PERFORMANCE OF FREQUENCY-DOMAIN MIMO EQUALIZATION FOR CYCLIC-PREFIXED SINGLE-CARRIER SPATIAL MULTIPLEXING

Mikel Mendicute, Jon Altuna, John Thompson and Vicente Atxa*

University of Mondragon
Loramendi 4, Mondragon, Spain
phone: +34 943739422, fax: +34 943739410
email: {mmendikute, jaltuna, batxa}@eps.mondragon.edu

*University of Edinburgh
Edinburgh EH93JL, UK
phone: +44 131 650 5585, fax: +44 131 650 6554
email: jst@ee.ed.ac.uk

ABSTRACT

Multiple-Input Multiple-Output (MIMO) signal processing combined with Orthogonal Frequency Division Multiplexing (OFDM) is a widely accepted solution to achieve the high bit rates that new communication standards require in frequency-selective wireless channels. In recent years, a new Frequency-Domain Equalized (FDE) Cyclic-Prefixed Single-Carrier (CPSC) block transmission system has been proven to achieve similar performance to OFDM for coded systems with the same overall complexity, avoiding its main drawbacks and becoming a candidate for future wireless standards. In this paper, a FDE CPSC spatial multiplexing system is analyzed and compared to OFDM. The applicability of different frequency-domain MIMO equalization and detection schemes is evaluated. Simulation-based performance results prove the potential of these techniques and highlight that FDE CPSC can equal or outperform OFDM in uncoded and high coding rate spatial multiplexing systems, as it has been shown in the literature for the SISO case.

1. INTRODUCTION

As new wireless communication network standards are being defined, the demand for higher throughput is growing enormously. Recent wireless local (WLAN) and metropolitan (WMAN) area network standards such as IEEE 802.11a, 802.11g, 802.16a and ETSI HIPERLAN/2 have chosen Orthogonal Frequency Division Multiplexing (OFDM) to overcome the wireless channel's high frequency selectivity. Newer not yet defined WLAN standards, such as 802.11n, aim to reach data rates of up to 300 Mbps. In order to achieve these high bit rates, Multiple-Input Multiple-Output (MIMO) signal processing techniques become necessary.

Two basic approaches have been studied to enhance OFDM with MIMO: Space-Time Coding (STC) and Spatial Multiplexing (SM) [1]. STC increases the diversity order of the communication system by coding over the different transmission antennas, which leads to a better performance. On the other hand, SM transmits independent data streams on each antenna simultaneously, thus allowing greater throughputs [2, 3].

Although OFDM has been included in many standards due to its simple equalization, alternative cyclic-prefixed or zero-padded single-carrier (SC) block transmission techniques have been proposed and developed in recent years [4, 5]. SC transmission avoids OFDM's three main drawbacks: Peak-to-average ratio of the signal power, frequency offset sensitivity and no multipath diversity in uncoded systems. The most simple and interesting of these SC techniques is the Frequency-Domain Equalized (FDE) Cyclic-Prefixed Single-Carrier (CPSC) transmission system. As it can be seen in Fig. 1, its overall complexity is similar to an OFDM system. As the equalization is done in the frequency domain, FFT and IFFT blocks are required in reception. The addition of the cyclic prefix avoids Inter-Block Interference (IBI) and transforms the linear convolution of the signal and the channel into circular, i.e., a product in the frequency domain. This system has been proven to fairly outperform OFDM for uncoded systems and yields similar BER performance in coded transmissions [4]. A very interesting feature of CPSC lies on the fact that the main complexity belongs to the receiver part, so it can be combined with OFDM, allowing asymmetric systems where most of the complexity resides at one side (base station, access point, etc.), as it has been proposed in new 802.16 proposals [6].

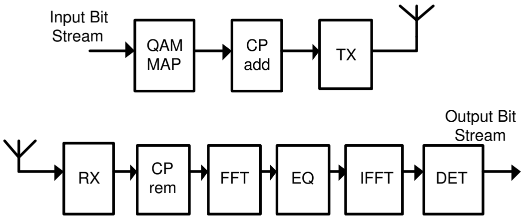

Figure 1: Wireless FDE CPSC transmission and reception systems.

This paper discusses the frequency-domain equalization and detection of CPSC spatial multiplexing systems. The applicability of the main MIMO equalization techniques is analyzed and their implementation is compared to OFDM based MIMO systems. The BER performances of both systems are evaluated with simulations.

The layout of this paper is as follows: Section 2 details the evaluated MIMO equalization techniques. In Section 3 the FDE CPSC-based MIMO transmission and reception systems are introduced. Section 4 shows the most important simulation results and some conclusions are drawn in Section 5.

2. MIMO EQUALIZATION AND DETECTION TECHNIQUES

Fig. 2 shows a single-carrier spatial multiplexing system for a frequency-flat channel with additive white Gaussian noise (AWGN). This model will be extended to a frequency-

---

Mr. Mendicute's work is partially sponsored by the Dpt. of Education, Universities and Research of the Basque Government through a Researcher Training Grant.

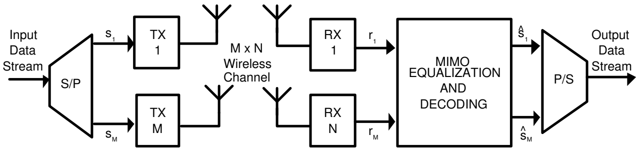

Figure 2: Flat MIMO system

selective channel case in Section 4. The system has M transmit and N receive antennas. The path loss from antenna m to antenna n is represented as h_{nm}. The sampled baseband system of a flat MIMO channel can be represented in matrix notation as:

$$\mathbf{r} = \sqrt{\frac{E_s}{M}}\tilde{\mathbf{H}}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{r}$ is an $Nx1$ vector containing the signal received in each antenna, $\mathbf{s}$ is an $Mx1$ vector with the signals transmitted simultaneously at each transmission branch and $\mathbf{n}$ is an $Nx1$ vector containing channel's AWGN. E_s is the transmitted signal power and $\tilde{\mathbf{H}}$ is a NxM channel matrix, defined as:

$$\tilde{\mathbf{H}} = \begin{pmatrix} h_{11} & h_{12} & ... & h_{1M} \\ h_{21} & h_{22} & ... & h_{2M} \\ : & : & : & : \\ h_{N1} & h_{N2} & ... & h_{NM} \end{pmatrix}$$

2.1 Linear non-OSIC MIMO detection

Maximum Likelihood (2) should be the optimal detection method for spatial multiplexing. However, it is computationally too complex due to the calculation of the output of S^M possible input vectors, where S is the number of symbols in the constellation.

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \left\| \mathbf{r} - \sqrt{\frac{E_s}{M}}\tilde{\mathbf{H}}\mathbf{s} \right\|_F^2 \tag{2}$$

MMSE (Minimum Mean Squared Error) and ZF (Zero-Forcing) [7] are simpler solutions, based in the product:

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{r} \tag{3}$$

where

$$\mathbf{G}_{ZF} = \sqrt{\frac{M}{E_s}}\tilde{\mathbf{H}}^{+} = \sqrt{\frac{M}{E_s}}(\tilde{\mathbf{H}}^{*}\tilde{\mathbf{H}})^{-1}\tilde{\mathbf{H}}^{*} \tag{4}$$

$$\mathbf{G}_{MMSE} = \sqrt{\frac{M}{E_s}}(\tilde{\mathbf{H}}^{*}\tilde{\mathbf{H}} + \alpha\mathbf{I}_{MxM})^{-1}\tilde{\mathbf{H}}^{*} \tag{5}$$

with $\alpha = NoisePower/SignalPower$. * and $^+$ stand for conjugate transpose and pseudo-inverse respectively. MMSE offers much better results than ZF in AWGN channels, but its performance is still quite far from optimal for $N = M$ [7].

2.2 Non-linear V-BLAST (OSIC) detection

An OSIC (Ordered Successive Interference Cancellation) based algorithm has been proposed in [8] to overcome the limitations of previous methods with relatively low complexity and has been named ZF-V-BLAST detection. It is based in the iteration of three steps:

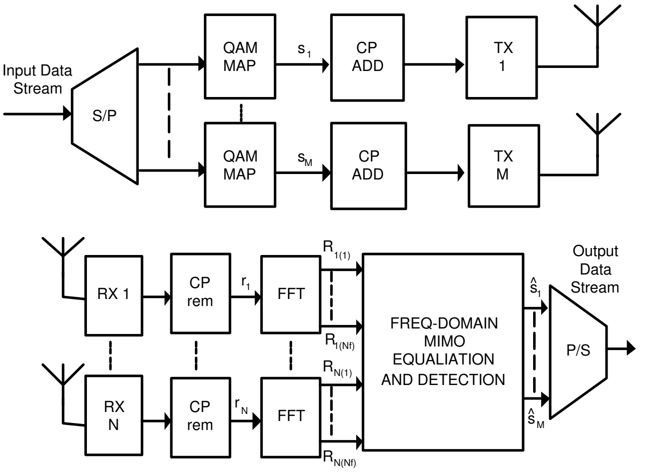

Figure 3: Frequency-Domain Equalized MIMO spatial multiplexing transmission and reception system.

1. Ordering: Determine the transmit antenna k with greatest SNR from the estimated channel, selecting the row $(\mathbf{G})_k$ of smallest norm of the nulling matrix $\mathbf{G}$:

$$k = \arg\min_{k} \|(\mathbf{G})_k\|^2 \quad \mathbf{G} = \mathbf{G}_{ZF} \tag{6}$$

2. Nulling and slicing: Detect transmitted symbol at antenna k $\hat{s}_k$:

$$y_k = ((\mathbf{G})_k)^T\mathbf{r} \tag{7}$$

$$\hat{s}_k = \arg\min_{\hat{s}_k} \|\hat{s}_k - y_k\| \tag{8}$$

3. Cancellation: Remove the effect of the detected symbol from received signal vector and its column $\tilde{\mathbf{H}}_k$ in the channel matrix:

$$\mathbf{r} = \mathbf{r} - \sqrt{\frac{E_s}{M}}\tilde{\mathbf{H}}_k\hat{s}_k \tag{9}$$

$$\tilde{\mathbf{H}}_{nm} = 0 \quad \forall m = k \tag{10}$$

Note that the diversity order increases by one in each iteration. Other V-BLAST detection algorithms such as MMSE-VBLAST or SOMLD (Successive Ordered Maximum Likelihood Detection) differ from ZF-V-BLAST in the calculation of the G matrix in (6) and the ordering criterion [3, 7].

3. FREQUENCY-DOMAIN EQUALIZED CYCLIC-PREFIXED SINGLE-CARRIER MIMO SYSTEM

The baseband flat MIMO channel model in (1) can be easily extended for a L tap frequency-selective channel:

$$\mathbf{r(k)} = \sqrt{\frac{E_s}{M}}\sum_{l=1}^{L}\tilde{\mathbf{H}}(l)\mathbf{s}(k-l) + \mathbf{n}(k) \tag{11}$$

The addition of the cyclic prefix avoids IBI and transforms the linear time convolution of the input signal and the channel response into a circular convolution, thus the model in (11) can be modelled as a product in frequency domain. This way a flat MIMO channel is obtained for each one of the Nf FFT frequency points of Fig. 3:

$$\mathbf{R}(f) = \sqrt{\frac{E_s}{M}}\mathbf{H}(f)\mathbf{S}(f) + \mathbf{N}(f) \tag{12}$$

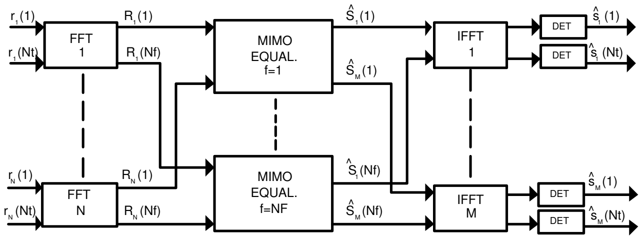

Figure 4: FDE MIMO equalization and detection. A MIMO equalizer is employed at each frequency.

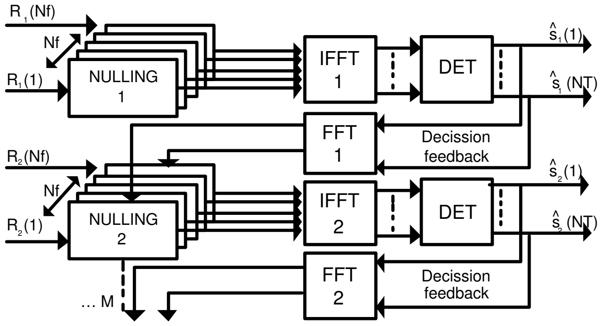

Figure 5: MIMO V-BLAST per-antenna signal detection and decision feedback.

R, **H**, **S** and **N** are the Nf-point FFT transforms of **r**, $\tilde{\mathbf{H}}$, **s** and **n**, respectively. Note that the MIMO equalization is not done on the information bearing symbols $\mathbf{s}(k)$, but on the frequency-domain FFT points $\mathbf{S}(f)$, so an IFFT is required before the symbol detection. This means that all symbols from an antenna must be equalized before they can be detected. This is particularly important for V-BLAST detection, as it will be shown later.

For the MIMO detection techniques that do not require any decision feedback, such as ZF or MMSE, the frequency-domain MIMO detection requires Nf MIMO equalizers, i.e., one for each of the FFT output points, as it can be seen in Fig. 4.

For MIMO equalization techniques requiring detected symbol feedback, such as ZF-V-BLAST or MMSE-V-BLAST, FDE CPSC implies several problematic changes because the IFFT and the FFT operations appear in the feedback chain as it can be seen in Fig. 5. Equations (7) and (8), slicing and cancellation, become:

$$\hat{\mathbf{s}}_k = \arg\min_{\hat{\mathbf{s}}_k} \|\hat{\mathbf{s}}_k - \mathit{ifft}(\hat{\mathbf{S}}_k)\| \tag{13}$$

$$\mathbf{D}_k = \mathit{fft}(\hat{\mathbf{s}}_k)$$

$$\mathbf{R}(f) = \mathbf{R}(f) - \sqrt{\frac{E_s}{M}}\mathbf{H}_k(f)\mathbf{D}_k(f) \tag{14}$$

where $\mathbf{s}_k$ is a whole block transmitted from antenna k in the time domain. **R** and $\hat{\mathbf{S}}$ are frequency-domain received and equalized signal and $\mathbf{D}_k$ is the FFT transform of the detected symbol block $\hat{\mathbf{s}}_k$. As it can be deduced from (13) and (14), the equalization and detection must be done per antenna and in a whole block basis due to the IFFT and FFT operations involved in the feedback chain. Thus, the antenna ordering process in (6) will be done for a whole time block, instead of per each FFT point in the frequency domain, like in OFDM, as it would be desirable to reduce the effects of frequency selectivity. This per-block ordering can slightly reduce the performance improvement of the OSIC (V-BLAST) algorithm.

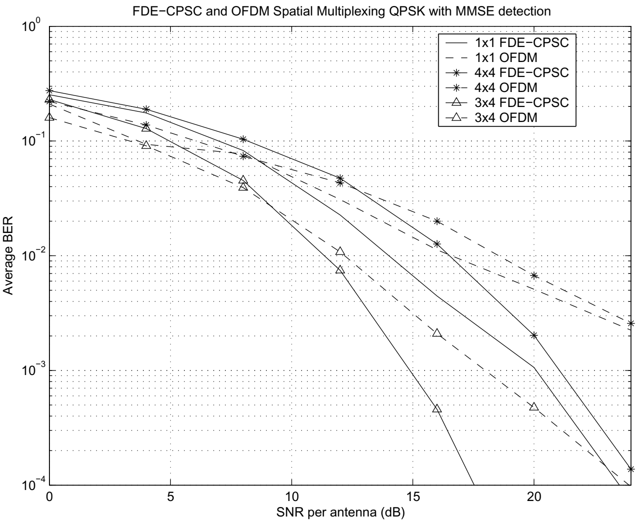

Figure 6: BER performance for uncoded QPSK MIMO FDE CPSC and MIMO OFDM with MMSE equalization.

4. SIMULATION RESULTS

4.1 Simulated System Parameters

Two different spatial multiplexing transmission systems are simulated: MIMO FDE CPSC and MIMO OFDM. The main simulation parameters are based on the Hiperlan/2 standard. For OFDM, this means a 20 MHz bandwith channel with 64 subcarriers, 48 of which are used to carry information symbols and 4 are reserved for pilot tones. The OFDM symbol duration is 4 us, 0.8 of which are the cyclic prefix. For CPSC, blocks of 64 symbols are sent with a cyclic prefix of 16 symbols at each transmission branch. The symbol duration is 50 ns and a 64-point FFT is employed in equalization. QPSK modulation is analyzed. Both uncoded and coded transmissions are compared. Perfect channel knowledge is assumed at reception.

A stochastic MIMO Rayleigh frequency-selective channel [9] is used with an rms delay spread of 100ns (channel model B from Hiperlan/2 specification). The discrete channel impulse responses have exponentially decaying power taps and there is no antenna correlation. Three antenna layouts are compared in order to evaluate how the antenna number and diversity affect BER performance: 1x1 (SISO), 4x4 and 3x4.

4.2 Results

The BER performance versus SNR per receive antenna is depicted in Fig. 6 for uncoded QPSK with several antenna setups with no correlation and MMSE detection technique. MIMO-FDE-CPSC (continuous line) clearly outperforms MIMO-OFDM (dotted line) in uncoded transmissions, as suggested in [4, 5] for SISO systems. When $N = M$ the BER performance degrades slightly with MMSE as the num-

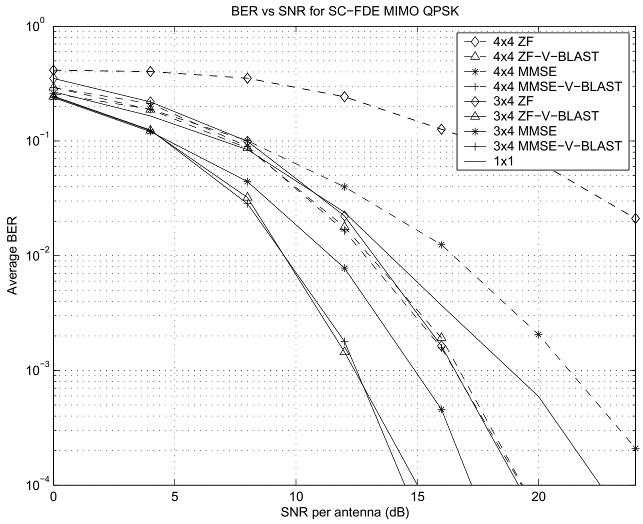

Figure 7: BER performance of the analyzed MIMO detection schemes for 3x4 and 4x4 uncoded FDE-CPSC systems.

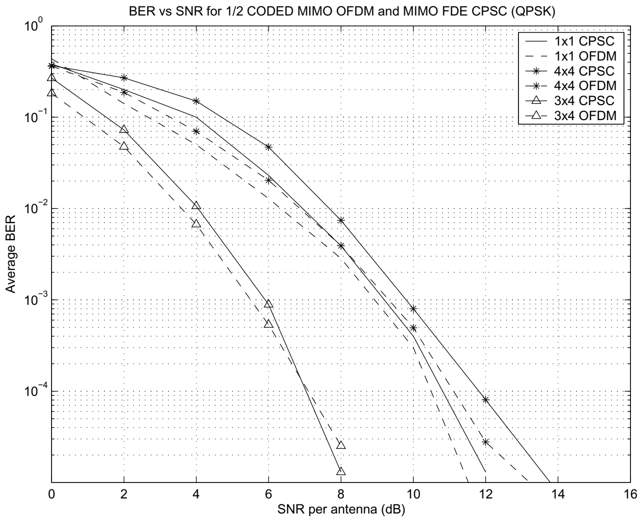

Figure 8: BER performance comparison for coded MIMO FDE CPSC and MIMO OFDM with MMSE equalization.

ber of transmission and reception antennas grows, but very good BER results can be obtained if reception diversity is increased. For example, 3x4 antenna setup outperforms SISO BER transmitting at a three times higher bitrate.

Fig. 7 shows the BER performance of the MIMO equalization schemes analyzed in Section 2 for 4x4 and 3x4 FDE CPSC systems. V-BLAST algorithms are the ones which offer the best performance. The diversity obtained by the OSIC algorithm allows even to outperform the SISO results for high SNR. The effect of including the FFT and IFFT in the decision feedback of V-BLAST detection systems does not degrade their BER performance. MMSE-V-BLAST algorithm performs slightly better than ZF-V-BLAST.

In order to compare OFDM and CPSC, Fig. 8 shows the BER performance results for MIMO coded systems. Transmitted symbols are per-antenna coded (PAC) with an interleaved 1/2 rate convolutional (Hiperlan/2 standard) code. QPSK modulation and frequency-domain MMSE detection are employed. Coding and interleaving give frequency diversity to OFDM, whose BER performance becomes similar or better to CPSC only for low coding rate frequency-selective systems.

5. CONCLUSION

We have analyzed the extension and applicability of fundamental MIMO equalization and detection techniques to FDE CPSC systems. ZF, MMSE, ZF-V-BLAST and MMSE-V-BLAST algorithms have been evaluated for CPSC and compared to MIMO-OFDM. BER performance simulations for uncoded and coded QPSK spatial multiplexing systems have been obtained and compared to OFDM for a frequency-selective Hiperlan/2 standard channel. It has been shown that MIMO equalization can be easily adapted to frequency-domain equalized single-carrier transmission and that this communication scheme can equal or outperform MIMO-OFDM in uncoded and high coding rate systems.

REFERENCES

[1] D. Gesbert, M. Shafi, D. Shiu, P. J. Smith, “From Theory to Practice: An overview of MIMO Space-Time coded Wireless Systems” *IEEE Journal on Selected Areas in Communcations*, vol. 21 no. 3 pp. 281-303, April 2003.

[2] A. Paulraj, R. Nabar, D. Gore, *Introduction to Space-Time Wireless Communications*, Cambridge University Press, 2003.

[3] A. Van Zelst, T.C.W. Schenk, “Implementation of a MIMO OFDM based Wireless LAN system” *to be published in IEEE Transactions on Signal Processing.*

[4] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar and B. Eidson, “Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems” *IEEE Communications Magazine*, vol. 40, pp. 58–67, Apr. 2002.

[5] Z. Wang, X. Ma and G.B. Giannakis, “OFDM or Single-Carrier Block Transmissions?” *To appear in IEEE transaction on Communications, 2004.*

[6] A. Benjamin-Seeyar, “PHY Layer System Proposal for Sub 11 GHz BWA” *IEEE 802.16 Broadband Wireless Access Working Group, 2001.* http://www.iee802.org/16/.

[7] R. Böhnke, D. Wübben and V. Kühn, “Reduced complexity MMSE detection for BLAST Architectures”, *IEEE 2003 Global Communications Conference (Globecom’2003), San Francisco, California, USA, December 1-5, 2003.*

[8] G. Foschini, “Layered Space-Time architecture for wireless communications in a fading environment when using multi-element antennas” *Bell Labs Technical Journal*, Autumn 1996.

[9] K.I. Pedersen, J.B. Andersen, J.P. Kermoal, P.E. Mogensen, “A stochastic Multiple-Input Multiple-Output radio channel model for evaluation of space-time codes” *Proc IEEE VTC 2000 Fall*, Boston, 2000.