3-D TEXTURE MODEL BASED ON THE WOLD DECOMPOSITION

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ABSTRACT

This paper deals with the parametric modelling of three-dimensional (3-D) stochastic processes. Based on the 3-D Wold-like decomposition theory, any 3-D discrete homogeneous process can be represented as a sum of three mutually orthogonal components: a completely non deterministic, a “remote past” type deterministic and an evanescent component. The evanescent component can be further decomposed into two orthogonally components called evanescent of type 1 and evanescent of type 2. The aim of this paper is to propose a new parametric models able to describe both the spectral support and spatial structure of each component.

1. INTRODUCTION

The Wold decomposition theorem for two-dimensional (2-D) stochastic processes [1, 3] stipulates that a homogeneous stochastic process can be decomposed into two orthogonal components, named deterministic field and completely non-deterministic (CND) one. Moreover, it has been shown that the deterministic is further decomposed into mutually orthogonal half-plane deterministic field and generalized evanescent components. Thus, in the 2-D case, in tackling the problem of texture analysis, Francos et al. [4], have proposed explicit parametric models of each component. These models have been applied to segmentation and indexation of textured images [5]. A main challenge in Wold- based image modelling is to develop an efficient and robust Wold decomposition algorithm. Some methods have been proposed in the literature for extracting the three components from textured images [6, 9].

The 2-D Wold decomposition overcomes the fact that the classical parametric AR like model cannot totally take into account all the information contained in some structured textured image. Indeed, the 2-D AR model permits only to model homogeneous field having absolutely continuous spectral density. However, the information concerning the orientation or periodical behaviour is provided by the evanescent component [8], [10].

Recently, in [11] the authors describe and analyse the theoretical structure of multidimensional Wold decomposition with respect to lexicographic-type orders for homogeneous discrete process. Especially, in the 3-D case, which is important for applications to textured image blocks processing, they showed that, the evanescent component can be decomposed into two orthogonally sub-evanescent components, with spectral measures of a special type. Our aim in this paper is to propose a new parametric models able to describe both the spectral and spatial caracteristics of these components.

The paper is organized as follows: in Section **2** we present some definitions and notations for the 3-D random field. In Section **3** we briefly recall the 3-D Wold-like decomposition theory. In Section **4** we present the 3-D texture model and illustrative synthetic textures examples of each Wold component. Finally, in Section **5** we provide our conclusion.

2. NOTATIONS AND DEFINITIONS

Consider a zero mean real valued 3-D stationary stochastic process $y=\left\{y(m,n,t);\ (m,n,t)\in \mathrm{Z}^3\right\}$ and let H be the Hilbert space spanned by the family $\{y(m,n,t)\}$ and denoted by

$$H=\overline{sp}\{\ y(m,n,t),(m,n,t)\in Z^3\ \}.$$

For a fixed total order $\le$ on Z^3, define the space $H_{(m,n,t)}$ called the past of the element $y(m,n,t)$, i.e. the subspace of H given by

$$H_{(m,n,t)}=\overline{sp}\{y(j,k,l)/(j,k,l)<(m,n,t)\}\subset H\ .$$

We call $u(m,n,t)$ the innovation of the process y at the point (m,n,t), i.e. the difference between $y(m,n,t)$ and its orthogonal projection $\hat{y}(m,n,t)$ on its past $H_{(m,n,t)}$:

$$u(m,n,t)=\hat{y}(m,n,t)-y(m,n,t).$$

To define the concepts of CND and evanescent process, consider the following subspaces:

- H_{cnd}, generated by the innovations field, i.e.

$$H_{cnd}=\overline{sp}\{\ u(m,n,t),(m,n,t)\in Z^3\ \}$$

- $H_{\det}$, the orthogonal complementary of H_{cnd}, i.e. $H_{\det}=H\Theta H_{cnd}$

- $H_{-\infty}$, the subspace spanned by the intersection of all the Hilbert space $H_{(m,n,t)}$, and called the remote past space,

$$H_{-\infty} = \bigcap_{(m,n,t)\in Z^3} H_{(m,n,t)} .$$

2.1 Definitions

A 3-D homogeneous stochastic process y is respectively called:

- *Non–deterministic* if $E[u(m,n,t)^2]>0$ for at least one (m,n,t) i.e. if the process of its innovations doesn't vanish, and deterministic otherwise. The symbol $E[\ ]$ denotes the expected value.
- *Completely non-deterministic* if $H = H_{cnd}$ i.e. the space of process coincides with the space generated by its innovations field.
- *Deterministic of $H_{-\infty}$ type* (or "*remote past*" process) if $H_{cnd} = \{0\}$ and $H = H_{-\infty}$ i.e. its space coincide with its remote past space.
- *Evanescent field* if $H_{cnd} = \{0\}$ and $H_{-\infty} = \{0\}$ i.e. it has neither innovations nor remote past.

Let us recall that a 3-D homogeneous process is CND if and only if its spectral measure is absolutely continuous and its density is log-integrable [1].

3. THE 3-D WOLD-LIKE DECOMPOSITION

In [11] the authors present and analyse the structure of n-dimensional Wold decompositions theory for n-D homogeneous discrete process with respect to a fixed total order on Z^n. In the particular case of three dimensional process (n = 3) they state the following theorem:

Theorem: Let $y = \{y(m,n,t)\}$ be a 3-D homogeneous stochastic process. There is a unique orthogonal decomposition

$$y = w + h + e ,$$

such that w is a CND, h is remote past type deterministic and e is evanescent processes.

Moreover it is shown in [11] that the evanescent component itself can be decomposed into two orthogonal components called the type 1 and type 2 evanescent ones:

$$e = e_1 + e_2 .$$

Consequently, any 3-D homogeneous random field can be uniquely represented as follows:

$$y = w + h + e_1 + e_2 . \qquad (1)$$

In addition, if F_y, F_w, F_h, F_{e_1} and F_{e_2} are the spectral measures of y, w, h, e_1, and e_2 respectively, then F_y can be uniquely represented as a sum of mutually singular measures:

$$F_y = F_w + F_h + F_{e_1} + F_{e_2} \qquad (2)$$

where F_w is absolutely continuous , F_h, F_{e_1} , and F_{e_2} are singular with respect to the Lebesgue measure. Thus, the spectral measure of each deterministic component is concentrated on a set of Lebesgue zero in the frequency space.
From (2) we conclude that the decomposition of the completely nondeterministic and the deterministic components of a 3-D regular homogeneous random field can be achieved by performing a spectral Lebesgue decomposition, i.e. by separating the singular and the absolutely continuous components of the spectral measure of the random field.
The decomposition in (1) can be in fact produced for any total order on Z^3. In this paper, we are particularly interested by the spatial structure and spectral support of evanescent fields relatively to one fixed total order.

4. THE 3-D TEXTURE MODEL

The 3-D texture field is assumed to be a finite realization of a 3-D homogeneous random field with a mixed spectral distribution. Thus, in the context of texture modelling, the orthogonal property of the three components in (1) leads to independent models of each one separately.

4.1 Evanescent field of the type 1

The spectral distribution function (SDF) of this evanescent component is a linear combination of separable measures $dF_{e_1}(.,.,.)$ given by

$$dF_{e1}(\omega,\nu,\eta) = f(w,v)d\lambda_2(w,v)dF_1^s(\eta) \qquad (3)$$

where F_1^s, is a one-dimensional singular measure, and $f(w,v)$ is a two-dimensional spectral density function. It is the product of a 2-D spectral density function and a one-dimensional singular spectral mesaure F_1^s. In other words, the spectral distribution function associated whith each evanescent field of the type 1 is absolutely continuous in two dimensions and is singular in the orthogonal one. For practical applications we can exclude singular-continuous spectral distribution functions from the framework of our treatment. Thus, the distribution F_1^s can be approximated by a linear combination of 1-D Dirac "functions". Consequently, the evanescent component of type 1 can be modelled by a linear combination of separable models, given by the product of 2-D completely nondeterministic process in two dimensions and 1-D sinusoidal in the orthogonal dimension as follows:

$$e_1(n,m,t) = \sum_{i=1}^{I_1}\left[\ s_i(n,m)\cos(2\pi v_i t) + t_i(n,m)\sin(2\pi v_i t)\ \right]$$

where $\{s_i(.,.)\}$, $\{t_j(.,.)\}$ are mutually orthogonal 2-D CND processes of identical autocorrelation function. They can be

modelled as a 2-D AR models. v_i is the 1-D frequencies of the i[th] elementary evanescent component of type 1.
In Fig 1 (a), we show the 3-D texture synthesized by this model with one component. Visualy this texture is random looking in two dimensions and structured in the third orthogonal dimension. The spectral measure of the 3-D evanescent field of type 1 is supported by parallel planes in the frequency space as shown in Fig. 1 (b).

4.2 Evanescent field of the type 2

The SDF $dF_{e_2}(.,.,.)$ of an 3-D evanescent field of type 2 is given by

$$dF_{e_2}(\omega,\nu,\eta) = g(w)\, d\lambda_1(w) dF_2^s(\nu,\eta)$$

where F_2^s, is a two-dimensional singular measure and $g(w)$ is a one-dimensional spectral density function. It is absoluetly continuous in one dimension and is singular in two other orthogonal dimensions. The 2-D singular spectral distribution function noted F_2^s can be approximated by a linear combination of 2-D Dirac "functions". Then, the 3-D evanescent field of type 2 can be modelled as a countable sum of randomly 2-D sinusoids in two dimensions, all modulated by 1-D completely nondeterministic process in the orthogonal dimension:

$$e_2(m,n,t) = \sum_{i=1}^{I_2} [\ s_i(m)\cos(2\pi(nv_i + \eta_i t) + t_i(m)\sin(2\pi(v_i n + \eta_i t)\ ]$$

where I_2 is the number of components, (ν_i, η_i) is the i[th] 2-D frequencies, $\{s_i(n)\}$ and $\{t_i(n)\}$ are mutually orthogonal 1-D CND processes of identical autocorrelation function.
A 3-D texture synthesized by this model, where $\{s_i(n)\}$ and $\{t_i(n)\}$ are modelled by 1-D AR, is presented in Fig 2 (a). It is structured in two dimensions and random looking in the third orthogonal dimension. The corresponding spectral measure is carried by parallel lines in the frequency space as in Fig.2 (b).

4.3 Harmonic component

The spectrum support of the remote past type deterministic field h, with respect a fixed total order, has several possible geometrical forms. Then, its spectral distribution function cannot be easily approximated and its parametric modelling is difficult. To alleviate this problem, a generalized Wold decomposition is necessary. This decomposition has been developed recently in [12] with respect to all lexicographic-type orders in Z^3.
However, one of the remote past type deterministic processes is the harmonic field represented by

$$h(n,m,t) = \sum_{p=1}^{P} [C_p Cos\,(n\,w_p + m\,v_p + t\,\mu_p) + D_p\, Sin\,(n\,w_p + m\,v_p + t\,\mu_p)]$$

where the triplet (w_p, v_p, μ_p) are the 3-D frequencies and the coefficients $\{C_p, D_p\}$ are real valued, mutually orthogonal random variables with $E[C_p^2] = E[D_p^2] = \sigma_p^2$.
In Fig 3 (a) we show the 3-D texture synthesized by a 3-D harmonic model with two components. This model generates periodic textures in all directions. The support of the corresponding spectral density function contains four isolated points as shown in Fig 3 (b).

4.4 Completely nondeterministic component

Since the spectral measure F_u is absolutely continuous with respect to the Lebesgue measure, the CND component can be modelled by a 3-D autoregressive model (3-D AR) given by the relationship

$$w(m,n,t) = \sum_{(j,k,l)\in D} a(j,k,l)\, w(m-j, n-k, t-l) + u(m,n,t)$$

where, $\{u(m,n,t)\}$ is the 3-D white innovations field described in section 2, and D is a subset of Z^3 called the support of the AR model. That means that a pixel (m,n,t) is a weighted sum of its neighbours. Depending on the form of this neighbour, two causal regions of support have been defined: the quarter space (QS) and the non symmetrical half space (NSHS). In the case of a QS support, the model is given by

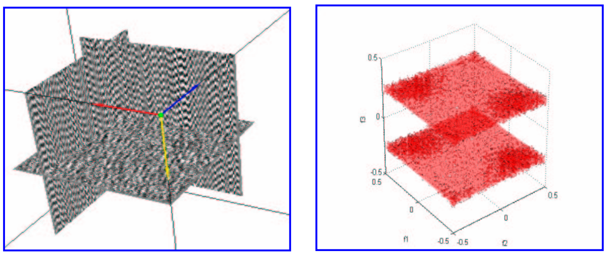
$$w(m,n,t) = -\sum_{k_1=0}^{p_1} \sum_{k_2=0}^{p_2} \sum_{\substack{k_3=0 \\ (k_1,k_2,k_3)\neq(0,0,0)}}^{p_3} a_{k_1,k_2,k_3} w(m-k_1, n-k_2, t-k_3) + u(m,n,t)$$

where, the coefficients $\{a_{k_1,k_2,k_3}\}$ are the transversal model's parameters and the triplet (p_1, p_2, p_3) denotes the model order. This class of models generate the random purely unstructured 3-D textures as in Fig 4.

5. CONCLUSION

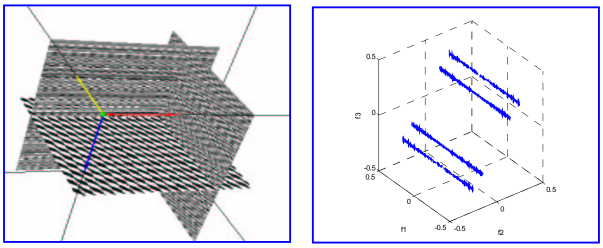
This paper is the sequel to [11, 12] where a 3-D Wold-like decomposition theory is developed. Based on this theory, we have proposed new explicit parametric models of each Wold component. These models will be employed for modelling, analysis, synthesis of a wide variety of 3-D homogeneous textures types found in natural image blocks. Indeed, from the illustrative synthetic textures examples presented here, we notice that the perceptual characteristics of the resulting mutually orthogonal components can be described as "randomness" for the purely non deterministic, "periodicity in all directions" for the harmonic field and "periodicity in particular directions" for the evanescent components. However, a decomposition algorithm for extracting all components from a natural 3-D texture is needed for practical application.

FIGURES



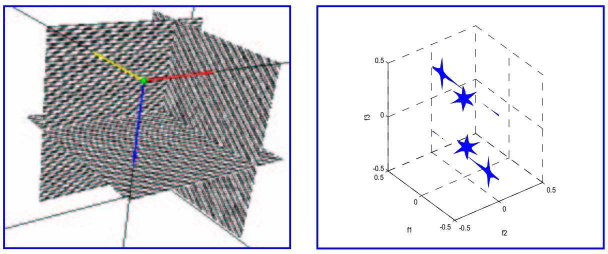
(a) (b)

Figure 1: (a) the (120th, 100th, 90th) slices from a 3-D synthesized textures by evanescent field of type 1 with one component.
(b) : Spectral representation by Fourier magnitude.



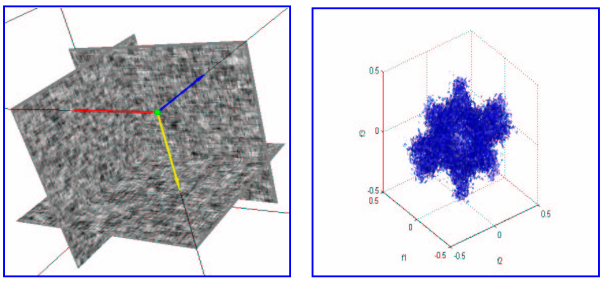
(a) (b)

Figure 1: (a) the (120th, 100th, 90th) slices from a 3-D synthesized textures by evanescent field of type 2 with one component.
(b): Spectral representation by Fourier magnitude.



(a) (b)

Figure 3: (a) the (120th, 100th, 90th) slices from a 3-D synthesized textures by harmonic field with two components.
(b): Spectral representation by Fourier magnitude.



(a) (b)

Figure 1: (a) the (120th, 100th, 90th) slices from a synthesized textures by a Gaussian 3-D QSAR model.
(b): Spectral representation by 3-D Fourier magnitude.

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