A SYMBOL BY SYMBOL CLUSTERING BASED BLIND EQUALIZER

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ABSTRACT

A new blind symbol by symbol equalizer is proposed. The operation of the proposed equalizer is based on the geometric properties of the two dimensional data constellation. An unsupervised clustering technique is used to locate the clusters formed by the received data. The symmetric properties of the clusters labels are subsequently investigated in order to label the clusters. Following this step, the received data are compared to clusters and decisions are made on a symbol by symbol basis by assigning to each data the label of the nearest cluster. The performance of the proposed equalizer is better compared to the performance of a CMA-based blind equalizer.

1. INTRODUCTION

Intersymbol Interference (ISI) is a major impairment in today's high bit rate communication systems [9]. Channel equalizers used in the receiver part aim to suppress the effect of ISI. In most of the cases the communication channel is unknown and the design of the equalizer is performed on the basis of a known training sequence of information bits. However, there are many cases that the transmission of a training sequence is not possible or desirable. This mode of equalizer design is known as *blind*.

Blind channel equalization is a challenging task and has been the focus of intense research effort. Recently, an interest has risen on approaches based on data clustering techniques [3], [6], [10].

In this paper a novel blind cluster based symbol by symbol equalizer is proposed. The equalizer extracts the information needed to perform data detection from the clusters formed by the received data. The whole process involves a simple symbol by symbol decision procedure.

The cluster based blind channel estimation algorithm consists of two steps: a) data clusters are first estimated via an unsupervised learning technique and b) labeling of the estimated clusters is performed by unravelling the information hidden in the geometry of the clusters constellation in the two dimensional space. That is, for data generated by bipolar alphabets (assumed in this paper) the clusters are arranged in pairs of clusters with the right sided cluster labeled as $+1$ and the left sided cluster labeled as -1. This property is called property of symmetric labels and it is used in order to label the clusters. Determination of the appropriate pairs of clusters is obtained by using the results of [2] concerning the properties of the convex hull of the two dimensional clusters constellation. The edjes of the convex hull are used to recover the channel taps. However, in contrast to [2], in this paper we need only absolute values of the channel taps and there is no need to find the specific permutation of the channel taps.

When channel estimation is completed the received data are compared to clusters and a closest neighbor rule [12] is utilized to achieve data detection on a symbol by symbol basis. That is, the currently observed data is classified by assigning to it the label associated with the nearest cluster.

The paper is organized as follows. Section 2 presents the system description and the properties of the 2-dimensional clusters constellation. Section 3 describes the proposed symbol by symbol blind equalizer. In Section 4 simulation results are given and finally, in Section 5 conclusions are drawn.

2. CLUSTERS CONSTELLATION PROPERTIES

The received signal $g(t)$ of an ISI and noise impaired linear system is written as:

$$g(t) = \sum_{i=0}^{L} h(i)I(t-i) + w(t), \tag{1}$$

where $I(t)$ is an equiprobable sequence of transmitted data taken from a binary alphabet, i.e., $I(t) \in \{x_k, k = 1,2\}$, $h(i)$ is the channel impulse response and $w(t)$ is an Additive White Gaussian Noise (AWGN) sequence. Eq. (1) can also be written as:

$$g(t) = c(t) + w(t), \tag{2}$$

where $c(t)$ is the noisless channel output sequence which is a discrete values signal with 2^{L+1} different elements.

In this paper, the neccessary information for data detection is extracted from the geometric structure created by the received data in the two dimensional space.

Consider the 2 x 1 vector of successively received samples:

$$\mathbf{g}(t) = [g(t)\ g(t-1)]^T. \tag{3}$$

In the absence of noise, $\mathbf{g}(t)$ is associated with $Q = 2^{L+2}$ points in the 2-dimensional space. Each point corresponds to one of the 2^{L+2} possible realizations of the sequence of transmitted bits: $(I(t),\ldots,I(t-L-1))$. If the received data is corrupted by AWGN, then the randomness of noise leads to the formation of a *cluster* around each point. Each cluster is represented by a suitably chosen *representative*, which

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corresponds to the noiseless channel response vector in the 2-dimensional space, i.e.,

$$\mathbf{c}(t) = [c(t)\ c(t-1)]^T, \quad (4)$$

with

$$\mathbf{c}(t) \in \{\mathbf{c}_k = [c_{k1}\ c_{k2}]^T, k = 1, ..., Q\}.$$

Each cluster representative, $\mathbf{c}_k$, corresponds to a specific sequence of transmitted data denoted as: $(I_{k0}I_{k1}, ..., I_{kL}I_{k(L+1)})$ and the two components of $\mathbf{c}_k$, c_{k1} and c_{k2} are written as:

$$c_{k1} = \sum_{l=0}^{L} I_{kl}h(l), \ \ c_{k2} = \sum_{l=1}^{L+1} I_{kl}h(l-1). \quad (5)$$

Each cluster is characterized by a *label*, X_k, which is defined as the value of the corresponding emitted data, i.e.,

$$X_k = I_{kd} = I(t-d),$$

with d an appropriate choosen delay and $X_k \in \{x_i, i = 1, 2\}$.

For a linear channel, the edjes E_i, $i = 1, .., 2L+4$, of the convex hull, H, of the two dimensional data constellation contain information related to the channel taps [2]. That is, every edje E_i of H is parallel to some vector $\mathbf{u}_i$, where:

$$\mathbf{u}_0 = [h(0)\ 0]^T, \mathbf{u}_1 = [h(1)\ h(0)]^T, ..., \mathbf{u}_{L+1} = [0\ h(L)]^T$$

and E_i has length $2|\mathbf{u}_i|$. Actually, there are two edjes parallel to each vector $\mathbf{u}_i$.

Moreover, for the edjes of the convex hull the following Theorem is shown.

Theorem 1 *For each unique edje, E_i, (i=1,...,L+2) of the convex hull there are $Q/2$ pairs of clusters such that each cluster of a pair defines the endpoint of a line parallel to the edje E_i. The length of each line is equal to the length of E_i.*

Proof: Consider two clusters: $\mathbf{c}_k = [c_{k1}\ \ c_{k2}]^T$ and $\mathbf{c}_j = [c_{j1}\ \ c_{j2}]^T$ with corresponding transmitted sequences $(I_{k0}....I_{k(L+1)})$ and $(I_{j0}...I_{j(L+1)})$ with

$$I_{kl} = \begin{cases} I_{jl} & l \neq i \\ -I_{jl} & l = i \end{cases}$$

where $l = 0, ..., L+1$ and $i \in \{0, ..., L+1\}$. Then, according to Eq. (5):

$$|c_{k1} - c_{j1}| = 2|h(i)| \ and \ |c_{k2} - c_{j2}| = 2|h(i-1)|.$$

Consequently, for each specific i, $i \in \{0, ..., L+1\}$, there are $Q/2$ such pair of clusters, in the two dimensional constellation being separated by distance equal to $2|\mathbf{u}_i|$. The corresponding transmitted sequences of the two clusters of each pair are the same except the value of data I_{ki} and I_{ji} (i.e., $I(t-i)$).

Definition: By now, we will call *pair of clusters* two clusters, $\mathbf{c}_k$ and $\mathbf{c}_j$, sharing the same data except the value of I_{ki} and I_{ji} respectivelly, and being separated by distance equal to $2|\mathbf{u}_i|$.

For the values of I_{ki} and I_{ji} the following Theorem holds.

Theorem 2 *(Property of labels symmetry): The values of the labels of I_{ki} and I_{ji} are ordered in ascending (or desceding) order, that is, for all the pairs parallel to E_i, the right sided cluster has $I(t-i) = +1$ and the left sided $I(t-i) = -1$ (or the opposit).*

Proof: The clusters $\mathbf{c}_k$ and $\mathbf{c}_j$ form a pair of clusters and

$$c_{k1} = I_{ki}h(i) + \sum_{l \neq i} I_{kl}h(l), \quad c_{j1} = I_{ji}h(i) + \sum_{l \neq i} I_{jl}h(l)$$

Since, $\sum_{l \neq i} I_{kl}h(l) = \sum_{l \neq i} I_{jl}h(l)$ then:

$$if \ \ c_{k1} < c_{j1}, \ \ then \ \ I_{ki} < I_{ji}.$$

That is, the right sided cluster has label +1 and the left cluster has label -1. This is true with the assumption that $h(i) > 0$, if $h(i) < 0$ then the opposite order takes place, i.e., if $c_{k1} < c_{j1}$ then $I_{ki} > I_{ji}$.

Example: Figure 1 represents the clusters formed in the two dimensional space by bipolar transmitted data when the channel impulse responce is $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$. In the figure appear also the convex hull and the values of the transmitted data: $I(t)I(t-1)I(t-2)I(t-3)$. It can be easily seen that the lengths of the convex hull edjes are: [0.6 0], [1.6 0.6], [0.6 1.6] [0 0.6] (each length corresponds to two edjes). In the figure are also plotted the lines which are parallel to $E_i = [1.6\ 0.6]$. All the respective pairs of clusters (endpoints of the parrallel lines) are labelled by the value of $I(t-1)$. From the figure is seen that all the right sided clusters have label $I(t-1) = 1$ and the left sided clusters have label $I(t-1) = -1$.

From the figure is also seen that the clusters constellation is characterized by pairs of clusters with distance between them equal to [0.6 0]. The labels of these pair are: $I(t) = +1$, for every right sided cluster and $I(t) = -1$, for every left sided cluster. In the same way, we can observe that there are 8 pairs with distance between them: [0.6 1.6] and labels $I(t-2) = +1$ (right clusters) and $I(t-2) = -1$ (left clusters). Finally, the pairs with distance [0 0.6] have labels $I(t-3) = +1$ (right clusters) and $I(t-3) = -1$ (left clusters).

3. SYMBOL BY SYMBOL BLIND CLUSTERING EQUALIZER

Two major steps compose the operation of the proposed blind equalizer. First, clusters estimation takes place and then follows the signal detection procedure. The block diagram of the proposed equalizer appears in Figure 2.

A cluster-based blind channel estimation algorithm consists of two steps [10]:

a) clusters representatives are first estimated via an unsupervised learning technique and

b) labeling of the estimated clusters is achieved.

In the proposed equalizer the two dimensional clusters representatives are identified by means of the Neural Gaz algorithm [8].

The proposed labeling algorithm aims to the characterization of each specific cluster according to the respective value of the transmitted data $I(t-d)$, where d is an unknown delay. It is known that a nonzero lag, d, generally permits a better equalization performance [1]. In this algorithm, it is chosen the delay which corresponds to the maximum tap of the channel impulse response, since, that way, we impose the biggest separation among the two classes (+1, -1). For many channels this leads to two separable decision regions. This is important for a symbol by symbol equalizer as it makes its decisions much more robust to the errors.

According to Section 2, the clusters labels in the 2-dimensional space have a symmetric distribution. Thus, according to their position on the constellation we can label the clusters.

The proposed algorithm is the following. First, we found the convex hull and consequently the channel taps (absolute values). Note also, that there is no need to discover the exact permutation of the channels taps (which is needed in [2]). Then, the maximum channel tap, $|h(d)|$, is chosen. Following this step, we seek for the $Q/2$ clusters pairs that are separated by the maximum distance in the horizontal (first) component. Obviously, the clusters of all these pairs are separated by distance $[2|h(d)|\ 2|h(d-1)|]$. Note, that if $d = 0$ then the pair's distance is $[2|h(0)|\ 0]$. Then, the cluster of each pair, lying to the left is labeled as -1 and the cluster of each pair lying to the right is labelled as +1. Note that, the ambuigity in the descending or ascending order of the labels is solved by using differential encoding [11].

Since labeling is completed a simple decision rule is adopted. For the received data vector $\mathbf{g}(t)$ is found its distance from each cluster, i.e.,

$$r_i = |\mathbf{g}(t) - \mathbf{c}_i)|,\ i = 1, ..., Q. \tag{6}$$

The label of the closest cluster determines the decision for the currently observed data $g(t)$.

Actually, there is no need to perform all the comparisons described in Eq. (6). Only a small number of comparisons need to be performed since data are first compared with the midle cluster (clusters are assumed ordered in the output of the unsupervised clustering algorithm). Next comparison is limited to the half of clusters and the incoming data is compared with the midle cluster of this section and so on. This procedure gives rise to a very small amount of total comparisons. Consequently, the computational complexity of the decision step is very small.

The algorithm for linear channels appears in Table 1.

4. SIMULATION RESULTS

In this experiment data are assumed bipolar and a non minimum phase channel is used with transfer function:

$$H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}.$$

The Signal to Noise Ratio (SNR) is defined as

$$SNR = 10log(S_g/S_w)$$

where, S_g and S_w are the signal power and noise power respectivelly.

The labeling of the clusters is performed by assuming the clusters pairs having the maximum (horizontal) distance between them (which is [1.6 0.6] according to Figure 1). That is, the labeling is based on the value of the tranmitted data $I(t-1)$.

In this experiment the performance of the proposed equalizer is compared to the performance of a blind equalizer based on the Godard algorithm (Constant Modulus Algorithm, CMA) [5]. The performance of the two equalizers appears in Figure 3. Clearly, the performance of the proposed blind symbol by symbol equalizer outperforms the performance of the CMA equalizer.

The above results have been verified on a variety of channels and proove the effectiveness of the algorithm. The application of the equalizer to M-ary data and complex data and to non linear channels is the subject of future work.

5. CONCLUSIONS

A new blind cluster based symbol by symbol equalizer is proposed. The equalizer consists of 3 steps: a) clusters identification through an unsupervised learning algorithm, b) labeling by unravelling the symmetric properties of labels in the 2-dimensional clusters constellation and c) symbol by symbol data detection.

The performance of the equalizer is superior to the performance of a CMA-based blind equalizer for linear channels.

It is also of great interest the fact that the computational complexity of the decision step of the proposed equalizer is very small. This feature can be very usefull in cases of static channels, where in steady state we can have a robust equalizer with very small complexity.

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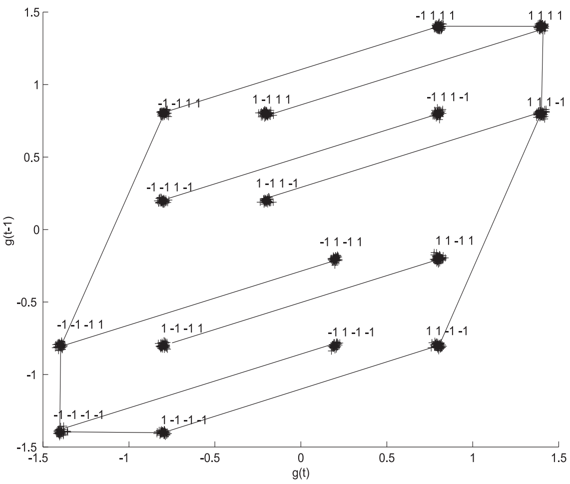

Figure 1: Clusters formed by the linear channel with: $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$ and binary data. Convex hull of the constellation and pairs of clusters defining lines parallel to the edge E_i.

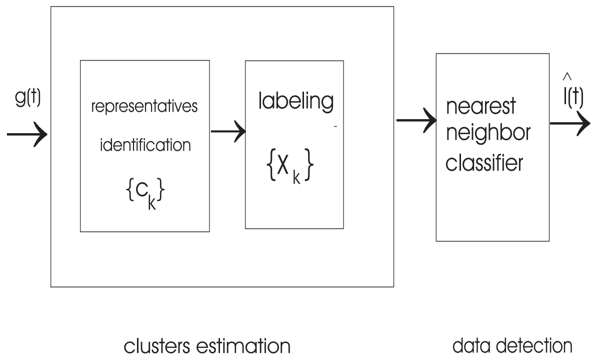

Figure 2: The proposed blind symbol by symbol cluster based equalizer.

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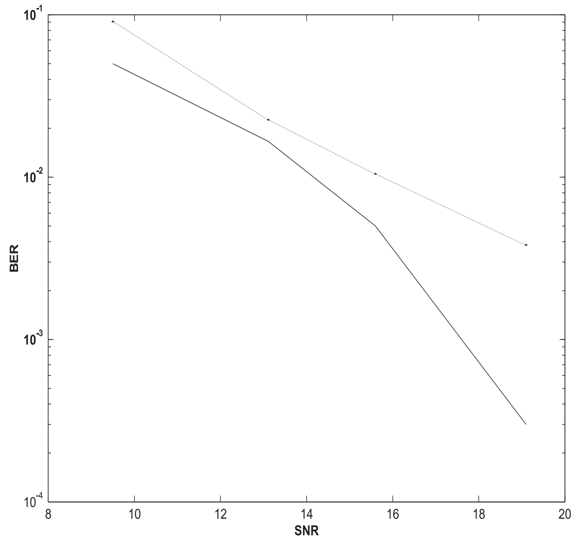

Figure 3: Performance Comparison. Linear Channel : $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$, ’-’: proposed blind equalizer, ’.’: blind CMA equalizer.

Symbol by Symbol Blind Equalizer for Linear Channels
A. Estimation of the two dimensional clusters
A1. Estimation of the clusters representatives, $\mathbf{c}_k$, $k = 1, ..., Q$
Unsupervised learning
A2. Labeling of clusters
a) Find the convex hull, H, and determine $\vert h(i)\vert$, $i = 0, ..., L$ b) Choose the max tap $\vert h(d)\vert$ c) Find the couples of clusters with maximum horizontal distance i.e., pairs having distance between them: $[2\vert h(d)\vert \; 2\vert h(d-1)\vert]$ (for $d = 0$ the distance is $[2\vert h(0)\vert \; 0]$) d) Label the (pair of) clusters: left cluster -1, right cluster +1
B. Data detection
Nearest neighbour rule.

Table 1: Symbol by symbol clustering based blind equalizer