ROBUST BLIND CHANNEL SHORTENING IN IMPULSIVE NOISE ENVIRONMENTS

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ABSTRACT

This paper proposes a robust blind channel shortening algorithm for multicarrier modulation systems. The algorithm is based on the sum-absolute autocorrelation minimization (SAAM) of the effective channel outside a window of desired length. The algorithm approaches the maximum shortening SNR (SSNR) solution of [1] under Additive White Gaussian Noise (AWGN) conditions, is computationally less expensive, and robust to non-Gaussian impulsive noise environments than a latest reported blind adaptive channel shortening algorithm (SAM). Due to the mostly minimum phase nature of Asymmetric Digital Subscriber Digital Lines (ADSL) channel impulse responses, a "left" initialization scheme is suggested for blind adaptive channel shortening equalizers which further enhances the robustness of SAAM to impulsive noise. Further studies have also been undertaken showing that the bit rates achieved by SAM in AWGN conditions can be improved and approach the maximum SSNR solution of [1] too.

1. LITERATURE OVERVIEW

Discrete multitone (DMT) is a special multicarrier modulation (MCM) technique and has attracted considerable attention as a practical and viable technology for high-speed data transmission over spectrally shaped noisy channels such as digital subscriber lines (DSLs). MCM for wireless channels is typically referred to as Orthogonal Frequency Division Multiplexing (OFDM). In MCM, if the length of the impulse response of the channel is greater than the cyclic prefix (CP) plus one, inter-block interference (IBI) is introduced by the data spreading between adjacent blocks. Also because the orthogonality between the sub-carriers is lost, inter-(sub)carrier (ICI) interference is introduced.

A common technique is to introduce a time-domain equalizer (TEQ) in the receiver front end which shortens the channel to the CP-length plus one to combat such IBI/ICI. The resulting shortened channel is, then, equalized by a bank of one-tap frequency-domain equalizers (FEQs). Most approaches to TEQ design are non-adaptive, have high complexity, and require training or channel estimation [3]. Certain applications, such as multipoint or broadcast network configuration in telephone connectivity, digital transmission of TV signals, and the nature of the mobile networked environment preclude extensive training and mandate blind equalization [6]. In fact the VDSL (Very High Speed DSL) standard does not allow for training sequences for equalization. In addition, no periodic transmission of a training signal is necessary, therefore, the advantage in terms of channel capacity by blind channel shortening becomes twofold.

In [2] a blind adaptive method is proposed. It equalizes the channel to a single spike rather than shortening it, moreover, its computational complexity is also high [3]. A low complexity, blind, adaptive TEQ algorithm known as MERRY is proposed in [3], but its parameter updates are performed only once per symbol [4]. In [4] a blind adaptive algorithm for channel shortening in ADSL has been proposed. This algorithm minimizes the sum of the squared autocorrelations of the output of the TEQ outside a CP-length window. This algorithms updates more frequently than the MERRY algorithm but at a significantly higher complexity. Unlike MERRY, it is not sensitive to synchronization error.

It is well known from the classic work in [5] that least-squares (LS) estimators are very sensitive to the tail behavior of the probability density of measurement errors (represented here by the additive noise). Their performance depends significantly on the Gaussian assumption, and even a slight deviation of the noise density from the Gaussian distribution can, in principle, cause a substantial degradation of the LS estimate. Therefore the SAM algorithm of [4] can be robustified to impulse noise by minimizing the sum of the absolute values (instead of sum of squared values) outside a window of desired length.

The remainder of the paper is organized as follows. Section 2 gives the system model and the notations. Section 3 and 4 discuss the SAAM cost function and the adaptive algorithm. Section 5 models the impulsive noise. Section 6 presents the existence of a good initialization for blind adaptive equalizers for ADSL channels. Section 7 provides simulations results, and a discussion on the results, and section 8 concludes the paper.

2. SYSTEM MODEL

The system model is shown in Fig. 1. The signal $x(n)$ is a white, zero-mean, wide-sense stationary (W.S.S), real and unit variance source sequence transmitted through the linear finite-impulse response (FIR) channel $\mathbf{h}$. $v(n)$ is a zero-mean, i.i.d., noise sequence uncorrelated with the source sequence and has a variance σ_v^2. The received signal $r(n)$ is

$$r(n) = \sum_{k=0}^{L_h} h(k)x(n-k) + v(n)$$

and $y(n)$, the output of the TEQ given by

$$y(n) = \sum_{k=0}^{L_w} w(k)r(n-k) = \mathbf{w}^T \mathbf{r}_n \quad (1)$$

where $\mathbf{w}$ is the impulse response vector of the equalizer and $\mathbf{r}_n = [r(n)\ r(n-1)\ \dots\ r(n-L_w)]^T$. L_h, L_c, and L_w are the order of the channel, shortened channel and equalizer respectively. We denote $\mathbf{c} = \mathbf{h} \star \mathbf{w}$ as the shortened or effective channel. For multicarrier ADSL applications we assume that $2L_c < N_{fft}$ holds, N_{fft} being the FFT size [4].

3. SAAM

Our algorithm is a blind adaptive implementation of [1]. For the effective channel $\mathbf{c}$ to have zero taps outside a window of size $(v+1)$, its autocorrelation values should be zero outside a window of size $(2v+1)$. Now the autocorrelation sequence of the effective channel is given by

$$R_{cc}(l) = \sum_{k=0}^{L_c} c(k)c(k-l)$$

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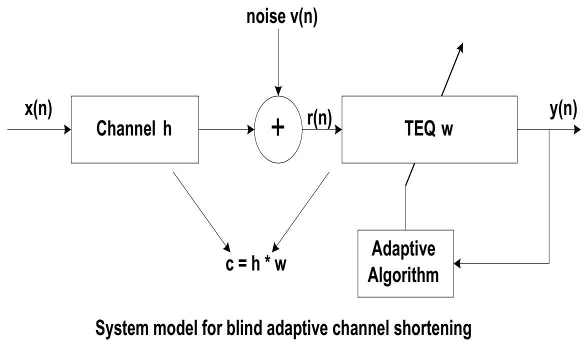

Figure 1: System model for blind adaptive channel shortening.

and for a shortened channel, we must satisfy

$$R_{cc}(l) = 0, \forall\, |l| > v$$

we, therefore, define a cost function, J_{v+1} based upon the sum of the absolute values of the auto-correlation of the effective channel, i.e.,

$$J_{v+1} = \sum_{l=v+1}^{L_c} |R_{cc}(l)| \quad (2)$$

Note that in equation (2) only the one sided autocorrelation values are taken into account because of the symmetry property of the autocorrelation sequence. The trivial solution of $\mathbf{c} = 0$ or $\mathbf{w} = 0$ can be avoided by imposing a norm constraint on the effective channel response, for instance $\|\mathbf{c}\|_2^2 = 1$ or on the equalizer i.e., $\|\mathbf{w}\|_2^2 = 1$. The optimization problem can then be stated as

$$\mathbf{w}^{opt} = \arg_{\mathbf{w}} \min_{\|\mathbf{c}\|_2^2 = 1} J_{v+1}$$

The autocorrelation sequence of the output $y(n)$ is given by

$$\begin{aligned} R_{yy}(l) &= E[y(n)y(n-l)] \quad (3) \\ &= E[(\mathbf{c}^T \mathbf{x}_n) + \mathbf{w}^T \mathbf{v}_n)\ (\mathbf{x}_{n-l}^T \mathbf{c} + \mathbf{v}_{n-l}^T \mathbf{w})] \end{aligned}$$

given the stated conditions on $x(n)$ and $v(n)$, equation (3) can be written as [4]

$$R_{yy}(l) = R_{cc}(l) + \sigma_v^2 R_{ww}(l) \quad (4)$$

so that we can approximate our cost function, denoted as $\hat{J}_{v+1}$, in equation (2) as

$$\begin{aligned} \hat{J}_{v+1} &= \sum_{l=v+1}^{L_c} |R_{yy}(l)| \\ &= \sum_{l=v+1}^{L_c} |R_{cc}(l) + \sigma_v^2 R_{ww}(l)| \quad (5) \end{aligned}$$

The second term being added is very small due to its multiplication with σ_v^2 and we presume that under practical SNR scenarios, we can drop the hat on J_{v+1} so that $\hat{J}_{v+1} \cong J_{v+1}$. For this cost function we need the length of the channel $\mathbf{h}$ to determine L_c which is fortunately known because the CSA test loops have almost all of their energy in 200 consecutive taps [9]. Our algorithm only requires the output of the TEQ and is, in that sense, blind.

4. BLIND ADAPTIVE ALGORITHM

The steepest-descent type algorithm to minimize J_{v+1} is

$$\mathbf{w}^{new} = \mathbf{w}^{old} - \mu \nabla_{\mathbf{w}} (J_{v+1}) \quad (6)$$

where μ is the step size and $\nabla_{\mathbf{w}}$ is the gradient evaluated at $\mathbf{w} = \mathbf{w}^{old}$. A moving average implementation is used to realize the instantaneous cost function

$$J_{v+1}^{inst}(k) = \sum_{l=v+1}^{L_c} \left| \sum_{n=kN}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right| \quad (7)$$

Here N is a design parameter which determines a tradeoff between the algorithm complexity and a good estimate of the expectation. Our algorithm of equation (6), therefore, can be written as (8) given at top of the next page which, using equation (1), takes the form of equation (9), also given at the top of next page. The equalizer vector $\mathbf{w}$ must be normalized at every iteration to ensure that $\|\mathbf{c}\|_2^2 = 1$. The introduction of *sign* function in equation (9) reduces the computational complexity of the implementation as compared to the SAM algorithm of [4].

5. GAUSSIAN-MIXTURE NOISE MODEL

Impulse noise is a principal source of degradation in DSL transmission systems and it is one of the most difficult transmission impairment to suppress. Measurement data show that the impulse noise events are longer than the maximum error-correcting capacities of the default interleaved forward error-correction (FEC) provided within current ANSI standards [11]. To model the behavior of impulse noise, we use a two term Gaussian-mixture model [12]. The probability density function of the noise model has the form

$$(1-p)\mathcal{N}(0, \sigma_v^2) + p\mathcal{N}(0, k^2\sigma_v^2) \quad (10)$$

with $\sigma > 0$, $0 \le p \le 1$, and $k \ge 1$. Here the $\mathcal{N}(0, \sigma_v^2)$ term is the AWGN with zero mean and variance σ_v^2 and the parameter p is the probability of contribution from the impulsive component (Gaussian with zero mean and variance $k^2\sigma_v^2$). By varying p and k, we simulate the effect of different shapes of impulsive noise on our algorithm performance.

6. "LEFT" EQUALIZER INITIALIZATION

For blind adaptive equalization, the location of the single spike initialization should be driven according to the center of the mass of the channel impulse response [6]. The equalizer can, therefore, be "left", "center", or "right" initialized. If we do not have any *a prior* knowledge about the channel impulse response, a center spike is a good strategy. This is motivated by being able to concentrate on the equalization of the minimum as well as the non-minimum phase response of the channel, and because it may lead to benefits in terms of FIR filter implementation simplicity [7]. CSA loop channels are mostly minimum phase channels with very few zeros outside the unit circle (see Fig. (2)). For such channels, the center of the mass of the impulse response is on the "left" as shown in Fig. (3). Using this knowledge, we suggest the "left" initialization strategy alongside the SAAM-cost function. With this initialization, we will generally be more robust to impulsive noise too because we would, then, be also using the equalizer to its fullest potential. Simulation results support this approach.

7. SIMULATION RESULTS

The Matlab code used for simulations is available in [8]. It was modified to simulate SAAM and the impulsive noise environments. Standard ADSL downstream parameters were chosen: The CP was 32, the FFT size was 512, the equalizer length was 16, the averaging window size was 32, and the channel was CSA test loop 1 available at [9]. These parameters were also chosen as to compare the results with those of SAM. The noise power was set such that $\sigma_x^2 \|\mathbf{h}\|^2 / \sigma_v^2 = 40$ dB. This is a typical value of SNR in ADSL environments. Total 75 symbols, comprising of 544 samples each, were used. We employed the moving average implementation of the SAAM algorithm given in equation (9) with unit norm constraint on

$$\mathbf{w}^{k+1} = \mathbf{w}^k - \mu \sum_{l=v+1}^{L_c} \left[sign \left\{ \sum_{n=kN}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right\} \times \left\{ \nabla_{\mathbf{w}} \left(\sum_{n=kN}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right) \right\} \right] \tag{8}$$

$$\mathbf{w}^{k+1} = \mathbf{w}^k - \mu \sum_{l=v+1}^{L_c} \left[sign \left\{ \sum_{n=kN}^{(k+1)N-1} \frac{y(n)y(n-l)}{N} \right\} \times \left\{ \sum_{n=kN}^{(k+1)N-1} \left(\frac{y(n)\mathbf{r}_{n-l} + y(n-l)\mathbf{r}_n}{N} \right) \right\} \right] \tag{9}$$

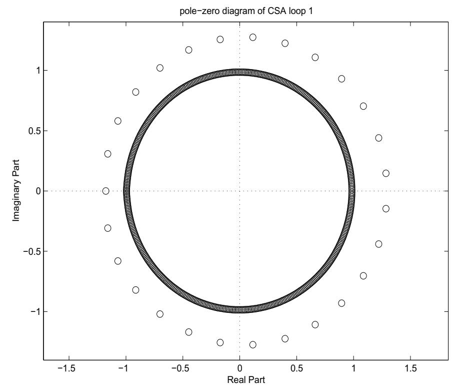

Figure 2: Pole-zero diagram of CSA loop 1.

the equalizer vector $\mathbf{w}$. The remainder of the explanation is related to the figures individually.

The impulse responses of the original and the shortened channel are shown in Fig. (3). The initialization was "left" (single first spike). Fig. (4) shows our equalizer designed after SAAM converges. In Fig. (5), we have shown the learning curve of our cost function and the bit rate as a function of the averaging block number. It is noteworthy that the bit rate approaches the maximum SSNR solution of [1] and that the cost function and the bit rate are a smooth function of each other i.e., the SAAM minima and the bit rate maxima appear to be located in close proximity. Careful selection of parameters for the SAM algorithm also leads to the same performance measures under AWGN (see Fig. (6) dashed-dot curve). Fig. (6) shows under AWGN, the achievable bit rate versus SNR for SAM, SAM-optimized, SAAM with center spike initialization, SAAM with "left" (first spike) initialization, and the maximum SSNR algorithm of [1]. The step sizes used for the adaptive algorithms are 5, 1.05, 0.0003, and 0.00016 respectively. The bit rate is calculated using 6-dB margin and a 4.2-dB coding gain [10]. The performance of SAAM is slightly below that of SAM-optimized, this is expected because robust algorithms are not optimal for the pure AWGN environment. They are designed for special conditions (impulsive noise, in fact Laplacian). However, SAAM with first spike initialization outperforms SAM-optimized showing existence of such a good initialization for such channels.

In Fig. (7) we have simulated the effect of impulsive noise on the quasi-achievable (quasi as we use the same bit rate calculation, to a first approximation, on the basis that the impulses occur infrequently. These results are for a impulsive contribution of only %1) bit rate by the above adaptive algorithms. The value of k is changed from 10 to 100 to simulate the amplitude of the impulsive spikes from 20 dB to 40 dB, while keeping p their contribution factor at 1%. The SNR is 40dB. As expected with increasing k the degradation in the SAM algorithm is more than in the other two algorithms. At these parameters settings the practical values of k are between 40 to 100 (from 30 to 40 dB), and the plot for SAM-opt algorithm is showing a much decrease in bit rate in this range. To illustrate it more, the convergence behavior is shown in Fig. (8) applying impulses of 40 dB higher than the AWGN and at a contribution factor of 1%. This is typical level of impulse noise in ADSL environments where the spikes can totally eliminate the signal for a hundred of microseconds [11]. It is evident from the Fig. (8) that the behavior of the SAM algorithm deteriorates, whereas that of SAAM (both center spike as well as first spike) remains smooth at higher bit rates. The results were, in general, similar for the 8 CSA loop channels.

8. CONCLUSION AND FUTURE WORK

We have proposed a new robust blind adaptive channel shortening algorithm and have shown significant improvement in bit rate even when the channel is corrupted by impulsive noise. Blind algorithm of SAM is also optimized to yield better bit rates in AWGN.Future work will consider the use of other impulsive noise models like α-stable distributions.

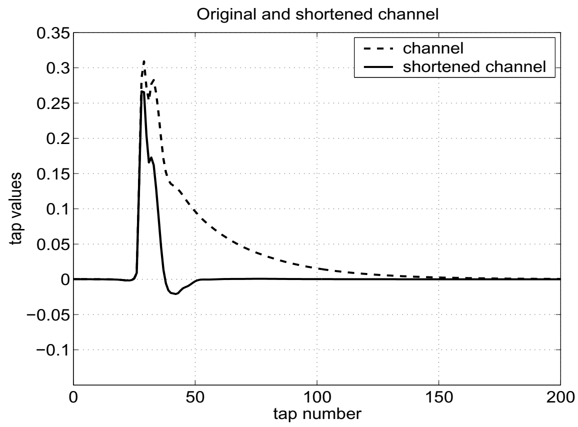

Figure 3: Original and the shortened channel.

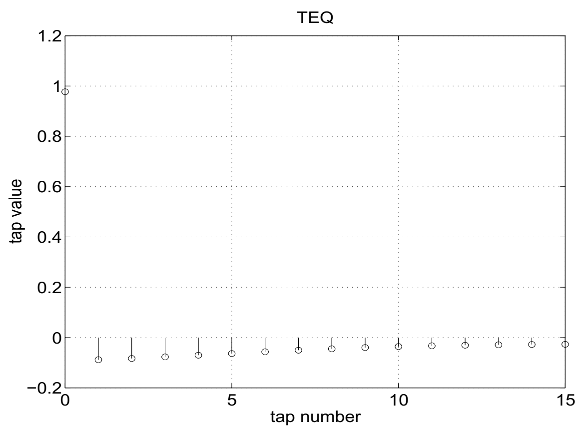

Figure 4: 16- tap TEQ

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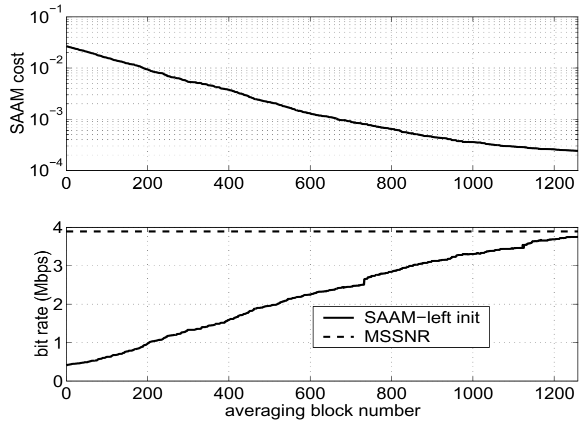

Figure 5: SAAM cost and bit rate versus averaging block number at 40 dB SNR.

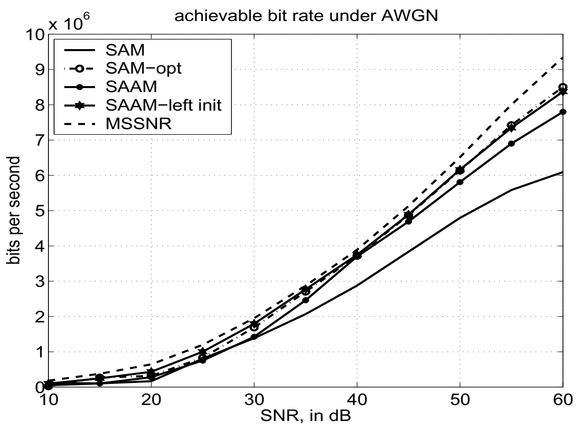

Figure 6: Achievable bit rate versus SNR for white noise.

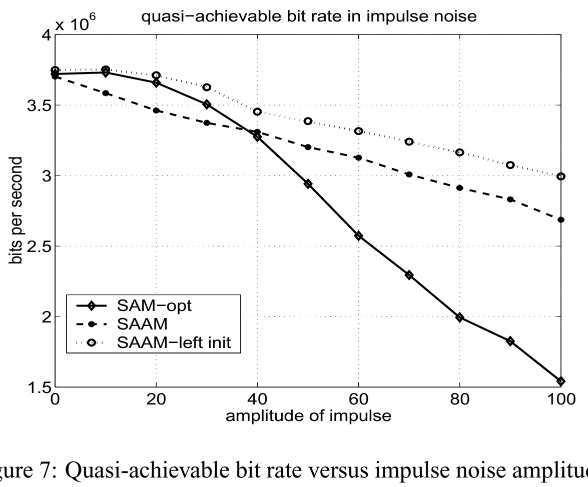

Figure 7: Quasi-achievable bit rate versus impulse noise amplitude.

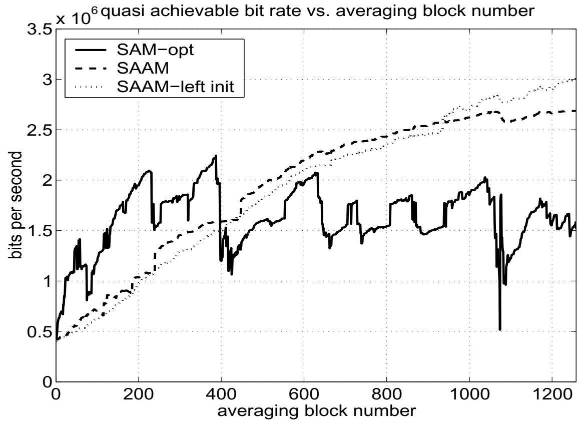

Figure 8: Quasi-achievable bit rate versus averaging block number, at 40 dB SNR and 40 dB 1% impulse noise.

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