algorithm to a general case in which the motion of the sources is arbitrary and a specific case in which the motion of sources is described by a linear polynomial model. Because of the simple structure of the gain matrix, the suggested procedures are easy to implement. Furthermore, extension of our approaches to broadband signals is straightforward. Numerical experiments showed that our approaches provide excellent results in a slowly changing environment. When the DOA parameter changes fast or two source directions cross with each other, the procedure derived for a linear polynomial model has a better performance than the general procedure.

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