BLIND EQUALISATION AND CARRIER OFFSET COMPENSATION FOR BLUETOOTH SIGNALS

Charles Tibenderana and Stephan Weiss

Communications Research Group, School of Electronics & Computer Science,
University of Southampton, UK
{ct02r,s.weiss}@ecs.soton.ac.uk

ABSTRACT

Bluetooth systems operating in medium to large sized rooms can suffer multipath distortion, which may be compounded by presence of frequency offsets permitted by the Bluetooth standard. Frequency errors can undermine common training based equalisation techniques. This paper establishes the effect of severe multipath indoor propagation and frequency errors on Bluetooth reception, and demonstrates the effectiveness of the constant modulus algorithm in performing channel equalisation when a frequency offset exists. A novel stochastic gradient based algorithm for frequency correction is also introduced and assessed.

1. INTRODUCTION

Gaussian Frequency Shift Keying (GFSK) is the modulation scheme selected for the Bluetooth physical layer. Bluetooth is a low cost wireless standard focused on wire replacement, Local Area Network (LAN) access points, and Personal Area Networks. Frequency Hopping is used for multiple access, with each channel occupying 1 MHz bandwidth, while its specified range is 10m. Despite low cost of Bluetooth transceivers, with the advent of Software Defined Radios (SDR), a common hardware platform implementing relatively complex standards such as IEEE 802.11b wireless LAN will have extra capacity when running Bluetooth. We are therefore researching about the possibility of using the extra computational capacity to realise a high performing Bluetooth receiver, which is robust to adverse conditions typical of Bluetooth.

We have already considered adopting a high-performance Continuous Phase Frequency Shift Keying (CPFSK) receiver [1], which enables near optimal detection in AWGN, but requires a prohibitively large filter bank. By reducing the computational requirements of the standard high-performance receiver by almost 90% we have taken steps towards making it a practical option [2]. But this method is vulnerable to signal adversities such as multipath distortion and carrier frequency errors, both of which are common in Bluetooth systems, and it is these problems that we address in this paper.

Many Bluetooth uses are indoor applications, and hence, the signal is reflected and scattered by walls and objects within the room or building en route to the receiver, resulting in time-shifted versions of the same signal forming a distorted composite signal that is "seen" by the receiver. Although damage caused by this distortion may be minimal for small rooms, substantial degradation occurs in large enclosed areas where the delayed components take longer to arrive and may not be sufficiently attenuated. This problem will become prevalent if pleas to increase the operation range are heeded.

Strategies have been suggested to tackle dispersive channels in Bluetooth using decision feedback equalisers [3, 4], but they will be undermined by frequency errors. Other more common equalisation techniques that rely on a training sequence to minimise the Minimum Mean Square Error (MMSE) may not be capable of tracking fast changes caused by frequency errors [5], and are not recommended for point-to-multipoint networks such as bluetooth because of the requirement for the control unit to interrupt transmission to retrain a tributary receiver that may have experienced a change in channel conditions, or that was not online during the initial training procedure [6]. In this paper we demonstrate the effectiveness of the Constant Modulus Algorithm (CMA) as an equalisation technique for Bluetooth in presence of carrier frequency offsets.

The necessity for cheap transceivers motivates the Bluetooth Special Interest Group to allow up to 75 KHz initial frequency errors [7], Research has shown that performance deteriorates significantly even when operating within this range [8], more so, in the high-performance CPFSK receiver where frequency errors propagate through an observation interval of K bit periods, thereby trading off robustness to Gaussian noise with immunity to a carrier offset.

Work done to address the problem of frequency errors in Continuous Phase Modulated signals, of which GFSK forms a subset, can be categorised as being training based, or blind methods. Notable research on blind algorithms is reported in [9], where the result $z[n] = r[n] \cdot r^*[n-M]$ is fed to an error estimating function during the adaptation process. Propositions in [9] rely heavily on the ability of the receiver to determine M which would represent a maximum phase shift of $\frac{\pi}{2}$ in the transmit signal. This is not easily attained in bluetooth where the modulation index $h \in (0.28, 0.35)$ [7].

Therefore, in this paper we present a novel algorithm based on gradient descent techniques, which converges under conditions specified in [9], without the necessity for the receiver to know the precise transmitter modulation index. The algorithm is derived analytically and assessed via simulation.

The structure of the paper is as follows: After the introduction in Sec. 1, a signal and system model is developed in Sec. 2. The CMA is discussed in Sec. 3, while the stochastic gradient based algorithm for carrier offset correction is derived in Sec. 4. Our simulations are described in Sec. 5, before concluding in Sec. 6.

2. SIGNAL AND SYSTEM MODEL

The transmitted signal $s[n]$, is GFSK modulated with a modulation index (h) of 0.35, while the bandwidth-time product

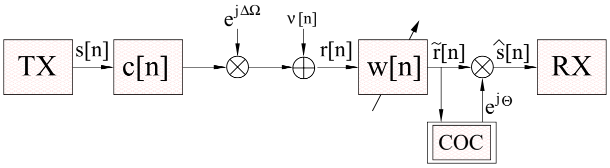

Figure 1: System model.

(K_{BT}) of the Gaussian pre-modulation filter was 0.5. Our signal flow graph is depicted in Fig. 1.

GFSK generally modulates a multilevel symbol $p[k]$, which here is assumed to be binary, $p[k] \in \{\pm 1\}$. This bit sequence is expanded by a factor of N and passed through a Gaussian filter with impulse response $g[n]$ of length $L_g N$, thus having a support of L_g bit periods (K_{BT}=0.5 results in $L_g = 3$), yielding a continuous instantaneous angular frequency signal

$$\hat{\omega}[n] = 2\pi h \sum_{k=-\infty}^{\infty} p[k] g[n-kN] \quad ,$$

where k and n stand for the symbol and chip indices respectively. The phase of the baseband version of the transmitted signal,

$$s[n] = \exp\{j \sum_{\nu=-\infty}^{n} \hat{\omega}[\nu]\} = \prod_{\nu=-\infty}^{n} e^{j\hat{\omega}[\nu]} \quad ,$$

is determined as the cumulative sum over all previous frequency values $\hat{\omega}[n]$.

Assuming the signal is corrupted by a dispersive stationary channel impulse response (CIR) $c[n]$, a carrier frequency offset that causes an excess phase shift of $\Delta\Omega$ between samples, and additive white Gaussian noise (AWGN) $v[n]$, the received signal can be expressed as

$$r[n] = \sum_{\lambda=0}^{L_c-1} c[\lambda]\, s[n-\lambda]\, e^{j\Delta\Omega n} + v[n] \quad , \tag{1}$$

L_c being the length of the CIR. If a vector of equaliser coefficients, defined as

$$\mathbf{w}[n]^H = [\, w_0[n], w_1[n], \cdots, w_{L_w-1}[n]\,] \quad ,$$

converges towards the inverse of the CIR, then deconvolution of the CIR takes place via

$$\tilde{r}[n] = \sum_{\lambda=0}^{L_w-1} \mathbf{w}_\lambda[n] r[n-\lambda] \quad , \tag{2}$$

ideally resulting in a version of the received signal which is corrupted only by a frequency offset and AWGN. Assuming accurate knowledge of the frequency offset, attained by adaptive processing of $\tilde{r}[n]$, we would be able to compensate for frequency errors by multiplying $\tilde{r}[n]$ by a derotating phasor $e^{j\Theta n}$ and extract $\hat{s}[n]$, the sum of transmitted signal and another Gaussian process.

Our receiver was a reduced-complexity version [2] of the high-performance CPFSK receiver [1], which provides near optimal detection in AWGN by selecting the matched filter output with the largest magnitude according to

$$\hat{p}[k] = \arg\max_i \left| \sum_{n=0}^{KN-1} \hat{s}[kN-n] \cdot s^*_{i,j}[-n] \right| \quad , \tag{3}$$

where $s^*_{i,j}[-n]$ are 2^K, K-bit long matched filter responses.

3. CHANNEL EQUALISATION

We wish to equalise adverse channel effects in presence of frequency errors. This would rule out phase dependent equalisation algorithms that would misinterpret the resulting phase changes as a rapidly varying channel, and hence, would never converge. The Constant Modulus Algorithm, is suitable because it is insensitive to signal phase [10, 11], the logic being that a dispersive channel cannot produce any phase errors that are not "seen" as amplitude deviations from the ideal symbol constellation. A price paid for neglecting phase information is a slower convergence to the ideal coefficients. The nonconvex CMA cost function is

$$J[n] = \mathcal{E}\{|\tilde{r}[n]|^2 - 1\}^2 \quad ,$$

whereby $\mathcal{E}\{\cdot\}$ is the expectation operator, $\tilde{r}[n]$ is the equaliser output, and the expected magnitude of the received signal samples is 1. Equaliser coefficients can be adjusted via a stochastic gradient search [12] according to

$$\mathbf{w}[n+1] = \mathbf{w}[n] - \mu_w \nabla[n] \quad ,$$

where μ_w is a step size and $\nabla[n]$ is the instantaneous estimate of the gradient of $J[n]$, given by

$$\nabla[n] = \mathbf{r}^*[n]\, \tilde{r}\,[n]\, (\, |\tilde{r}\,[n]|^2 - 1\,) \quad ,$$

in which $\mathbf{r}[n]$ is a vector of received signal samples

$$\mathbf{r}[n]^T = [\, r_0[n], r_1[n], \cdots, r_{L_w-1}[n]\,] \quad .$$

4. CARRIER FREQUENCY OFFSET CORRECTION

An estimation of the carrier offset can be based on the received signal in (2) by denoting

$$\begin{aligned} \mathcal{E}\{\tilde{r}[n]\tilde{r}^*[n-M]\} &= \mathcal{E}\{s[n]s^*[n-M]\}\, e^{j\Delta\Omega M} + \\ &\quad \mathcal{E}\{s[n]v^*[n-M]\}\, e^{j\Delta\Omega n} + \\ &\quad \mathcal{E}\{v[n]s^*[n-M]\}\, e^{-j\Delta\Omega(n-M)} + \\ &\quad \mathcal{E}\{v[n]v^*[n-M]\} \qquad (4) \\ &= e^{j\Delta\Omega M} \quad . \qquad (5) \end{aligned}$$

Due to the independence and zero mean of $s[n]$ and $v[n]$, the second and third term in (4) will be zero. By selecting M sufficiently large, the autocorrelation term of the noise in (4) vanishes. Since the instantaneous frequency accumulated over M samples of the transmitted signal $s[n]$ will either rotate in a positive or negative direction but on average be zero, we have $\mathcal{E}\{s[n]\, s^*[n-M]\} = 1$. Hence the simplification in (5). Note that the detection of the carrier frequency offset is independent of any other receiver functions.

4.1 Cost Function

We create a modified receiver input

$$\hat{s}[n] = \tilde{r}[n]\, e^{j\Theta n} \quad , \tag{6}$$

i.e. modulating by Θ, to match the carrier offset $\Delta\Omega$. In order to determine Θ, we can use the following constant modulus (CM) cost function,

$$\chi[n] = |\mathcal{E}\{\hat{s}[n]\hat{s}^*[n-M]\} - 1|^2 \quad . \tag{7}$$

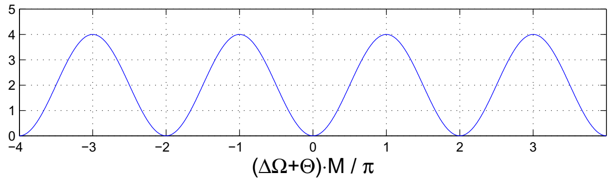

Figure 2: Cost function χ .

Inserting (6) and (5) into (7) yields

$$\chi[n] = (e^{j(\Delta\Omega+\Theta)M} - 1)(e^{j(\Delta\Omega+\Theta)M} - 1)^* = 2 - 2\cos((\Theta + \Delta\Omega)M) \quad (8)$$

$$\text{with } \chi[n] = 0 \longleftrightarrow \Theta = \frac{2\pi k}{M} - \Delta\Omega \quad (9)$$

Fig. 2 confirms the assertion in (9), however we are interested in the solution for $k = 0$ only, for which the cost function provides a unique minimum under the condition,

$$-\pi < (\Theta + \Delta\Omega)M < \pi \quad , \quad (10)$$

similar to [9]. Hence, a trade-off exists for the selection of M between decorrelating the noise in the receiver and not exceeding the bounds in (10).

4.2 Stochastic Gradient Method

Within the bounds of (10), Θ can be iteratively adapted over time based on gradient descend techniques [12] according to

$$\Theta[n+1] = \Theta[n] - \mu_\Theta \frac{\partial \chi}{\partial \Theta} \quad (11)$$

with a suitable step size parameter μ_Θ. A stochastic gradient can be dependent on an instantaneous cost $\hat{\chi}[n]$ by omitting expectations in (7) and assuming small changes in Θ:

$$\frac{\partial \chi}{\partial \Theta} = \frac{\partial}{\partial \Theta}(\hat{s}[n]\hat{s}^*[n-M] - 1) \cdot (\hat{s}[n]\hat{s}^*[n-M] - 1)^* = -2M\Im\{\hat{s}[n]\hat{s}^*[n-M]\left(\hat{s}[n]\hat{s}^*[n-M] - 1\right)^*\} \quad (12)$$

5. SIMULATION AND RESULTS

Effectiveness of CMA for equalisation in Bluetooth, and carrier frequency correction using the algorithm derived in Sec. 4, will be evaluated in the following.

5.1 Simulation Model

Fig. 1 shows a flow graph of our simulation model. The transmitter produces a GFSK modulated signal as specified in Sec. 2, with parameters $K_{BT} = 0.5$, $h = 0.35$, and N=2, to simulate Bluetooth. The channel $c[n]$, shown in Fig. 3(a), was derived via discretisation of a Saleh-Valenzuela indoor propagation model [13], and has a Root Mean Square (RMS) value of approximately 300 ns, thereby typifying a medium to large sized office [14, 15] in which Bluetooth transceivers would normally operate. The spectrum of the CIR in Fig. 3(b) shows 6 dB drop approximately every 2 MHz, which will affect a Bluetooth signal. The equaliser used 20 coefficients, it was updated by the CMA of Sec. 3, and its first coefficient was initialised to unity. The carrier offset compensation block (COC) mechanises equations (11) and (12) to correct frequency errors. In order to minimise steady state error, μ_Θ was adjusted once per 1000 iterations by

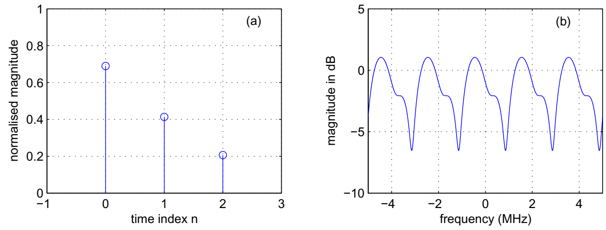

Figure 3: Modulus of the channel impulse response (a), and its spectrum (b).

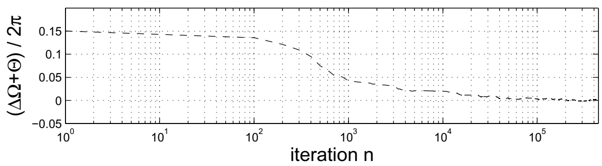

Figure 4: Corrected carrier frequency trajectory under dispersive conditions with equalisation via CMA. Channel RMS≃ 300, $M = N = 2$, $h = 0.35$, $K_{BT} = 0.5$, $\mu_\Theta = 0.0005$, $\mu_{CMA} = 0.002$.

$$\mu_\Theta = \mu_\Theta^{initial} . \bar{\nabla}_{\Theta\chi} \quad ,$$

where $\bar{\nabla}_{\Theta\chi}$ is the mean result of (12), obtained over the last 1000 iterations. Our receiver was a high-performance CPFSK detector [2, 1], with variable observation interval of K bit periods .

5.2 Convergence

In the experiments reported in this Sub-section $\mu_\Theta^{initial} = 0.005$ and μ_w=0.002. The dispersive channel described above was included, but no extra time was allowed for the equaliser to converge . The initial carrier angular frequency error $\Delta\Omega = 0.3\pi$, to represent the maximum allowable initial frequency error between a Bluetooth transceiver pair. Results plotted in Fig. 4, show that the speed of frequency offset correction is somewhat inversely proportional to the magnitude of the error, with relatively quick dash to within 0.1π of the carrier angular frequency and then a more gradual convergence to the ideal.

5.3 Bit Error Performance

BER performance curves portrayed in Fig. 5 were obtained under dispersive conditions, or an initial frequency offset of $\Delta\Omega = 0.3\pi$ similar to Sec. 5.2. It confirms that while increasing the observation interval K improves performance in Gaussian noise, it increases receiver susceptibility to frequency errors, which degrades performance substantially. Dispersion due to the indoor channel model caused more than 3 dB loss at 10^{-3} BER —the maximum BER allowed in Bluetooth— when $K = 9$, and caused much more degradation for $K = 3$. $K = 9$ was affected more by a carrier offset than $K = 3$. Hence, large K is better for AWGN and multipath, while small K is less vulnerable to frequency errors.

Obviously the dispersive channel combined with a angular frequency error of 0.3π would result in an error rate close to 0.5 whatever the value of K. However, the use of CMA with the stochastic gradient frequency correction algorithm brings performance of the system, under these conditions, to within 0.3 and 1.9 dB of the theoretical MMSE solution for $K = 9$, and $K = 3$ respectively (see Fig. 6).

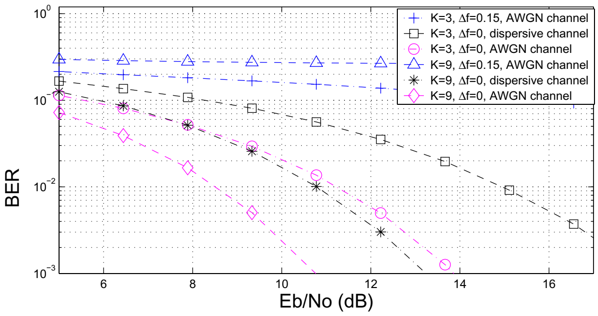

Figure 5: BER of a high-performance matched filter detector used to receive a GFSK signal in presence of carrier frequency offset ($\Delta\hat{f}$), or with a dispersive channel. No equalisation or frequency correction. $N = 2$, $h = 0.35$, $K_{BT} = 0.5$.

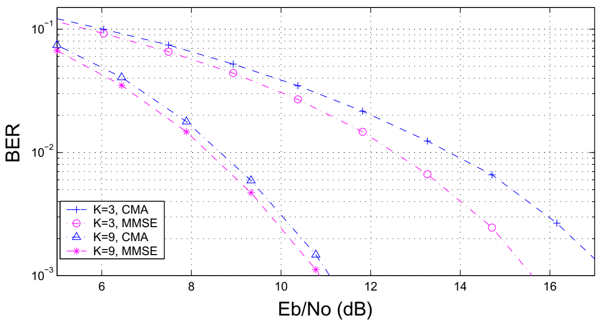

Figure 6: BER of a high-performance matched filter detector used to receive a GFSK signal in presence of carrier frequency offset ($\Delta\hat{f}$), and a dispersive channel. With equalisation and frequency correction. Channel RMS$\simeq$ 300, $N = M = 2$, $h = 0.35$, $K_{BT} = 0.5$, $\mu_{\Theta} = 0.005$.

6. CONCLUSION

Multipath propagation will degrade performance of Bluetooth transceivers in large enclosed areas, and this problem will be compounded by the presence of significant frequency offsets between the transmitter and receiver (allowed in the standard). Frequency errors could render more conventional channel equalisation methods ineffective. The CMA is insensitive to frequency errors, and we have demonstrated that it can achieve channel equalisation and significant BER improvement, leaving the frequency error correction to be done further along the signal processing chain.

For frequency error correction, we have derived an algorithm based on the stochastic gradient of the received signal modulated by a derotating phasor. This method is independent of other receiver functions, and does not require precise knowledge of the modulation index.

A Bluetooth transmission subjected to the largest initial frequency error allowed by the standard, and a channel model with highest RMS for a medium to large sized room, was processed by a the CMA and the frequency error correction algorithm. BER improved to within 0.3 dB and 1.9 dB of the theoretical MMSE solution when using a high-performance CPFSK receiver with observation intervals of 3 and 9 bits respectively. The remaining mismatch between the MMSE and CMA performances in Fig. 6 was found to be due to the CMA's sensitivity to strong correlation in the phase of $s[n]$. While for small N, the resulting performance loss is tolerable, future work will address this problem for large values of $N \gg 1$.

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