VARIABLE LEAKAGE FXLMS ALGORITHMS FOR NONLINEAR ACTIVE CONTROL APPLICATIONS

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ABSTRACT

When linear adaptive algorithms are used in active control applications, they do not consider the system nonlinearities in the update equations. On the other hand, such an application class is inherently nonlinear due to the nature of the system components. So, in such an environment a nonlinear adaptive algorithm, including information about the system nonlinearities, would be the most indicated for performance. However, this requires the knowledge of the model of the nonlinearities, which is mostly unavailable. In this paper, we propose the use of the linear FXLMS algorithm with variable leakage as a way for overcoming the need of a prior knowledge of the system nonlinearities. Thus, we show that: (i) the performance of the linear leaky FXLMS algorithm versions with variable leakage can approximate to that of the nonlinear ones; (ii) the knowledge about the system nonlinearities is no longer needed. Therefore, the proposed algorithm represents an interesting alternative for practical implementation in the presence of saturation-type nonlinearities. We confirm the conjectures made through numerical examples.

1. INTRODUCTION

In the last few years, the area of active control of sound and vibration has experimented a considerable advance in both research and applications. It has been benefited by the continuous improvement of digital signal processors, permitting more sophisticated implementation with efficient control algorithms. In this area the linear filtered-X LMS (FXLMS) [1] is a widely used algorithm. Such popularity is mainly due to its simple implementation and robustness. An issue that is frequently disregarded when an active control system is studied is its inherent nonlinear nature, mainly due to its components (power amplifiers, sensors, actuators, etc.). To illustrate this, an active noise control application scheme is shown in Fig. 1. Fig. 1(b) presents the block diagram of such an application, where the system nonlinearities are concentrated in the block denoted by $f[\cdot]$ (nonlinear function). In this study, we are considering saturation-type nonlinearities. In general, the effects of the nonlinearities are more detectable when low-cost components are used, which usually present a more restrictive linear region. In this case, by implementing the nonlinear FXLMS algorithm (NL-FXLMS), i.e., taking into account the nonlinear function to determine the gradient, we obtain the best performance in terms of MSE [2]. In this way, the gradient expression has now an additional term, which results from the derivative of the nonlinear function [3]. Then in order to implement the NL-FXLMS algorithm, the knowledge or estimate of such a function is required. This represents a limiting factor for practical use of this algorithm. Recently, it has been shown that using a suitable leakage factor the performance of the conventional FXLMS algorithm with leakage (CL-FXLMS) can approximate to that of the NL-LMS [3], [4]. We use here the denomination conventional to emphasize the fact that the nonlinearity effect is disregarded when the gradient of the instantaneous cost function is computed. In doing so, all the expressions that govern the adaptive algorithm are obtained by not allowing for the system nonlinearities.

The introduction of leakage in the LMS algorithm in linear applications is a well-known solution against finite-precision effects of the processors or to insufficient spectral excitation of the input signal, among other factors [5]. Since the effect of leakage (under a persistent input signal) is to restrict the adaptive filter weights, one can expect that the magnitude of the adaptive filter output is in some sense controlled. Now, turning our attention to a nonlinear system, we can take advantage of that effect to keep the operating point of a given component away from its nonlinear region. In this way, undesired behavior (typically, degradation of the system performance or even divergence of the adaptive algorithm, due to the overdrive of some system component) is avoided [1], [6]. Fig. 2 depicts the behavior of the steady-state MSE of the CL-FXLMS algorithms, in presence of system nonlinearities as in Fig. 1(b), for several leakage values. From this figure, we can notice that there exists an optimum leakage factor for which the performance of the algorithms with leakage is similar to that of the NL-FXLMS. Such an optimum leakage can be experimentally determined [3] or analytically obtained for a specific working condition that, in addition, requires the knowledge of the nonlinear function [4]. To overcome such a limitation of the CL-FXLMS algorithms, we propose the use of a self-adapted leakage factor. In doing so, two problems are solved. We can cope with time-varying conditions, i.e., the optimal leakage is self-adjusted; this is not the case in [4]. Secondly, the knowledge about the system nonlinearities is no longer required due to controlling effect of leaking the algorithm. In this paper, we derive recursive expressions to perform the leakage factor adjust. In this way, two new algorithms are proposed, namely, the leaky FXLMS and MOV-FXLMS algorithms with variable leakage, denoted by VL-FXLMS and VLMOV-FXLMS, respectively. Simulation results are presented to assess the proposed algorithms.

2. WEIGHT UPDATE EQUATIONS

In this section, we derive the equations for the VL-FXLMS and VLMOV-FXLMS algorithms as well as for the NL-FXLMS algorithm, which is used for comparison purposes. To this end, we consider a typical set-up of an active noise control application, as depicted in Fig. 1(b). In this figure $d(n)$ and $e(n)$ represent the primary and error signals, respectively; $\mathbf{x}(n)$ is the reference signal, and $z(n)$ is a zero-mean Gaussian measurement noise with variance σ_z^2 and uncorrelated with any other signal in the system. In the adaptation path, we have in series with the adaptive filter a linear block represented by an FIR filter given by $\mathbf{s} = [s_0\, s_1 \cdots s_{M-1}]$ followed by a nonlinear memoryless block denoted by $f[\cdot]$, which represents the saturation-type nonlinearities (concentrated) of the system. Both blocks in tandem represent the power amplifier. The loudspeaker is another typical nonlinear element in this kind of application. It has a type of nonlinearity with memory, which complicates the mathematical treatment [7]. Thus, for simplicity, we assume here that the loudspeaker is a linear component.

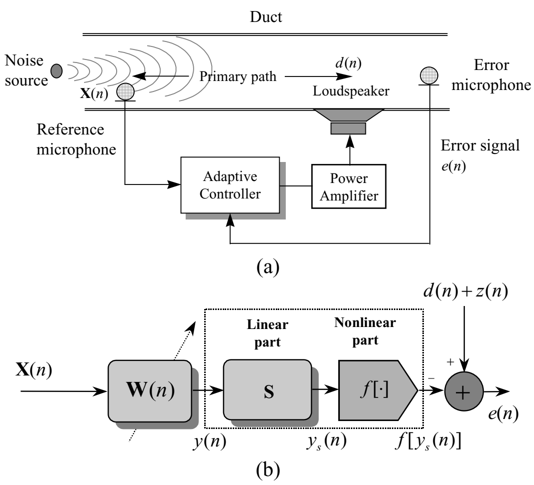

Fig. 1. Basic active noise control set-up in a duct. (a) Physical set-up, (b) block diagram.

2.1. Weight-update equation for the NL-FXLMS algorithm

To obtain the weight update equation, we use the stochastic gradient algorithm, given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu}{2}\nabla J_{\mathbf{w}}(n)\,, \tag{1}$$

where $J_{\mathbf{w}}(n) = e^2(n)$ is the instantaneous cost function in which the error signal is obtained from Fig. 1(b), resulting in the following expression

$$e(n) = d(n) - f[y_{\mathrm{s}}(n)] + z(n)\,. \tag{2}$$

Note that the error signal is a nonlinear function of the adaptive weight vector. The output of the linear block, $y_{\mathrm{s}}(n)$, is given by

$$y_{\mathrm{s}}(n) = \sum_{i=0}^{M-1} s_i y(n-i) = \sum_{i=0}^{M-1} s_i \mathbf{x}^{\mathrm{T}}(n-i)\mathbf{w}(n-i)\,. \tag{3}$$

Hence, by determining the gradient of the cost function, after squaring (2), we obtain

$$\nabla J_{\mathbf{w}}(n) = \frac{\partial e^2(n)}{\partial \mathbf{w}(n)} = -2e(n)f'[y_{\mathrm{s}}(n)]\frac{\partial y_{\mathrm{s}}(n)}{\partial \mathbf{w}(n)}\,, \tag{4}$$

where the term $f'[y_{\mathrm{s}}(n)]$ denotes the differentiation of $f[y_{\mathrm{s}}(n)]$ with respect to $y_s(n)$. Now, by differentiating (3) with respect to $\mathbf{w}(n)$, substituting it into (4), and the resulting expression into (1), the weight update expression reads

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n) f'[y_{\mathrm{s}}(n)]\mathbf{x}_{\mathrm{f}}(n)\,, \tag{5}$$

where

$$\mathbf{x}_{\mathrm{f}}(n) = \sum_{i=0}^{M-1} s_i \mathbf{x}(n-i)\,. \tag{6}$$

Note from (5) that an additional term has appeared, as compared with the typical linear counterpart expression. Such a term is a consequence of the nonlinear function $f[\cdot]$ [3].

2.2. VL-FXLMS algorithm

As stated before, the linear leaky FXLMS (LFXLMS) is frequently used in practical implementations. There are two typical forms of leaking an adaptive algorithm [8]. In the first, a penalty term proportional to the quadratic norm of the filter weight vector $\mathbf{w}(n)$ is added to the classical cost function $e^2(n)$. Thus,

$$J_{\mathrm{L1}}(n) = e^2(n) + \gamma \mathbf{w}^T(n)\mathbf{w}(n)\,, \tag{7}$$

where $\gamma \geq 0$ is a weighting factor denoted as the leakage factor. To obtain the weight update expression we disregard the system nonlinearities in the derivation of the updating equation. In other words, we use the conventional weight update equation with leakage determined for a linear system, however, applied now to a nonlinear environment. In doing so, the updating expression is obtained by [3]

$$\mathbf{w}(n+1) = \nu(n)\mathbf{w}(n) + \mu e(n)\mathbf{x}_{\mathrm{f}}(n)\,, \tag{8}$$

where

$$\nu(n) = 1 - \mu\gamma(n)\,. \tag{9}$$

To implement an adjusting expression for the leakage factor, the stochastic gradient rule is used. In this way, the leakage is adjusted according to the negative of the gradient of the squared estimation error with respect to the leakage factor. Thus,

$$\gamma(n) = \gamma(n-1) - \frac{\rho}{2}\frac{\partial e^2(n)}{\partial \gamma(n-1)}\,, \tag{10}$$

where ρ is a positive constant. The second term of (10) is computed as follows

$$\frac{\partial e^2(n)}{\partial \gamma(n-1)} = \left[\frac{\partial e^2(n)}{\partial \mathbf{w}(n)}\right]^{\mathrm{T}} \frac{\partial \mathbf{w}(n)}{\partial \gamma(n-1)}\,, \tag{11}$$

where

$$\frac{\partial e^2(n)}{\partial \mathbf{w}(n)} = -2e(n)\mathbf{x}_{\mathrm{f}}(n)\,. \tag{12}$$

Note that in (12) the nonlinear function is disregarded when the derivative is computed; and secondly, from (8),

$$\begin{aligned}\frac{\partial \mathbf{w}(n)}{\partial \gamma(n-1)} &= \frac{\partial\{[1-\mu\gamma(n-1)]\mathbf{w}(n-1) + \mu e(n-1)\mathbf{x}_{\mathrm{f}}(n-1)\}}{\partial \gamma(n-1)} \\ &= -\mu\mathbf{w}(n-1).\end{aligned} \tag{13}$$

From (11), and by substituting (12) and (13) into (10), the update equation becomes

$$\gamma(n) = \gamma(n-1) - \rho\mu e(n)\mathbf{w}^{\mathrm{T}}(n-1)\mathbf{x}_{\mathbf{f}}(n) \tag{14}$$

2.3. VLMOV-FXLMS algorithm

The second form of algorithm leaking is to penalize the cost function $e^2(n)$ by using the instantaneous power of a specific signal in the adaptive path [8]. In this way, from Fig. 1(b), we can choose to use one of the signals $y(n)$, $y_{\mathrm{s}}(n)$ or $f[y_{\mathrm{s}}(n)]$. However, among those, the signal $y(n)$ is preferred, since it is an available signal in the system, being the output of the adaptive filter. The other signals, $y_{\mathrm{s}}(n)$ and $f[y_{\mathrm{s}}(n)]$, are virtual, inasmuch as in a real control system they are not readily available. Thus, by using $y(n)$, the instantaneous cost function is now given by

$$J_{\mathrm{L2}}(n) = e^2(n) + \gamma\, y^2(n) = e^2(n) + \gamma \mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n)\mathbf{w}(n)\,. \tag{15}$$

By determining the gradient of $J_{\mathrm{L2}}(n)$, and again disregarding the nonlinearity in the derivation of the updating equation, we obtain

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\, e(n)\mathbf{x}_{\mathrm{f}}(n) - \mu\gamma(n)y(n)\mathbf{x}(n)\,. \tag{16}$$

For the derivation of the adaptive expression of $\gamma(n)$, (10) is again used. Here, the main difference is in the derivative of the weight vector with respect to the leakage factor. In this case, from (16), such a derivative is given by

$$\frac{\partial \mathbf{w}(n)}{\partial \gamma(n-1)} = -\mu y(n-1)\,\mathbf{x}(n-1). \tag{17}$$

By using (12) and (17), the recursive expression to update the leakage factor is obtained by

$$\gamma(n) = \gamma(n-1) - \rho\mu e(n) y(n-1)\mathbf{x}^{\mathrm{T}}(n-1)\mathbf{x}_{\mathrm{f}}(n) \tag{18}$$

In the simulation results section, we compare the performance of the adaptive controller by implementing (5), (8), and (16).

2.4 Nonlinearities Model

The nonlinearities considered in this paper are saturation-type ones, which represent an extensive class of nonlinear systems [9]. Such a characteristic can be modeled by the scaled error function [10], expressed as

$$f(r) = \int_0^r \mathrm{e}^{-\frac{t^2}{2\sigma^2}} dt\,, \tag{19}$$

with derivative given by $f'(r) = \mathrm{e}^{-\frac{r}{2\sigma^2}}$. The parameter σ is a constant factor that controls the extent of the linear region of $f(r)$. The scaled error function is an attractive model for the system nonlinearities because it makes further mathematics easy [10].

3. SIMULATION RESULTS

In this section, the performance of the NL-FXLMS, VL-FXLMS, and VLMOV-FXLMS algorithms, represented by (5), (8), and (16), respectively, are compared. For this purpose, the system depicted in Fig. 1(b) has been considered. For the examples, the primary signal $d(n)$ is obtained by passing white and colored signals $x(n)$, with variance $\sigma_x^2 = 1$, through an FIR filter representing the primary path given by $[\,0.0179, 0.1005, 0.279, 0.489, 0.586, 0.489, 0.279, 0.1005, 0.0179\,]^{\mathrm{T}}$. To study the algorithm behavior under a non-stationary environment (when convergence is achieved), we abruptly change the plant to $[\,0.2179, 0.3005, 0.4795, 0.6896, 0.586, 0.2896, 0.0795, \text{-}0.0995, \text{-}0.1821\,]^{\mathrm{T}}$. The linear filter $\mathbf{S}$ is $[\,0.7756, 0.5171, \text{-}0.3620\,]^{\mathrm{T}}$. The variance of the measurement noise $z(n)$ is equal to 0.0001. The comparison is carried out by using two values for the parameter σ^2, 0.6 and 1.2, in (19). In (10), $\rho = 1$ is used.

Example 1: For this example, a white input signal is used. The values for the step-size and σ^2 are: 0.005 and 1.2, respectively. Fig. 3 shows the MSE curves obtained from Monte Carlo simulation (average of 300 independent runs) for the NL-FXLMS algorithm and the proposed VL-FXLMS and VLMOV-FXLMS algorithms. The plant is changed at iteration 8000. From this figure, we can verify a similar behavior for the three algorithms. Note that the MSE curve of the VLMOV-FXLMS algorithm is closer to the NL-FXLMS than the corresponding to the VL-FXLMS one.

Fig. 4 illustrates the evolution of the leakage factor for both the VL-FXLMS and VLMOV-FXLMS algorithms. It can be noticed that when the adaptive algorithms reach the convergence, the leakage factor is also in steady-state condition.

Example 2: For this example, we use here the same step-size value as in the previous example, but now the nonlinear function is obtained by using $\sigma^2 = 0.6$, and a colored input signal is used. By using $\sigma^2 = 0.6$, a nonlinear block having a more restricted linear region is obtained. The colored signal is determined from an AR(2) process having an eigenvalue spread of 188.42. Figs. 5 and 6 are obtained by a similar way as Figs. 3 and 4, respectively. From Figs. 5 and 6, we can again verify that the proposed algorithms with variable leakage perform very well as compared with the NL-FXLMS one.

4. CONCLUSIONS

In this paper two variable leakage algorithms (VL-FXLMS and VLMOV-FXLMS) to be used in nonlinear active control environment are proposed. These algorithms are compared with the NL-FXLMS one, which uses the true gradient. From this study, the following conclusions are drawn: the NL-FXLMS algorithm always gives a better performance. However, this is achieved if the model of the nonlinearities is known, which represents a major drawback for a practical implementation. On the other hand, by using the proposed approaches, we can attain a similar performance as with the NL-FXLMS algorithm without the need to estimate the system nonlinearities.

5. REFERENCES

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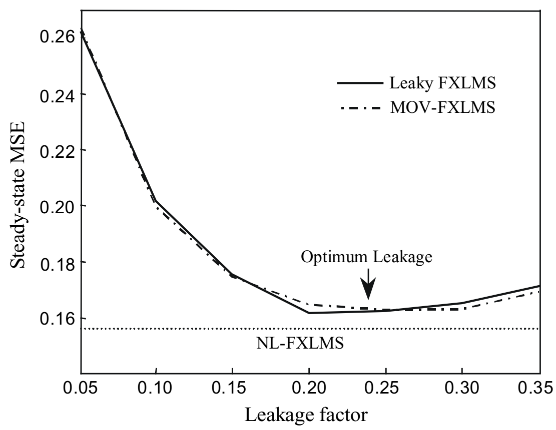

Fig. 2. Steady-state MSE *versus* leakage. Simulation results for $\sigma^2 = 0.6$.

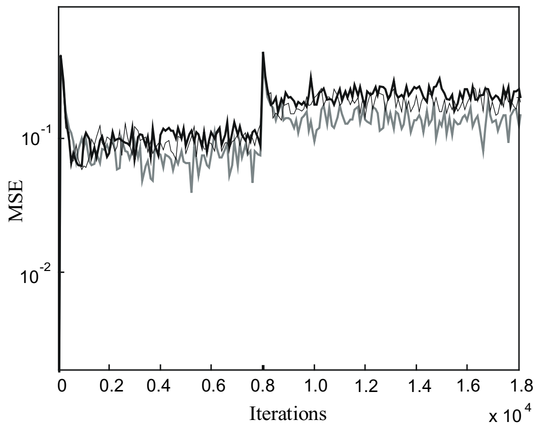

Fig. 3. Simulations comparing the MSE of the NL-FXLMS (gray line), VL-FXLMS (thick-dark line), and VLMOV-FXLMS (thin-dark line) algorithms (average of 300 independent runs).

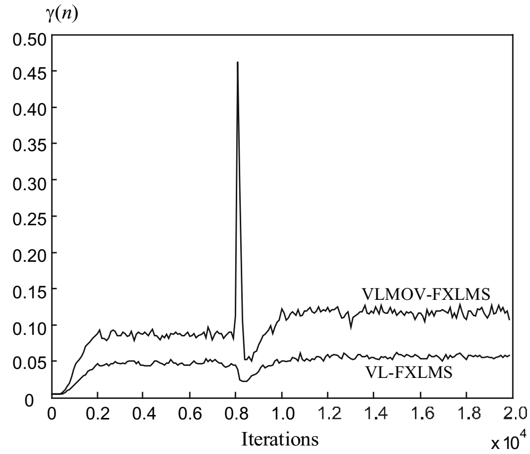

Fig. 4. Evolution of the time-varying leakage factor.

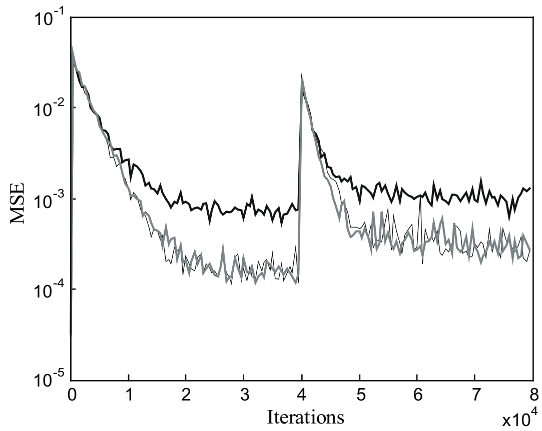

Fig. 5. Simulations comparing the MSE of the NL-FXLMS (gray line), VL-FXLMS (thick-dark line), and VLMOV-FXLMS (thin-dark line) algorithms (average of 300 independent runs).

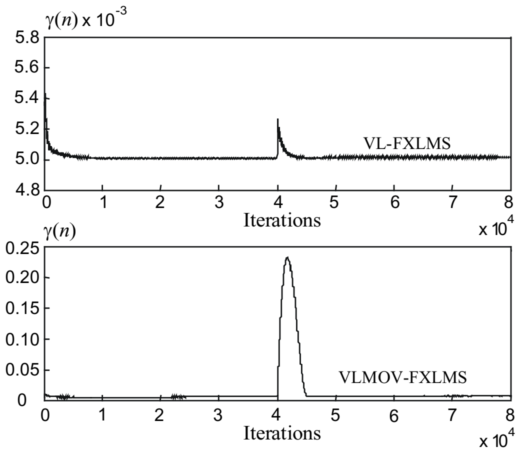

Fig. 6. Evolution of the time-varying leakage factor.