

# A FAST BLIND MULTIPLE ACCESS INTERFERENCE REDUCTION IN DS/CDMA SYSTEMS BY ADAPTIVE PROJECTED SUBGRADIENT METHOD

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## ABSTRACT

This paper presents a novel blind multiple access interference (MAI) suppression filter in DS/CDMA systems. The filter is adaptively updated by parallelly projecting them onto a series of convex sets. These sets are defined based on the received signal as well as *a priori* knowledge about the desired user's signature. In order to achieve fast convergence and good performance at steady state, the *adaptive projected subgradient method* (Yamada *et al.*, 2003) is applied. The proposed scheme also jointly estimates the desired signal amplitude and the filter coefficients based on a stochastic approximation of an EM type algorithm, following the original idea proposed by Park and Doherty, 1997. Simulation results highlight the fast convergence behavior and good performance at steady state of the proposed scheme.

## 1. INTRODUCTION

In DS/CDMA system, multiple users can transmit their signals through the same channel at the same time. Each user has its own signature and the receiver recovers the information by correlating the input signal with the known user's signature. At the receiver, the input signal is not only corrupted by noise, but also by Multiple-Access Interference (MAI) caused by the correlation among the users' signatures. Even if the cross-correlation among the users' signatures is kept low, a conventional matched filter cannot recover satisfactorily the desired information if the signal power from the desired user is much weaker than the interfering users. Such problem is known as near-far problem and power control can be applied in order to overcome this undesired effect [1, 2]. The main goal of power control is to keep the same power level of all users seen by the receiver, but its disadvantage is that the overall multiple-access and antijamming properties of the system is decreased [3]. Besides, in wireless environments the power levels often vary drastically. Whether power control is being used or not, another way of tackling the near-far problem is to use near-far resistant filters.

Several near-far resistant adaptive schemes have been reported [4–7]. The receiver can first use a training sequence and then switch to a decision direct mode in order to minimize a minimum-mean-square error [7]. But, in high throughput systems, blind schemes are more desirable.

Blind schemes are needed when a training sequence in a predefined time slot is not available or not desirable. As in blind schemes the additional overhead imposed by training sequences is absent, the throughput of the overall system is increased. The main burden with conventional blind schemes in comparison to non-blind schemes is that their convergence is normally poor and not comparable with those of non-blind schemes [6].

A simple set-theoretic blind scheme was presented in [4] and shows better performance at steady state than the blind scheme in [6]. Unfortunately, its speed of convergence is still poor compared with the one proposed in [6]. In relatively fast time-varying conditions, i.e., when the users' power changes drastically as in wireless communications, fast algorithms are necessary, otherwise the receiver will not be able to achieve good performance.

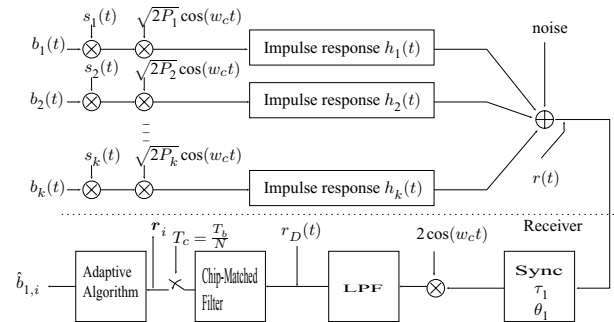


Figure 1: DS/CDMA system model

Recently a novel set-theoretic adaptive filtering method named *Adaptive Projected Subgradient Method* was developed [8–10]. The method offers an unified view for broad range of set-theoretic adaptive signal processing and can realize fast, efficient and robust filtering. For simplicity, in this paper we study a MAI reduction problem in nondispersive channels [4–7, 11] and propose a novel blind MAI suppression filter by combining some ideas in [4] and [8–10]. The filter is adaptively updated by parallelly projecting them onto a series of convex sets based on desired signal properties and another convex set based on the user's signature. Information about the signal amplitude is necessary, so the algorithm jointly estimates the desired signal amplitude and the filter coefficients.

The numerical examples show that the proposed algorithm achieves fast convergence, while attaining good performance at steady state. Results are even comparable to some non-blind schemes.

## 2. PRELIMINARIES

### A. System Model

This section briefly introduces a continuous-time DS/CDMA system and its equivalent discrete-time representation. Figure 1 describes the considered receiver. It is an asynchronous binary phase-shift keying (BPSK) short-code DS/CDMA system.

Let us define:

$$s_k(t) = \sum_{n=0}^{N-1} c_k[n] \varphi_{T_c}(t - nT_c),$$

where  $s_k(t)$  is the  $k$ th user's signature waveform in time domain,  $c_k[n] \in \{-1, +1\}$  is the  $n$ th spreading code chip of the  $k$ th user,  $\varphi_{T_c}(t)$  is the chip waveform with unity energy defined on  $[0, T_c]$ ,  $T_c$  is the chip period, and  $N = \frac{T_b}{T_c}$  is the processing gain, given that  $T_b$  is the bit period.

We also define:

$$b_k(t) = \sum_{i=-\infty}^{+\infty} b_{k,i} \varphi_{T_b}(t - iT_b),$$

where  $b_k(t)$  is the information-bearing baseband signal,  $b_{k,i} \in \{-1, 1\}$  is the  $i$ th data bit of the  $k$ th user,  $\varphi_{T_b}(t)$  is the data pulse shape with unity energy.

Throughout this paper, the first user ( $k = 1$ ) is the desired one. Each user modulates the baseband signal, hence producing at the input of the receiver, under the assumption that transmission is distortionless

$$r(t) = \sum_{k=1}^K \alpha_k \sqrt{2P_k} b_k(t - \tau_k) s_k(t - \tau_k) \cos(\omega_c t - \theta_k) + n(t),$$

where  $n(t)$  is the noise,  $\alpha_k$  is the attenuation due to path losses of the  $k$ th user,  $\omega_c$  is the angular carrier frequency,  $\theta_k$  is the phase of user  $k$ ,  $P_k$  is the transmitted power of the  $k$ th user and  $K$  is the number of users at the same time [4].

The receiver synchronizes with the first user and recovers the baseband signal again, with the help of a low pass filter (LPF). As we are synchronized with user 1 ( $\tau_1 = 0$  and  $\theta_1 = 0$ ), the resulting signal is given by:

$$r_D(t) = A_1 b_1(t) s_1(t) + \sum_{k=2}^K A_k b_k(t - \tau_k) s_k(t - \tau_k) + n_{LP}(t),$$

where

$$A_k = \alpha_k \sqrt{2P_k} \cos(\theta_k - \omega_c \tau_k),$$

where  $n_{LP}$  is the filtered noise.

Then, this signal is chip-matched filtered [1] and sampled every  $T_c$  seconds. Such operation can be described by an  $N$ -dimension vector at the  $i$ th bit interval:

$$\mathbf{r}_i = A_1 b_{1,i} \mathbf{s}_1 + \sum_{m=2}^M A_m \bar{b}_{m,i} \bar{\mathbf{s}}_m + \mathbf{n}_i, \quad (1)$$

where  $\mathbf{n}_i$  is the sampled noise,  $\mathbf{s}_1 = [c_1[0] \ c_1[1] \ \dots \ c_1[N-1]]^T$  is the signature vector given by the desired user's chips,  $\bar{\mathbf{s}}_m$  and  $\bar{b}_{m,i}$  are the interference vectors and interfering symbols generated by interfering users' parameters such as associated data symbols and spreading vectors.  $M - 1$ , the number of interference vectors, can range from  $K - 1$  to  $2(K - 1)$ , due to relative delays of the  $K - 1$  interfering users [5]. The summation in Eq. (1) is the result of the MAI. An adaptive filter is used to suppress this undesirable interference.

The adaptive filter  $\mathbf{h}$  is also a  $N$ -dimensional vector. The decision on the received bit is made from  $\hat{b}_{1,i} = \text{sign}[\mathbf{h}^T \mathbf{r}_i]$ , i.e., it is obtained from the inner product between the filter vector  $\mathbf{h}$  and the received signal vector  $\mathbf{r}_i$ .

## B. Adaptive Projected Subgradient Method

A function  $\Theta : \mathbb{R}^N \rightarrow \mathbb{R}$  is said to be *convex* if  $\forall \mathbf{x}, \mathbf{y} \in \mathbb{R}^N$  and  $\forall \nu \in (0, 1)$ ,  $\Theta(\nu \mathbf{x} + (1 - \nu) \mathbf{y}) \leq \nu \Theta(\mathbf{x}) + (1 - \nu) \Theta(\mathbf{y})$ . Let  $\Theta$  be a continuous convex function. The *subdifferential* of  $\Theta$  at  $\mathbf{y}$  is the set of all the *subgradients* of  $\Theta$  at  $\mathbf{y}$ :

$$\partial\Theta(\mathbf{y}) := \{\mathbf{a} \in \mathbb{R}^N \mid \Theta(\mathbf{y}) + \langle \mathbf{x} - \mathbf{y}, \mathbf{a} \rangle \leq \Theta(\mathbf{x}), \forall \mathbf{x} \in \mathbb{R}^N\}$$

A set  $C \subset \mathbb{R}^N$  is *convex* provided that  $\forall \mathbf{x}, \mathbf{y} \in C$ ,  $\forall \nu \in (0, 1)$ ,  $\nu \mathbf{x} + (1 - \nu) \mathbf{y} \in C$ . For any nonempty closed convex set  $C \subset \mathbb{R}^N$ , the *projection operator*  $P_C : \mathbb{R}^N \rightarrow C$  maps  $\mathbf{x} \in \mathbb{R}^N$  to the unique vector  $P_C(\mathbf{x}) \in C$  such that  $d(\mathbf{x}, C) := \min_{\mathbf{y} \in C} \|\mathbf{x} - \mathbf{y}\| = \|\mathbf{x} - P_C(\mathbf{x})\|$ .

With the above definitions, the goal of the *Adaptive Projected Subgradient Method* is to asymptotically minimize a certain sequence of non-negative convex functions over a closed convex set.

**Algorithm 1** (*Adaptive Projected Subgradient Method* [9, 10]) Let  $\Theta_n : \mathbb{R}^N \rightarrow [0, \infty)$  ( $\forall n \in \mathbb{N}$ ) be a sequence of continuous convex functions and  $K \subset \mathbb{R}^N$  a nonempty

closed convex set. For an arbitrarily given  $\mathbf{h}_0 \in K$ , the adaptive projected subgradient method produces a sequence  $(\mathbf{h}_n)_{n \in \mathbb{N}}$  by

$$\mathbf{h}_{n+1} := \begin{cases} P_K \left( \mathbf{h}_n - \mu_n \frac{\Theta_n(\mathbf{h}_n)}{\|\Theta_n'(\mathbf{h}_n)\|} \Theta_n'(\mathbf{h}_n) \right), \\ \text{if } \Theta_n'(\mathbf{h}_n) \neq \mathbf{0} \\ \mathbf{h}_n \quad \text{otherwise} \end{cases}, \quad (2)$$

where  $\Theta_n'(\mathbf{h}_n) \in \partial\Theta(\mathbf{h}_n)$ ,  $0 \leq \mu_n \leq 2$ .

**Proposition 1** (*Adaptive Projected Subgradient Method* [10]) The sequence  $(\mathbf{h}_n)_{n \in \mathbb{N}}$  generated by (2) satisfies the following:

(a) (*Monotone approximation*) Suppose that

$$\mathbf{h}_n \notin \Omega_n := \{\mathbf{h} \in K \mid \Theta_n(\mathbf{h}) = \Theta_n^*\} \neq \emptyset,$$

where  $\Theta_n^* := \inf_{\mathbf{u} \in K} \Theta_n(\mathbf{u})$ . Then, by using  $\forall \mu_n \in (0, 2 \left(1 - \frac{\Theta_n^*}{\Theta_n(\mathbf{h}_n)}\right))$ , we have

$$\forall \mathbf{h}^{*(n)} \in \Omega_n, \|\mathbf{h}_{n+1} - \mathbf{h}^{*(n)}\| < \|\mathbf{h}_n - \mathbf{h}^{*(n)}\|.$$

(b) (*Boundedness, Asymptotic optimality*) Suppose

$$\exists N_0 \in \mathbb{N} \quad \text{s.t.} \quad \begin{cases} \Theta_n^* = 0, \quad \forall n \geq N_0 \quad \text{and} \\ \Omega := \bigcap_{n \geq N_0} \Omega_n \neq \emptyset. \end{cases}$$

Then  $(\mathbf{h}_n)_{n \in \mathbb{N}}$  by (2) is bounded. Moreover if we specially use  $\forall \mu_n \in [\epsilon_1, 2 - \epsilon_2] \subset (0, 2)$ , where  $\epsilon_1, \epsilon_2 > 0$ , we have  $\lim_{n \rightarrow \infty} \Theta_n(\mathbf{h}_n) = 0$  provided that  $(\Theta_n(\mathbf{h}_n))_{n \in \mathbb{N}}$  is bounded.

For other properties, e.g., the strong convergence of the method, see [10].

## 3. PROPOSED ADAPTIVE RECEIVER

In this section, we propose a blind adaptive receiver, which does not require any training sequence and only assume the knowledge on  $\tau_1, \theta_1$  and  $\mathbf{s}_1$ .

### 3.1 Filter Constraint Sets

Let's define useful closed convex sets corresponding respectively to the desired properties for the MAI suppression filter  $\mathbf{h} \in \mathbb{R}^N$ .

Suppose, as the first step, that the information about  $A_1$  is known. The problem about how to obtain this information will be addressed shortly. Taking into account the received signal in (1), the desired MAI suppression filter  $\mathbf{h}$  should belong to [4]:

$$\tilde{C}_A(i) := \{\mathbf{h} : E[|\mathbf{h}^T \mathbf{r}_i|] = A_1\}.$$

Unfortunately, a stochastic approximation  $C_A(i) := \{\mathbf{h} : |\mathbf{h}^T \mathbf{r}_i| = A_1\}$  is not a convex set, and thus the convex set theoretic schemes cannot be applied. We employ as its simple convex relaxation:

$$C_B(i) := \{\mathbf{h} : |\mathbf{h}^T \mathbf{r}_i| \leq A_1\}. \quad (3)$$

$C_B(i)$  is now a closed convex set and its projection is given by:

$$P_{C_B(i)}(\mathbf{h}) = \begin{cases} \mathbf{h} - (\mathbf{h}^T \mathbf{r}_i - A_1) \frac{\mathbf{r}_i}{\mathbf{r}_i^T \mathbf{r}_i}, & \text{if } \mathbf{h}^T \mathbf{r}_i > +A_1 \\ \mathbf{h} - (\mathbf{h}^T \mathbf{r}_i + A_1) \frac{\mathbf{r}_i}{\mathbf{r}_i^T \mathbf{r}_i}, & \text{if } \mathbf{h}^T \mathbf{r}_i < -A_1 \\ \mathbf{h}, & \text{otherwise.} \end{cases} \quad (4)$$

To avoid the null vector  $\mathbf{h} = \mathbf{0}$  as the adaptive filter, which eliminates not only the interference, but also the desired signal, we define one set that contains the signature  $\mathbf{s}_1$ :

$$C_s := \{\mathbf{h} : \mathbf{h}^T \mathbf{s}_1 = 1\}, \quad (5)$$

onto which the projection is given by

$$P_{C_s}(\mathbf{h}) = \mathbf{h} - (\mathbf{s}_1^T \mathbf{h} - 1) \mathbf{s}_1. \quad (6)$$

It is easy to see that an ideal filter also belongs to set  $C_s$ .

### 3.2 Proposed Scheme

Suppose that at time  $n$  we have an estimated filter  $\mathbf{h}_n$  for MAI suppression. To update the filter from  $\mathbf{h}_n$  to  $\mathbf{h}_{n+1}$  we may consider the following cost functions as the performance measure to be decreased:

$$\Theta_n(\mathbf{h}) := \begin{cases} \sum_{j=0}^{q-1} \frac{\omega_j^{(n)}}{L_n} \|\mathbf{h}_n - P_{C_B(n-j)}(\mathbf{h}_n)\| \cdot \\ \cdot \|\mathbf{h} - P_{C_B(n-j)}(\mathbf{h})\|, & \text{if } L_n \neq 0 \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

$\forall n \in \mathbb{N}$ ,  $q \in \mathbb{N}^*$  where  $\sum_{j=0}^{q-1} \omega_j^{(n)} = 1$ ,  $\{\omega_j^{(n)}\}_{j=0, \dots, q-1} \subset (0, 1]$  and  $L_n := \sum_{j=0}^{q-1} \omega_j^{(n)} \|\mathbf{h}_n - P_{C_B(n-j)}(\mathbf{h}_n)\|$ . Note that  $\Theta_n$  ( $n = 0, 1, 2, \dots$ ) is a sequence of continuous convex functions that uses not only the actual received signal vector, but also past ones. By decreasing it, we can find a filter closer to the intersection of the sets  $C_B(n-j)$ ,  $j = 0, \dots, q-1$ , which reduces MAI when the filter also lies in  $C_s$ . Thus we have to minimize asymptotically a sequence of non-negative cost functions  $\Theta_n$  ( $n = 0, 1, 2, \dots$ ), which reflects the sets  $C_B(i)$ , over the closed convex set  $C_s$ . As shown in Proposition 1, the *Adaptive Projected Subgradient Method* [9, 10] asymptotically minimizes such sequence.

For the function  $\Theta_n$  in (7), we have its subgradient

$$\Theta'_n(\mathbf{h}_n) = \begin{cases} \sum_{j=0}^{q-1} \frac{\omega_j^{(n)}}{L_n} (\mathbf{h}_n - P_{C_B(n-j)}(\mathbf{h}_n)) \in \partial\Theta_n(\mathbf{h}_n), \\ \text{if } L_n \neq 0 \\ 0 \in \partial\Theta(\mathbf{h}_n), & \text{otherwise.} \end{cases} \quad (8)$$

Applying Eqs. (7) and (8) to the scheme in (2) yields the following algorithm for  $K := C_s$ :

**Algorithm 2** Given  $q \in \mathbb{N}^*$ , let's use the sets  $C_{B(n)}$ ,  $\dots$ ,  $C_{B(n-q+1)}$  and  $C_s$  at time  $n$ . The value  $q$  corresponds to the number of parallel processors to be engaged at time  $n$ . In addition, let  $\omega_j^{(n)} > 0$ ,  $j = 0, \dots, q-1$ , satisfy  $\sum_{j=0}^{q-1} \omega_j^{(n)} = 1$ . For any  $\mathbf{h}_o \in \mathbb{R}^N$ , define a sequence  $(\mathbf{h}_n)_{n \in \mathbb{N}}$  by

$$\mathbf{h}_{n+1} = P_{C_s} \left( \mathbf{h}_n + \lambda_n \left( \sum_{j=0}^{q-1} \omega_j^{(n)} P_{C_B(n-j)}(\mathbf{h}_n) - \mathbf{h}_n \right) \right), \quad (9)$$

$P_{C_B(i)}$  and  $P_{C_s}$  are the projections defined in Eq. (4) and (6), respectively.  $\lambda_n \in [0, 2\mathcal{M}_n]$  is a relaxation parameter, where

$$\mathcal{M}_n = \begin{cases} \frac{\sum_{j=0}^{q-1} \omega_j^{(n)} \|P_{C_B(n-j)}(\mathbf{h}_n) - \mathbf{h}_n\|^2}{\|\sum_{j=0}^{q-1} \omega_j^{(n)} P_{C_B(n-j)}(\mathbf{h}_n) - \mathbf{h}_n\|^2}, \\ \text{if } \mathbf{h}_n \notin \bigcap_{j=0}^{q-1} C_B(n-j) \\ 1, & \text{otherwise.} \end{cases} \quad (10)$$

For convergence of the scheme in Eq. (9), see Proposition 1 (for more detailed discussion see [9, 10]). Also by Eq. (9),  $\mathbf{h}_n \in C_s$  is always granted. The scheme in [8] does not use  $P_{C_s}$ .

As in [4], we have the following simple recursive estimator for the amplitude.

$$A_{1,n+1} = \begin{cases} A_{1,n} - \gamma(A_{1,n} - \mathbf{h}_n^T \mathbf{r}_n), & \text{if } \mathbf{h}_n^T \mathbf{r}_n \geq 0 \\ A_{1,n} - \gamma(A_{1,n} + \mathbf{h}_n^T \mathbf{r}_n), & \text{otherwise,} \end{cases} \quad (11)$$

where  $A_{1,0} = 0$  and  $0 \leq \gamma < 1$  is a forgetting factor.  $A_{k,n}$  is the amplitude estimate of user  $k$  at  $n$ th iteration.

The resulting algorithm first updates  $A_{1,n}$  by Eq. (11) and, with this new estimation, it updates  $\mathbf{h}_n$  by Eq. (9).

## 4. SIMULATION RESULTS AND CONCLUDING REMARKS

Figures 2 and 3 compare the speed of the proposed algorithm in a near-far situation and asynchronous communication with the

Table 1: Adaptive Algorithms

| Algorithm | Adaptation rule  |
|-----------|--|
| NLMS      | $\mathbf{h}_{n+1} = \mathbf{h}_n - \mu(\mathbf{h}_n^T \mathbf{r}_n - b_{1,n}) \frac{\mathbf{r}_n}{\mathbf{r}_n^T \mathbf{r}_n}$<br>Assumption: $b_{1,n}$ is known (training sequence)  |
| GPA       | $\mathbf{h}_{n,1} = \begin{cases} \mathbf{h}_n - \mu(\mathbf{h}_n^T \mathbf{r}_n - A_1) \frac{\mathbf{r}_n}{\mathbf{r}_n^T \mathbf{r}_n}, & \text{if } \mathbf{h}_n^T \mathbf{r}_n > 0 \\ \mathbf{h}_n - \mu(\mathbf{h}_n^T \mathbf{r}_n + A_1) \frac{\mathbf{r}_n}{\mathbf{r}_n^T \mathbf{r}_n}, & \text{otherwise} \end{cases}$<br>$\mathbf{h}_{n+1} = \mathbf{h}_{n,1} - (s_1^T \mathbf{h}_{n,1} - 1) \mathbf{s}_1$<br>Assumption: $A_1$ and $\mathbf{s}_1$ are known   |
| SAGP      | $A_{1,n+1} = A_{1,n} + \gamma( \mathbf{h}_n^T \mathbf{r}_n  - A_{1,n})$<br>$\mathbf{h}_{n,1} = \begin{cases} \mathbf{h}_n - \mu(\mathbf{h}_n^T \mathbf{r}_n - A_{1,n+1}) \frac{\mathbf{r}_n}{\mathbf{r}_n^T \mathbf{r}_n}, & \text{if } \mathbf{h}_n^T \mathbf{r}_n > 0 \\ \mathbf{h}_n - \mu(\mathbf{h}_n^T \mathbf{r}_n + A_{1,n+1}) \frac{\mathbf{r}_n}{\mathbf{r}_n^T \mathbf{r}_n}, & \text{otherwise} \end{cases}$<br>$\mathbf{h}_{n+1} = \mathbf{h}_{n,1} - (s_1^T \mathbf{h}_{n,1} - 1) \mathbf{s}_1$<br>Assumption: $\mathbf{s}_1$ is known |
| OPM-GP    | $\mathbf{x}_{n+1} = \mathbf{x}_n - \mu[\mathbf{r}_n - (s_1^T \mathbf{r}_n) \mathbf{s}_1] \frac{\mathbf{h}_n^T \mathbf{r}_n}{\mathbf{r}_n^T \mathbf{r}_n}$<br>$\mathbf{h}_{n+1} = \mathbf{s}_1 + \mathbf{x}_{n+1}$<br>Assumption: $\mathbf{s}_1$ is known   |
| Proposed  | $A_{1,n+1} = \begin{cases} A_{1,n} - \gamma(A_{1,n} - \mathbf{h}_n^T \mathbf{r}_n), & \text{if } \mathbf{h}_n^T \mathbf{r}_n \geq 0 \\ A_{1,n} - \gamma(A_{1,n} + \mathbf{h}_n^T \mathbf{r}_n), & \text{otherwise} \end{cases}$<br>$\mathbf{h}_{n+1} = P_{C_s} \left( \mathbf{h}_n + \lambda_n \left( \sum_{j=0}^{q-1} \omega_j^{(n)} P_{C_B(n-j)}(\mathbf{h}_n) - \mathbf{h}_n \right) \right)$<br>Assumption: $\mathbf{s}_1$ is known  |

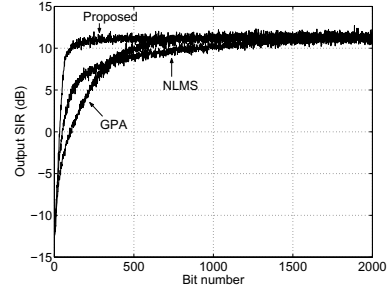


Figure 2: Output SIR curves for SNR = 15 dB,  $K = 6$ ,  $\gamma = 0.01$ ,  $\mu = 0.6$ ,  $\lambda_n = 0.2\mathcal{M}_n$ ,  $q = 64$ ,  $A_k = 10A_1$ ,  $k = 2, \dots, K$  and  $\omega_j^{(n)} = \frac{1}{q}$ ,  $j = 0, \dots, q-1$ .

normalized LMS (with training sequence), the OPM-based gradient projection (OPM-GP) [6], the generalized projection algorithm (GPA) [4] and the space alternating generalized projection with approximate EM mapping (SAGP) [4]. Table 1 summarizes the algorithms. The noise is assumed to be Gaussian. The performance characteristic is shown by the ensemble-averaged output-to-interference ratio (SIR), which at the  $n$ th iteration is calculated by:

$$\text{SIR}_n = \frac{\sum_{k=1}^U (\mathbf{h}_n[k]^T \mathbf{s}_1)^2}{\sum_{k=1}^U \left[ \frac{\mathbf{h}_n[k]^T (\mathbf{r}_n[k] - A_1[k] b_{1,n}[k] \mathbf{s}_1)}{A_1[k]} \right]^2},$$

where  $\mathbf{h}_n[k]$  and  $\mathbf{r}_n[k]$  are the respective vectors on  $k$ th realization.  $A_1[k]$  and  $b_{1,n}[k]$  are the transmitted bits and amplitude of the desired user at  $k$ th realization.  $U$  is the number of realizations.

Figure 2 compares the performance of the proposed algorithm with schemes that rely on training sequences or knowledge of the amplitude  $A_1$ . Figure 3 is a fair comparison with other schemes that have the same information as the proposed one. We set the number of realizations  $U = 500$ .  $\mathbf{r}_i = \mathbf{r}_1$  for  $i \leq 1$ . The number of past vectors processed  $q$  is 64.  $\mu = 0.6$ ,  $\lambda_n = 0.2\mathcal{M}_n$ ,  $\gamma = 0.01$  and  $\omega_j^{(n)} = \frac{1}{q}$ ,  $j = 0, \dots, q$ . The number of interfering users is  $(K-1) = 5$  and all users have amplitude 10 times greater than the amplitude of the desired signal  $A_1 = 1$ . The signal-to-noise ratio (SNR) is 15 dB. Signals are modulated by 31-length gold sequences, which were chosen randomly. For simulation simplicity, the path delays of users  $k = 2, \dots, 6$  are given by  $\tau_k = l_k T_c$ , where  $l_k$  is a uniformly random integer which satisfies

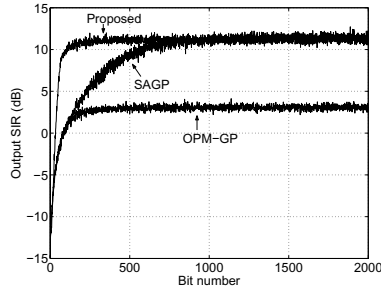


Figure 3: Output SIR curves for SNR = 15 dB,  $K = 6$ ,  $\gamma = 0.01$ ,  $\mu = 0.6$ ,  $\lambda_n = 0.2\mathcal{N}_n$ ,  $q = 64$ ,  $A_k = 10A_1$ ,  $k = 2, \dots, K$  and  $\omega_j^{(n)} = \frac{1}{q}$ ,  $j = 0, \dots, q-1$ .

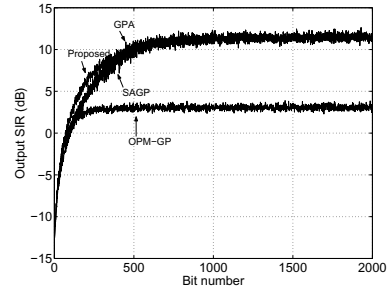


Figure 5: Speed improvement for small  $q$  over the fastest achievable speed with GPA and SAGP. SNR=15 dB,  $K = 6$ ,  $\gamma = 0.01$ ,  $\mu = 0.6$ ,  $\lambda_n = 0.3\mathcal{N}_n$ ,  $q = 3$ ,  $A_k = 10A_1$ ,  $k = 2, \dots, K$  and  $\omega_j^{(n)} = \frac{1}{q}$ ,  $j = 0, \dots, q-1$ .

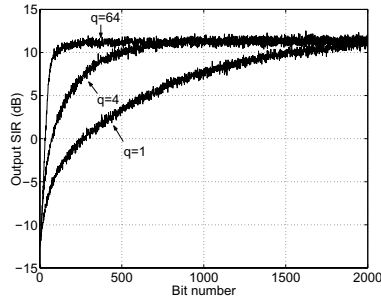


Figure 4: Output SIR curves for different values of  $q$ . SNR = 15 dB,  $K = 6$ ,  $\gamma = 0.01$ ,  $\lambda_n = 0.2\mathcal{N}_n$ ,  $A_k = 10A_1$ ,  $k = 2, \dots, K$  and  $\omega_j^{(n)} = \frac{1}{q}$ ,  $j = 0, \dots, q-1$ .

$0 \leq l_k T_c < T_b$ . [11] considers only a synchronous DS-CDMA system ( $l_k T_c = 0$ ,  $k = 2, 3, \dots$ ) and that is the reason it is not compared. For all algorithms,  $\mathbf{h}_0 = \mathbf{s}_1$  and, for OPM-GP,  $\mathbf{x}_0 = 0$ . By fixing all parameters and varying  $\mu$ , the value  $\mu = 0.6$  resulted in the fastest noticeable speed of convergence for GPA and SAGP. As for the SAGP, the parameter  $\gamma$  did not influence the results in a noticeable way in the sense of speed increases. Also, by varying the step size of the NLMS, it was not possible to achieve faster convergence than the proposed one and the same happened with the OPM-GP.

We observe that the speed of the proposed algorithm is unbeatable by the compared methods. Further performance increases can be achieved through proper selection of  $q$ ,  $\lambda_n$ ,  $\omega_j^{(n)}$  and  $\gamma$ . This fast numerical convergence is due to the fact that we use more information in parallel at the same time. Not only is the actual input sample vector used, but also past ones are used.

Regarding the parameter  $q$ , the higher it is, the more information we use at the same time. Therefore speed increases are expected if other parameters are kept the same. However, the performance at steady state is not necessarily the same. This is illustrated in Fig. 4.

Practically it may not be possible to have a large value for  $q$  and, for smaller values of  $\mu$ , GPA and SAGP may perform better at steady state if the right set of parameters is not properly chosen. However, speed improvements can be achieved even for small  $q$ , as illustrated in Fig. 5. Regarding the performance at steady state, one possible solution is to ignore past data when steady state is achieved, i.e.,  $q$  is set to 1. With such procedure the proposed method presents both desirable features: fast speed and good performance at steady state.

Suitable choice of the weighting coefficients  $\omega_j^{(n)}$  can provide even better results than demonstrated in the above examples, giving

even more flexibility to our method. The results show that the proposed scheme offers a reasonable alternative specially when speed of convergence is concerned and MAI is high. Finally, the additional computational complexity can be somehow alleviated by using processors in parallel, due to the inherently parallel construction of the summation in Eq. (9).

## REFERENCES

- [1] S. Glisic and B. Vucetic, *Spread Spectrum CDMA Systems for Wireless Communications*, Boston: Artech House, 1997.
- [2] D. M. Novakovick and M. L. Dukic, "Evolution of the power control techniques for DS-CDMA toward 3G wireless communication systems," *IEEE Comm. Surveys*, vol. 3, no. 4, 4th Quarter 2000.
- [3] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 32, no. 1, pp. 85–96, January 1986.
- [4] S. C. Park and J. F. Doherty, "Generalized projection algorithm for blind interference suppression in DS/CDMA communications," *IEEE Trans. on Circuits and Systems-II*, vol. 44, no. 6, pp. 453–460, June 1997.
- [5] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Comm.*, vol. 42, no. 12, pp. 3178–3188, December 1994.
- [6] M. Honig, U. Madhow, and S. Verdu, "Blind adaptive multiuser detection," *IEEE Trans. Inform. Theory*, vol. 41, no. 4, pp. 944–960, July 1995.
- [7] S. L. Miller, "An adaptive direct-sequence code division multiple-access receiver for multiuser interference rejection," *IEEE Trans. Comm.*, vol. 43, pp. 1746–1754, 1995.
- [8] I. Yamada, K. Slavakis, and K. Yamada, "An efficient robust adaptive filtering algorithm based on parallel subgradient projection techniques," *IEEE Trans. Signal Processing*, vol. 50, no. 5, pp. 1091–1101, May 2002.
- [9] I. Yamada, "Adaptive projected subgradient method – a unified view for projection based adaptive algorithms," *Journal of IEICE*, vol. 86, no. 8, pp. 654–658, 2003, in Japanese.
- [10] I. Yamada and N. Ogura, "Adaptive projected subgradient method and its applications to set theoretic adaptive filtering," in *Proceedings of the 37th Asilomar Conference on Signals, Systems and Computers*, November 2003.
- [11] X. M. Wang, W. S. Lu, and A. Antoniou, "A near-optimal multiuser detector for DS-CDMA systems using semidefinite programming relaxation," *IEEE Trans. Signal Processing*, vol. 51, no. 9, pp. 2446–2450, September 2003.