AN ALGEBRAIC IDENTIFICATION METHOD FOR THE DEMODULATION OF QPSK SIGNAL THROUGH A CONVOLUTIVE CHANNEL

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ABSTRACT

We present a straightforward algebraic setting for the demodulation of a QPSK (Quadrature Phase Shift Keying) signal over a convolutive and additive noise channel. Our method exploits the form of the continuous-time received signal without questioning the digital nature of the communication system. The developments rely on classical deterministic numerical analysis. No assumption is required for the statistical properties of the signals and noises. The proposed method estimates the channel using a very short training sequence (typically 5 symbols) and, afterwards, directly reconstructs the transmitted symbols using a very fast (on-line) estimation procedure. The simulation results show a good robustness to additive noise disturbance and to channel's order mismatch.

1. INTRODUCTION

Channel equalizers have become an integral part of today's communication systems. Equalization methods, including both training and (semi-) blind techniques, have now a long history, well documented in the litterature [1]. Some features of this history may be summarised as follows:

- An extensive use of highly developed probabilistic tools has become quite universal.
- Shannon's information theory and digital computers have imposed an almost exclusive analysis of discrete-time signals.

Now, in a probabilistic setting, some prior knowledge on the statistical distribution of the signals and noise disturbances, is a key assumption. And, performance of asymptotic nature with concomitant processing delays are inherent to almost any probabilistic approach. On the other hand, the "true" physical nature of the continuous-time signals is ignored when an early sampling is applied.

In this paper, we propose a new method for the detection of QPSK (Quadrature Phase Shift Keying) modulated symbols, transmitted through a dispersive and noisy channel. The method is based on a novel identification/estimation theory developped by the authors (due to lack of space, the theory is not pressented here. See [2, 3]). This theory, which is based on differential fields[1], ring theory, and operational calculus, leads to the following facts:

- No precise statistical knowledge of the noise is required[2].
- We are keeping the "true" physical nature of the continuous-time signals, which might be forgotten when sampling[3].
- There is no distinction between stationary and non-stationary signals.
- The computations of the estimates can be done on-line.

Applying this theory, we first identify a rational transfert function model for the channel using a very short training sequence (typically 5 symbols). Then, we directly reconstruct the transmitted symbols using a very fast (on-line) estimation procedure.

The system model is presented in section 2. Section 3 is devoted to the (continuous-time) channel identification, while section 4 presents the proposed demodulation method. Simulation results, including noise perturbation and channel undermodelling scenarios, are shown and discussed in section 5.

2. SYSTEM MODEL

Figure 1 shows the system in study. The method presented here may be applied to other signal modulation schemes. Let $\{\xi_k\}$ denotes the (complex) sequence of the transmitted symbols. The symbols are drawn from a finite alphabet. The transmitted signal, $u(t)$, has the following form:

$$u(t) = \sum_{k=0}^{\lfloor t/T \rfloor} \left\{ \Re\{\xi_k\}\cos(\omega_c t) + \Im\{\xi_k\}\sin(\omega_c t) \right\} g(t-kT) \tag{1}$$

where T is the symbol period, $g(t)$ is the shaping pulse and ω_c is the carrier frequency. The notation $\lfloor \cdot \rfloor$ stands for the integer part of the argument. To simplify the developments, we consider, without loss of generality, the complex baseband signal that we still denote by $u(t)$:

$$u(t) = \sum_{k=0}^{n} \xi_k g(t-kT), \quad \text{for } (n-1)T \leqslant t < nT \tag{2}$$

The channel output signal, $x(t)$, is given by:

$$x(t) = \int_0^t u(\tau)h(t-\tau)d\tau \tag{3}$$

where $h(t)$ is the impulse response of the channel. For the moment we will consider a system with no noise. We can also represent this same relation using the Laplace transform:

$$\hat{x}(s) = \hat{u}(s)H(s) \tag{4}$$

[1] Differential fields are already playing some rôle in non-linear control (see, *e.g.*, [4]).

[2] *Unknown but Bounded* and *Interval Analysis* are other ways of a complete different nature for avoiding probability and statistics in estimation. See, *e.g.*, [5], [6] and the references therein.

[3] Note that the *differentially flat* systems [4], which are so useful in practice, have also shed a new light on sampling in control.

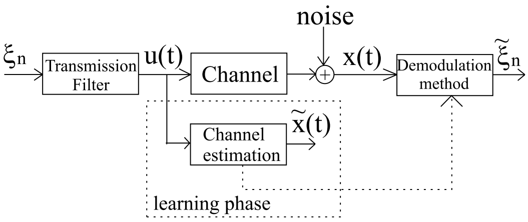

Figure 1: System model

where $\hat{x}$ and $\hat{u}$ are the Laplace transforms of $x(t)$ and $u(t)$ and $H(s)$ is the transfer function of the channel.

3. CHANNEL ESTIMATION

To estimate the channel, we consider a rational transfert function model $F(s)$ as in:

$$F(s) = \frac{b_0 + b_1 s + ... + b_M s^M}{-a_0 - a_1 s - ... - a_{N-1} s^{N-1} + s^N} \tag{5}$$

where $a_N = 1$ without loss of generality.

Let us first ignore the noise and start with a very simple case where $M = N = 1$ in order to present the basic principles of the method. We assumed that, at the receiver, $u(t)$ is known during one symbol period, that is, during an interval $(n-1)T \leq t < nT$. In this case, the received signal satisfies the differential equation:

$$\dot{x}(t) = a_0 x(t) + b_0 u(t) + b_1 \dot{u}(t), \tag{6}$$

which reads in the operational domain as

$$s\hat{x} - x_0 = a_0 \hat{x} + b_0 \hat{u} + b_1 (s\hat{u} - u_0) \tag{7}$$

where $u_0 = u((n-1)T)$ and $x_0 = x((n-1)T)$ denote the initial conditions. To determine the unknown parameters a_0, b_0 and b_1 we can differentiate (7) with respect to s recursively, in order to generate a system having the same number of equations and unknowns [2]. In this sense, we can also include the necessary differentiation for eliminating the dependence on the initial conditions. We will then differentiate (7) 3 times. The resulting system of equations is then:

$$(s\hat{x})^{(i)} = a_0 \hat{x}^{(i)} + b_0 \hat{u}^{(i)} + b_1 (s\hat{u})^{(i)}, \quad i = 1,2,3 \tag{8}$$

where the superscript (i) denotes the derivation of order i, with respect to s. As the differentiation in the time domain is a difficult and not robust operation to calculate numerically, we divide all equations by a factor s^γ, where γ is a constant greater than the highest power of s appearing in the system. Here, γ has to be greater then 1 and the resulting system:

$$\frac{(s\hat{x})^{(i)}}{s^\gamma} = a_0 \frac{\hat{x}^{(i)}}{s^\gamma} + b_0 \frac{\hat{u}^{(i)}}{s^\gamma} + b_1 \frac{(s\hat{u})^{(i)}}{s^\gamma} \tag{9}$$

contains only integral terms of the form

$$\widehat{U}_{ij} = \left(\frac{\hat{u}^{(i)}}{s^{\gamma-j}} \right) \text{ and } \widehat{X}_{ij} = \left(\frac{\hat{x}^{(i)}}{s^{\gamma-j}} \right) ; \; i = 0,1,2,3 \; ; \; j = 0,1 \tag{10}$$

To obtain the time domain analog U_{ij} and X_{ij} of the coefficients $\widehat{U}_{ij}$ and $\widehat{X}_{ij}$, we have to remember that we are in a time interval $(n-1)T \leqslant t < nT$ where $u(t)$ is known. In this interval, a change of variables gives: $\tilde{u}(\tau) = u((n-1+\tau)T)$ and $\tilde{x}(\tau) = x((n-1+\tau)T)$, where we have set $t = (n+\tau)T$ and so $0 \leqslant \tau < 1$. The time domain coefficients U_{ij} and X_{ij} are then obtained by using $\tilde{u}(\tau)$ and $\tilde{x}(\tau)$ in the following way [2]:

$$\begin{aligned} U_{ij} &= \frac{(-1)^i}{(\gamma-j-1)!} \int_0^\lambda (\lambda-\tau)^{\gamma-j-1} \tau^i \tilde{u}(\tau) d\tau \\ X_{ij} &= \frac{(-1)^i}{(\gamma-j-1)!} \int_0^\lambda (\lambda-\tau)^{\gamma-j-1} \tau^i \tilde{x}(\tau) d\tau \end{aligned} \tag{11}$$

where the integration time λ $(0 < \lambda < 1)$ may be chosen very small. This explains the fastness of the method.

Developing (9) and using (10) and (11), we finally obtain the estimates of the channel parameters from the solutions of the following system of equations:

$$\begin{aligned} X_{00} + X_{11} &= a_0 X_{10} + b_0 U_{10} + b_1 (U_{00} + U_{11}) \\ 2X_{10} + X_{21} &= a_0 X_{20} + b_0 U_{20} + b_1 (2U_{10} + U_{21}) \\ 3X_{20} + X_{31} &= a_0 X_{30} + b_0 U_{30} + b_1 (3U_{20} + U_{31}) \end{aligned} \tag{12}$$

The method shown above can be easily generalized. Considering $F(s)$ given by (5), (7) can be extended to:

$$\begin{aligned} & s^N \hat{x} - \left(s^{N-1} x_0 + s^{N-2} \dot{x}_0 + s^{N-3} \ddot{x}_0 + \ldots + x_0^{(N-1)} \right) \\ & = \; a_0 \hat{x} + \ldots + a_{N-1} (s^{N-1} \hat{x} - s^{(N-2)} x_0 - \ldots - x_0^{(N-2)}) \\ & \quad + b_0 \hat{u} + \ldots + b_M (s^M \hat{u} - s^{(M-1)} u_0 - \ldots - u_0^{(M-1)}) \end{aligned} \tag{13}$$

In this case, the number of derivatives needed to eliminate the dependence on the initial conditions is equal to the maximum between M and N. If we consider the channel to be proper, then $M \leqslant N$ and N derivations are sufficient. Given that we have $M+N+1$ variables, equation (13) has to be differentiated $2N+M$ times. Thus, the desired system of equations is obtained by taking the derivatives of orders N to $(2N+M)$. These equations have the following general form:

$$\begin{aligned} (s^N \hat{x})^{(i)} = a_0 \hat{x}^{(i)} + \ldots + a_{N-1} (s^{N-1} \hat{x})^{(i)} \\ + b_0 \hat{u}^{(i)} + \ldots + b_M (s^M \hat{u})^{(i)} \end{aligned} \tag{14}$$

where $i = N, \ldots, 2N+M$.

Dividing both sides of (14) by s^γ, where now $\gamma > N$, gives a system which reads in the time domain as:

$$\mathscr{P} \begin{bmatrix} a_0 \\ \vdots \\ a_{N-1} \\ b_0 \\ \vdots \\ b_M \end{bmatrix} = \mathscr{Q} \tag{15}$$

where the entries of the $(N+M+1) \times (N+M+1)$ and $(N+M+1) \times 1$ matrices $\mathscr{P}$ and $\mathscr{Q}$ are finite linear combinations of U_{ij} and X_{ij} in (11).

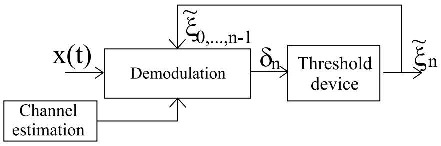

Figure 2: Demodulation method

4. DEMODULATION METHOD

Once we have found $F(s)$ from (15), one could use its inverse as a zero-forcing equalizer. Then sampling its output would allow one to recover the transmitted symbols. However, this assumes that not only $F(s)$ is minimum phase but also, the estimation is quite precise. Moreover, it is well known that zero-forcing equalization is not robust to noise perturbation. For these reasons, we rather exploit[4] the form of the continuous-time received signal together with the estimate of the channel to detect the transmitted symbol directly. As the simulation will show, the method is robust to noise and channel order mismatch.

To begin, denote by $f(t)$ the impulse response corresponding to the estimated channel model $F(s)$. Assuming that $F(s)$ is strictly proper, this impulse response may readily be computed as:

$$f(t) = \sum_{k=1}^{N} \lambda_k e^{-\omega_k t} \tag{16}$$

where the ω_k's are the poles of $F(s)$ and the λ_k's are constants. The received signal $x(t)$ in the n^{th} symbol period, $(n-1)T \leqslant t < nT$, then reads as

$$x(t) = \sum_{k=0}^{n} \xi_k \int_0^t g(\tau - kT) f(t-\tau) d\tau = \sum_{k=0}^{n} \xi_k f_k(t), \tag{17}$$

where we have set $f_k(t) = \int_0^t g(\tau - kT) f(t-\tau) d\tau$. Based on the previously detected symbols, the n^{th} transmitted symbol is estimated by the explicit formula:

$$\xi_n = \frac{\displaystyle\int_{(n-1)T}^{nT} x(t)dt - \sum_{k=0}^{n-1} \xi_k \int_{(n-1)T}^{nT} f_k(t)dt}{\displaystyle\int_{(n-1)T}^{nT} f_n(t)dt} \tag{18}$$

Remark 1. *Since it is necessary to use the previous decided symbols to obtain the current one, the proposed demodulation scheme may be viewed as a continuous-time decision feedback equalizer (DFE), as shown in figure 2 [8].*

5. DISCUSSIONS AND SIMULATION RESULTS

Starting with the channel transfer function estimation, as was seen in section 3, $u(t)$ has to be known at the receiver during one symbol period. Note that the knowledge of only one transmitted symbol is not sufficient to reconstruct $u(t)$ at the receiver. Indeed, the number of required symbols corresponds to the number of symbol period within the support of the shaping pulse $g(t)$. Here, the raised cosine function is considered for $g(t)$, with a support corresponding approximately to $5T$. A training sequence of 5 symbols is then necessary to obtain a good estimate of $u(t)$. The raised cosine roll-off factor was set to 0.33. The unknown channel was modeled by the non minimum phase transfer function:

$$H = \frac{s^2 - 1.5s - 1}{s^3 + 7.4s^2 + 17.44s + 27.2} \tag{19}$$

Table 1 compares the zeros and poles of the channel with the ones of $F(s)$, obtained using (15) with $M = 2$, $N = 3$ and $\gamma = 6$. The integrals were computed numerically using the classical trapezoidal rule, with a very small grid (10^5 points). (Higher order integration methods require much less points.) As it can be seen, the result is a very good approximation of $H(s)$.

	Zeros		Poles		
H(s)	-0.5	2	-1.2+2i	-1.2-2i	-5
F(s)	0.4965	2.0466	-1.1944 +1.9926i	-1.1944 -1.9926i	-4.9979

Table 1: Comparison between H(s) and F(s)

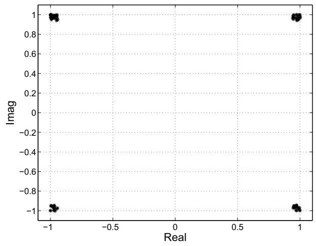

Figure 3: Recovered symbols after demodulation, no noise

Figure 3 shows the recovered symbols, ξ_n, after the demodulation process, for the transmission of 100 symbols, using the estimated channel $F(s)$ shown in table 1. This result was obtained using (18). We can see that the remaining intersymbol interference (ISI) is a very small.

In the above simulation we used M and N equal to the order of the numerator and denominator of the channel, but usually these values are not known. Simulations show that in the overmodelled case the zeros and poles in excess have large absolute valued real parts. Among them, the poles with negative real part have a negligeable effect on the global response of the system. Those having positive real parts obviously make the system unstable. However, as it is easy to identify them, it suffices to replace them by their opposite. An example is shown in table 2. We considered a minimum phase channel with two zeros and three poles and estimated it using $M = 1$, $N = 3$ ($F_1(s)$) and $M = 2$, $N = 4$ ($F_2(s)$). The first case, $F_1(s)$, corresponds to an undermodeling situation. Although the model order is not sufficient to completely eliminate the ISI in figure 5, we can note that the eye is completely open. For comparison, the channel output signal is shown in figure 4. In the second case, we consider an overmodeling setting. The poles and zeros of the channel are well estimated while the pole in excess has a large absolute value when compared to the others. As shown in figure 6,

[4]This idea has been succeesfully applied to the demodulation of CPM signals [7]

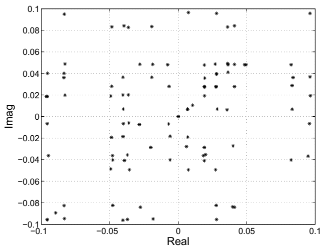

Figure 4: Channel output signal (table 2)

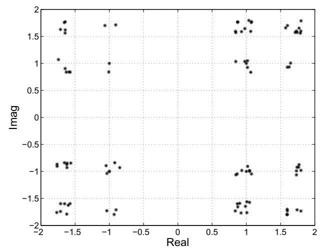

Figure 5: Recovered symbols for M smaller then necessary

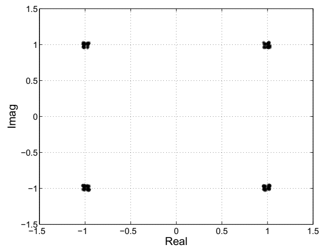

Figure 6: Recovered symbols for N greater then necessary

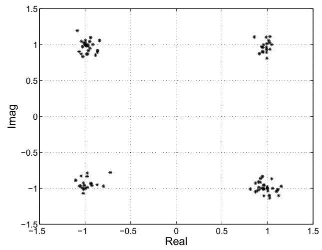

Figure 7: Recovered symbols, SNR=10 dB

which shows the constellation of the demodulated symbols, this excess pole does not have a significant effect.

	Zeros		Poles			
H(s)	-0.5	-3	-1.2+2i	-1.2-2i	-5	
$F_1(s)$	-0.716		-1.236 +2.780i	-1.236 -2.780i	-4.691	
$F_2(s)$	-0.546	-3.149	-1.233 +2.029i	-1.233 -2.029i	-4.989	-6990

Table 2: Channel estimation done with incorrect values of M and N

Simulation with noise perturbation is now considered. For the channel estimation step, mitigating the noise effect can be achieved using higher order derivatives in (14) and a higher value for γ (more iterated integrals). This has not been done here. Interested readers are invited to see [3] where the identification of a rational transfert function in noisy environment is presented. So, we consider in the simulation a zero-mean white Gaussian noise added to the received signal after the channel identification step. Figure 7 shows the recovered symbols after de demodulation process when the SNR (signal to noise ratio) is 10dB. The channel used was the one given by (19) and its estimation, $F(s)$, is shown in table 1. Again we considered the transmission of 100 symbols. We can see that the demodulation method had a very good performance.

6. CONCLUSION

In this paper we developed an equalization process using an uncommon point of view. We considered the system as being continuous, instead of sampling it. As the simulations have shown, the first results were very encouraging, showing a good performance of the system even in noisy conditions.

Acknowledgement

This work was partially supported by the *action spécifique* “Méthodes algébriques pour les systèmes de communications numériques”, RTP 24 - CNRS

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