PARAMETER ESTIMATION AND EQUALIZATION TECHNIQUES FOR MIMO FREQUENCY SELECTIVE CHANNELS WITH MULTIPLE FREQUENCY OFFSETS

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ABSTRACT

In this work, we focus upon the estimation of the channel parameters of a multiple input and multiple output (MIMO) model, allowing for more than one path between the transmit and the receive antennas, with each path possibly having different frequency offsets (FOs). Such a scenario may arise due to Doppler shifts associated with various multipaths. In particular, we derive an approximate maximum likelihood estimator (AML) for the channel parameters by exploiting the correlation property of the pilot signal. The resulting AML is realised as the sum of $n_T L$ one-dimensional maximizations, where n_T is the number of transmit antennas and L is the total number of paths, and is validated by comparison with the Cramér-Rao lower bound (CRLB), which is also derived. Furthermore, a minimum mean square error recursive (MMSER) equalizer is proposed which exploits the estimated parameters, the performance of which is shown to be much improved on an equalizer not accounting for FOs.

1. INTRODUCTION

A multipath wireless environment makes it difficult for the receiver to determine the transmitted signal, unless the receiver is provided with some form of diversity. Recent research in communication theory has shown that large gains in diversity, capacity and reliability of communications over wireless channels could be achieved by exploiting spatial diversity. Spatial diversity uses multiple antennas at both transmit and receive sides and will play a key role in future high rate wireless communication [1]. However, the performance of such MIMO systems may seriously degrade in the presence of FOs due to bad synchronization between the transmitter and the receiver as well as motion-induced Doppler shift, due to the relative motion between the mobile station and the local scatterers. Therefore, it is of importance to determine these FOs and to take them into account in the equalizer design. The MIMO flat fading channel with various FOs was addressed in [2]. In this paper, we extend this work for frequency selective channels with different FOs between each transmit and receive antenna and address a general framework. Such a scenario may arise due to the motion of a transmitter or a receiver with high speed, thereby resulting in distinct Doppler shifts for paths with different angles of arrival. Another important contribution of this paper is the design of a recursive equalizer to account for symbol-by-symbol variations of the channel due to multiple FOs.

2. PROBLEM STATEMENT

Consider a baseband MIMO communication system with n_T transmit and n_R receive antennas, where the signal between any two transmit and receive antennas has propagated through a total of L different paths, with each path possibly having different FOs. The received baseband signal at antenna k over one frame, can then be written as

$$r_k(n) = \sum_{l=1}^{n_T} \sum_{p=0}^{L-1} h_{kl}(p) e^{j\omega_{klp} n} x_l(n-p) + \varepsilon_k(n) \tag{1}$$

for $n = 0, \ldots, N-1$, $k = 1, \ldots, n_R$, where $h_{kl}(p)$ and ω_{klp} are the channel gains (CGs) and the FOs, respectively, between the receive antenna k and the transmit antenna l, for the multipath p; these are assumed to be quasi-stationary, i.e., they do not change significantly over the observed data frame, but may change between frames. Here, $x_l(n)$ is the training signal transmitted from the lth transmitter and $\varepsilon_k(n)$ is assumed an additive zero mean circular Gaussian noise with variance σ_ε^2 and n is the discrete time index. Let $\mathbf{X}_{lp} = \mathrm{diag}\{\ \mathbf{x}_{l,p}\ \}$, and

$$\begin{aligned}
\mathbf{x}_{l,p} &= \left[\ x_l(n-p) \ \cdots \ x_l(n-p-N+1)\ \right]^T \\
\mathbf{e}_{kl}(p) &= \left[\ e^{j\omega_{klp} n} \ \ldots \ e^{j\omega_{klp}(n-N+1)}\ \right]^T \\
\mathbf{h}_{kl} &= \left[\ h_{kl}(0) \ \cdots \ h_{kl}(L-1)\ \right]^T
\end{aligned}$$

where $(\cdot)^T$ denotes vector transpose. Here, $\mathbf{e}_{kl}(p)$ contains the FOs between the receive antenna k and the transmit antenna l, for the multipath p, $\mathbf{h}_{kl}$ is the vector of CGs between the receive antenna k and the transmit antenna l, and $\mathbf{X}_{lp}$ denotes the $N \times N$ diagonal matrix formed from the signal transmitted from the lth antenna at lag p. Further, suppose that

$$\begin{aligned}
\mathbf{V}_{kl} &= \left[\ \mathbf{X}_{l0}\mathbf{e}_{kl}(0) \ \cdots \ \mathbf{X}_{l(L-1)}\mathbf{e}_{kl}(L-1)\ \right] \\
\mathbf{h}_k &= \left[\ \mathbf{h}_{k1}^T \ \mathbf{h}_{k2}^T \ \cdots \ \mathbf{h}_{kn_T}^T\ \right]^T \\
\mathbf{V}_k &= \left[\ \mathbf{V}_{k1} \ \mathbf{V}_{k2} \ \cdots \ \mathbf{V}_{kn_T}\ \right]
\end{aligned}$$

Thus, (1) can be written in block form as

$$\begin{aligned}
\mathbf{r}_k &= \left[\ r_k(n) \ \cdots \ r_k(n-N+1)\ \right]^T \\
&= \mathbf{V}_k \mathbf{h}_k + \boldsymbol{\varepsilon}_k
\end{aligned} \tag{2}$$

Here, we are interested in estimating all the various CGs, $h_{kl}(p)$, and the FOs, ω_{klp}. Let

$$\omega_{kl} = \begin{bmatrix} \omega_{kl0} & \omega_{kl1} & \cdots & \omega_{kl(L-1)} \end{bmatrix}^T \quad (3)$$

$$\omega_k = \begin{bmatrix} \omega_{k1}^T & \omega_{k2}^T & \cdots & \omega_{kn_T}^T \end{bmatrix}^T \quad (4)$$

Then, the unknown parameter vector, θ_k, to be estimated can be written as $\theta_k = \begin{bmatrix} \mathbf{h}_k^T & \omega_k^T \end{bmatrix}$.

3. ESTIMATION OF CHANNEL GAINS AND FREQUENCY OFFSETS

To estimate the CGs and the FOs, we outline an AML, fully exploiting the structure of the transmitted pilot sequence. Since the noise, $\varepsilon_k(n)$, at the receive antenna is spatially uncorrelated, the parameters corresponding to each antenna k can be estimated independently from the signal received at each antenna. Considering (2), the log-likelihood function of $\mathbf{r}_k$ can be written as (ignoring the constant terms)

$$\ln p(\mathbf{r}_k|\ \theta_k) = \frac{1}{\sigma_\varepsilon^2}\left(\mathbf{r}_k - \mathbf{V}_k\mathbf{h}_k\right)^H\left(\mathbf{r}_k - \mathbf{V}_k\mathbf{h}_k\right) \quad (5)$$

where $(\cdot)^H$ denotes the conjugate transpose. Minimizing (5) with respect to $\mathbf{h}_k$ yields

$$\hat{\mathbf{h}}_k = (\mathbf{V}_k^H\mathbf{V}_k)^{-1}\mathbf{V}_k^H\mathbf{r}_k. \quad (6)$$

Then, inserting $\hat{\mathbf{h}}_k$ into (5) yields the cost function

$$J(\omega_{klp}) = \mathbf{r}_k^H\mathbf{r}_k - \mathbf{r}_k^H\mathbf{V}_k(\mathbf{V}_k^H\mathbf{V}_k)^{-1}\mathbf{V}_k^H\mathbf{r}_k \quad (7)$$

Note that minimizing (7) requires an $n_T L$-dimensional minimization. However, choosing $x_l(n)$ such that

$$E\left\{x_l^*(n-u)x_d(n-v)\right\} = \delta_{u-v}\delta_{l-d} \quad (8)$$

where δ_k denotes the Kronecker delta and $(\cdot)^*$ the complex conjugate, this minimization can be decoupled into $n_T L$ one-dimensional minimizations. Considering (8), we note that $\mathbf{V}_k^H\mathbf{V}_k$ will be dominated by the large diagonal terms, with almost negligible contribution from the off-diagonal terms. Thus, $\mathbf{V}_k^H\mathbf{V}_k \approx \sum_{n=0}^{N-1}|x_l(n)|^2\,\mathbf{I} = \kappa\mathbf{I}$, where κ is constant over the frame considered, enabling us to approximate the minimum of (7) as the maximum of

$$\begin{aligned} J'(\omega_{klp}) &= \mathbf{r}_k^H\mathbf{V}_k\mathbf{V}_k^H\mathbf{r}_k \\ &= \sum_{l=1}^{n_T}\sum_{p=0}^{L-1}\left|\sum_{n=0}^{N-1} r_k^*(n)x_l(n-p)e^{j\omega_{klp}n}\right|^2 \quad (9)\end{aligned}$$

This is a joint multi dimensional optimization problem. Consider estimation of FOs for an arbitrary path s from transmit antenna q to a receive antenna k . We could write the contribution of this path to (9) as

$$\begin{aligned}\psi_{kqs}(n) &= r_k^*(n)x_q(n-s)e^{j\omega_{kqs}n} \\ &= Q_{kq}(s) + c_1(n) + c_2(n)\end{aligned}$$

where

$$Q_{kq}(s) = h_{kq}^*(s)\left|x_q(n-s)\right|^2 \quad (10)$$

$$c_1(n) = \varepsilon_k^*(n)x_q(n-s)e^{j\omega_{kqs}n}$$

$$c_2(n) = \sum_{l=1}^{n_T}\sum_{\substack{p=0 \\ p\neq s|\ l=q}}^{L-1} h_{kl}^*(p)x_l^*(n-p)x_q(n-s)e^{j\Delta\omega_{klp}n}$$

where $\Delta\omega_{klp} = \omega_{kqs} - \omega_{klp}$. As $Ex_l(n-u)x_q(n-s) = 0$ for $u \neq v$, or $l \neq d$, we could show that

$$\begin{aligned}\left|\sum_{n=0}^{N-1}\psi_{kqs}(n)\right|^2 &\approx N^2|h_{kq}(s)|^2|x_q(n-s)|^2 \\ &+ N\sum_{l=1}^{n_T}\sum_{\substack{p=0 \\ p\neq s|\ l=q}}^{L-1}|h_{kl}(p)|^2|x_{kl}(n-p)|^2 \\ &= N^2\rho_s + N\rho_i \approx N^2\rho_s \ \ (For\ \ large\ \ N)\end{aligned}$$

Therefore, provided that the ratio between the signal component, (ρ_s), to the interfering components (ρ_i) is greater than $1/N$, we could reduce the multidementional joint maximization in (9) over all possible frequencies to the maximization of the following for each individual frequency

$$\hat{\omega}_{kqr} = \arg\max_{\omega}\left|\sum_{n=0}^{N-1} r_k^*(n)x_q(n-r)e^{j\omega n}\right|^2 \quad (11)$$

which can be efficiently evaluated using the fast Fourier transform (FFT). Once the FOs are estimated, the CGs, $\mathbf{h}_k$, can be estimated using (6).

4. CRAMÉR-RAO LOWER BOUND

In this section, we derive the CRLB for the problem at hand. Recalling (1), we write the received signal as

$$r_k(n) = u_k(n) + \varepsilon_k(n),\ k = 1,...,n_R,\ n = 0,...,N-1$$

and in vector form as

$$\mathbf{r} = \mathbf{u} + \varepsilon \quad (12)$$

together with the unknown desired parameters in

$$\eta \stackrel{\triangle}{=} \begin{bmatrix} \eta_1^T & \eta_2^T & \ldots & \eta_{n_R}^T \end{bmatrix}^T \quad (13)$$

where

$$\eta_k \stackrel{\triangle}{=} \begin{bmatrix} Re(\mathbf{h}_k)^T & Im(\mathbf{h}_k)^T & \omega_k^T \end{bmatrix}^T \quad (14)$$

and $Re(\cdot)$ and $Im(\cdot)$ denote the real and imaginary parts of a complex argument, respectively. Since the noise sequence $\varepsilon_k(n)$ is spatially and temporally uncorrelated, the Fisher information matrix (FIM) $\mathbf{F}$ for the estimation of η can be found using Bangs' formula (see, e.g., [3])

$$\mathbf{F}(k,l) = \frac{2}{\sigma_\varepsilon^2}Re\left(\frac{\partial\mathbf{u}^H}{\partial\eta_k}\frac{\partial\mathbf{u}}{\partial\eta_l{}^T}\right) \quad (15)$$

where, $k,l = 1,2,...,n_R$., and $\mathbf{F}(k,l)$ denotes the (k,l)th submatrix of $\mathbf{F}$, corresponding to the vector parameters, η_k and η_l. From (15), it can be noted that $\mathbf{F}(k,l) = \mathbf{0}$ whenever $k \neq l$, hence, there is a decoupling between the estimation error in parameters of two different antennas and hence, the FIM is block diagonal. If we suppose $\mathbf{F}_k = \mathbf{F}(k,k)$, the FIM has size $3n_TL \times 3n_TL$ for the estimation of η_k, then the elements of $\mathbf{F}_k$ can be found using

$$\frac{\partial u_k(n)}{\partial Re(h_{kl}(p))} = e^{j\omega klpn} x_l(n-p) \quad (16a)$$

$$\frac{\partial u_k(n)}{\partial Im(h_{kl}(p))} = je^{j\omega klpn} x_l(n-p) \quad (16b)$$

$$\frac{\partial u_k(n)}{\partial \omega_{klp}} = jnh_{kl}(p)e^{j\omega_{klp}n} x_l(n-p) \quad (16c)$$

Therefore, $\mathbf{F}_k$, with each submatrix having dimension $n_TL \times n_TL$ for the estimation of η_k, can be written as

$$\mathbf{F}_k = \frac{2}{\sigma_\varepsilon{}^2} \begin{bmatrix} Re(\mathbf{U}_k) & Im(\mathbf{U}_k) & Im(\mathbf{T}_k) \\ -Im(\mathbf{U}_k) & Re(\mathbf{U}_k) & Re(\mathbf{T}_k) \\ -Im(\mathbf{T}_k)^H & Re(\mathbf{T}_k)^H & Re(\mathbf{S}_k) \end{bmatrix}$$

where

$$\begin{aligned} \mathbf{U}_k &= \mathbf{W}_k^H \mathbf{W}_k \\ \mathbf{W}_k &= [\ \mathbf{p}_{k1}(0)\ \dots\ \mathbf{p}_{k1}(L-1)\ \dots\ \mathbf{p}_{kn_T}(L-1)\ ] \\ \mathbf{p}_{kl}(p) &= \mathbf{X}_{lp}\mathbf{e}_{kl}(p) \\ \mathbf{D}_n &= diag(0,1,\cdots,N-1) \\ \mathbf{D}_h &= diag(h_{k1}(0),\dots h_{k1}(L-1),\dots,h_{kn_T}(L-1)) \\ \mathbf{T}_k &= \mathbf{W}_k^H \mathbf{D}_n \mathbf{W}_k \mathbf{D}_h \\ \mathbf{S}_k &= \mathbf{D}_h^H \mathbf{W}_k^H \mathbf{D}_n^2 \mathbf{W}_k \mathbf{D}_h \end{aligned}$$

Note that there is a coupling in the estimation error between the channel parameters and the FOs. Recall that the CRLB is obtained as the inverse of the FIM, i.e.,

$$\mathbf{CRB}(\eta_k) = \mathbf{F}_k^{-1} \quad (17)$$

Therefore, the CRLB associated with $Re(h_{kl}(p)), Im(h_{kl}(p))$ and, ω_{klp}, will be respectively the $((l-1)L+p)$th, $((l-1)L+p+n_T)$th and $((l-1)L+p+2n_T)$th diagonal term, of the matrix in (17).

5. A RECURSIVE MMSE EQUALIZER DESIGN

We proceed to the design of a recursive MIMO equalizer. Let the temporal length of the equalizer be M. We write the equalizer input vector as

$$\mathbf{r}_M = [\ \mathbf{r}^T(n) \quad \cdots \quad \mathbf{r}^T(n-M+1)\ ]^T = \mathbf{H}_c\mathbf{x} + \varepsilon$$

where $\mathbf{H}_c$ is the $n_RM \times n_T(M+L-1)$ channel convolution matrix, and

$$\begin{aligned} \mathbf{r}(n) &= [\ r_1(n) \quad \cdots \quad r_{n_T}(n)\ ]^T \\ \mathbf{x} &= [\ \mathbf{x}^T(n) \quad \dots \quad \mathbf{x}^T(n-M-L+2)\ ]^T \\ \mathbf{x}(n) &= [\ x_1(n) \quad \dots \quad x_{n_T}(n)\ ]^T \end{aligned}$$

Using these definitions, the equalizer output can be written as $y(n) = \mathbf{w}^H\mathbf{H}_c\mathbf{x} + \mathbf{w}^H\varepsilon$. The MMSE equalizer is

$$\mathbf{w} = \left(\mathbf{H}_c\mathbf{H}_c^H + \frac{\sigma_\varepsilon{}^2}{\sigma_x^2}\mathbf{I}\right)^{-1} \mathbf{H}_c\mathbf{z}_v \triangleq \mathbf{R}^{-1}\mathbf{H}_c\mathbf{z}_v \quad (18)$$

where $\mathbf{z}_v$ is the $n_T(M+L-1) \times 1$ coordinate vector, only containing a non-zero element at position v, i.e.,

$$\mathbf{z}_v = [\ 0 \quad \cdots \quad 0 \quad 1 \quad 0 \quad \cdots \quad 0\ ]^T \quad (19)$$

Due to FOs, the channel convolution matrix $\mathbf{H}_c$ changes after every symbol. Therefore, it is necessary to update the equalizer at every symbol, which is typically computationally infeasible. We propose a computationally efficient recursive scheme, exploiting the structural movement of the submatrices in $\mathbf{R}$. To emphasize the fact that $\mathbf{R}$ changes at every symbol time n, we use a subscript n in (18), as follows

$$\mathbf{w}_n = \mathbf{R}_n^{-1}\mathbf{H}_c(n)\mathbf{z}_v \quad (20)$$

The matrices $\mathbf{R}_n$ and $\mathbf{R}_{n+1}$ can be written as follows

$$\mathbf{R}_n = \left[\begin{array}{c|c} \mathbf{G}_n & \mathbf{C}_n \\ \hline \mathbf{C}_n^H & \mathbf{B}_n \end{array}\right]$$

$$\mathbf{R}_{n+1} = \left[\begin{array}{c|c} \mathbf{D}_n & \mathbf{E}_n \\ \hline \mathbf{E}_n^H & \mathbf{G}_n \end{array}\right]$$

where $\mathbf{G}_n \in \mathscr{C}^{n_R(M-1)\times n_R(M-1)}$, $\mathbf{B}_n \in \mathscr{C}^{n_R\times n_R}$, $\mathbf{D}_n \in \mathscr{C}^{n_R\times n_R}$, $\mathbf{C}_n \in \mathscr{C}^{n_R(M-1)\times n_R}$, and $\mathbf{E}_n \in \mathscr{C}^{n_R\times n_R(M-1)}$. Note how the Hermitian matrix $\mathbf{G}_n$ moves from the top left corner to bottom right corner from time n to $n+1$. Further, if we know the inverse of $\mathbf{G}_n$, we could find the inverses of $\mathbf{R}_n$ and $\mathbf{R}_{n+1}$ using the matrix inversion lemma (see, e.g., [3]), yielding a computationally efficient update of $\mathbf{w}_n$. As $\mathbf{G}_n$ will not appear in $\mathbf{R}_{n+2}$, it can not be used to find the inverse of $\mathbf{R}_{n+2}$. Thus, the scheme so far only allows for a pairwise computational saving, still requiring inversion of $\mathbf{G}_{n+1}$, to find the inverse of $\mathbf{R}_{n+2}$ efficiently. However, further exploiting the structure one may compute the inverse of $\mathbf{G}_{n+1}$ efficiently from the inverse of $\mathbf{R}_{n+1}$ using the following lemma.

Lemma 1 *Let*

$$\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \quad (21)$$

Here, $\dim\{\mathbf{H}_{kl}\} = \dim\{\mathbf{Q}_{kl}\}$. *Then, provided the relevant inverses exist, the inverse of matrix* $\mathbf{Q}_{11}$ *can be written as the Schur complement of* $\mathbf{H}_{22}$*, i.e., (see, [4] for proof)*

$$\mathbf{Q}_{11}^{-1} = \mathbf{H}_{11} - \mathbf{H}_{12}\mathbf{H}_{22}^{-1}\mathbf{H}_{21} \quad (22)$$

Therefore, by starting at time n, we find the inverse of submatrix $\mathbf{G}_n$. The inverse of $\mathbf{R}_n$ and $\mathbf{R}_{n+1}$ is then found using the matrix inversion lemma. Once we have the inverse of $\mathbf{R}_{n+1}$, the inverse of $\mathbf{G}_{n+1}$ is found using (22). Further, the inverse of $\mathbf{G}_{n+1}$ can then be used to find the inverse of $\mathbf{R}_{n+2}$ and so on. We call this a forward and backward recursion method to find the inverse of the matrix $\mathbf{R}_n$ after every symbol. Thus, we need the explicit inverse of the submatrix $\mathbf{G}_n$ only once at the start, thereafter, we only need the inverse of $\mathbf{H}_{22}$ after every symbol significantly reducing the complexity of finding the inverse of $\mathbf{R}_n$ from $\mathscr{O}(n_R^3\ M^3)$ to $\mathscr{O}(n_R^3)$. If only one path exists between any two transmit and receive antennas, then the matrix $\mathbf{H}_c(n)\mathbf{H}_c^H(n)$ will be block diagonal, enabling the inverse to be found by taking the inverse of individual blocks in $\mathbf{H}_c(n)\mathbf{H}_c^H(n)$. Moreover, for single transmit and receive antenna schemes with distinct FOs for each path, $\mathbf{H}_{22}$ is only a scalar. Hence, for this case we do not require any explicit matrix inversion, whereas without the presented recursive method it would mean the inversion of an $M \times M$ matrix at every symbol[4].

6. SIMULATION

To illustrate the performance of the proposed estimator, a case using two transmit and two receive antennas is simulated. Here, it is considered that there are two paths between each transmit and receive antenna, allowing eight different paths and correspondingly eight different FOs. The FOs have been chosen to be of the order of 10^{-3}. This is reasonable as the maximum Doppler shift for a vehicular speed of 250 km/h (RA250 channels as defined in GSM standards) at a carrier frequency of 900 MHz is 1.3 KHz, which corresponds to 0.005 when normalised to the symbol rate of 270 KHz as in GSM. Using the assumption of a quasi-stationary channel, the CGs, $h_{kl}(p)$, and the FOs, ω_{klp}, remain constant throughout the training burst interval. The CGs are randomly chosen but normalized to have unit amplitude. Binary phase shift keying (BPSK) signals are used for pilot as well as for data signals, for data signals higher order modulation schemes could be used, the length of the pilot signal is 200 samples. In the simulation, parameters are estimated and compared with the corresponding CRLB. Figures 2 and 1 depict the variance of the estimator for the CGs and FOs, respectively, as a function of the noise variance indicating the near efficient performance obtained by the AML estimator. In order to demonstrate the benefits of employing FOs

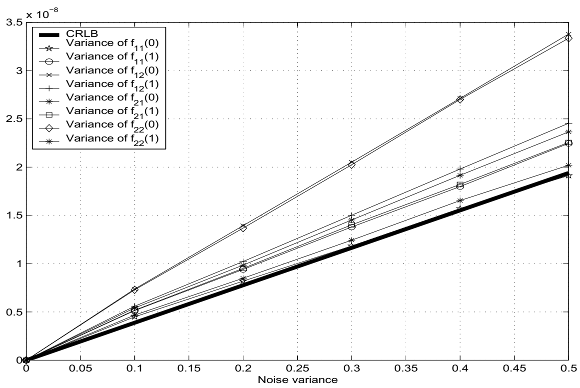

Figure 1: Comparison of the mean square estimation error of frequency offsets with the corresponding CRLB.

in equalization, we considered a two transmit and three receive antenna model with two multipaths between any two antennas. The length of the equalizer is set to $M = 4$, which is the minimum length required. Here, we consider three scenarios: in the first scenario, we set the FOs associated with each path to zero, and designed an equalizer based on the MMSE criterion. In the second scenario, we set the FOs as explained and exploit the proposed recursive equalizer. In the third, we considered a channel with FOs as in the second scenario, but designed the equalizer ignoring the FOs. The results depicted in Figure 3 show the superior performance of the proposed scheme over the equalizer not considering the effect of FOs.

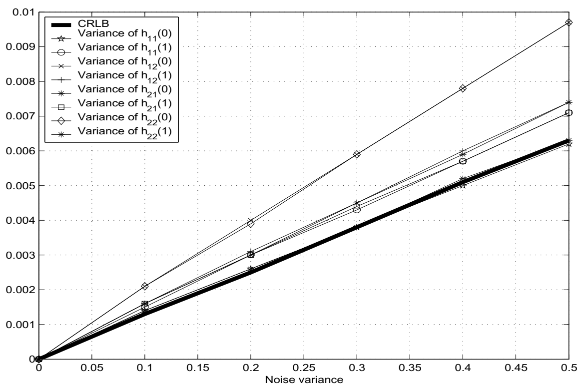

Figure 2: Comparison of the mean square estimation error of channel gain with the corresponding CRLB.

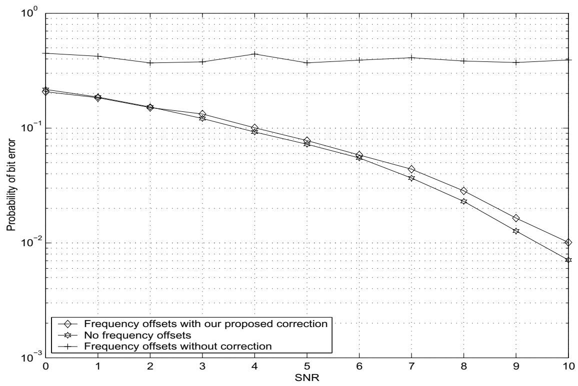

Figure 3: Probability of bit error versus SNR for different cases.

7. CONCLUSION

In this paper, we have addressed the CGs and FOs estimation problem for a frequency selective MIMO channel with distinct FOs for each path. By exploiting the correlation property of the transmitted pilot signal, we provided an approximation to the maximum likelihood estimator, and validated this approximation by comparing the performance of the estimator with CRLB. The estimator is found to be both computationally and statistically efficient. We also demonstrated the variation of the channel convolution matrix at every symbol and proposed a recursive equalizer that reduces the complexity significantly. Finally. our simulation results showed substantial improvement in the performance, when the frequency offsets are considered in the equalizer design, as compared to the equalizer not accounting FOs.

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