BLIND MULTI-USER EQUALISATION FOR A DISPERSIVE DS-CDMA DOWNLINK UNDER CARRIER FREQUENCY OFFSET CONDITIONS

Stephan Weiss [1], *Mahmoud Hadef* [1], *and Markus Rupp* [2]

[1] Communications Research Group, School of Electronics & Computer Science
University of Southampton, Southampton SO17 1BJ, UK
[2] Christian Doppler Pilotlabor for Design Methodology of Signal Processing Algorithms
Communications and RF Institute, Technical University of Vienna, Austria

ABSTRACT

We address blind multiuser detection in a DS-CDMA downlink channel in the presence of carrier frequency offset. The synchronous users are separated by re-establishing orthogonality of their spreading sequences in a shared equaliser at the chip level. The adaptation algorithm is based on a constant modulus criterion of the various users, for which a stochastic gradient descent algorithm can be derived. We show that the resulting filtered-error filtered-regressor algorithm requires modifications in order to cope with carrier offset, and propose an combined blind multiuser equaliser with blind carrier frequency offset estimation. Simulations demonstrating the algorithm's convergence and BER performance are presented.

1. INTRODUCTION

In a DS-CDMA downlink scenario, transmission over a dispersive channel destroys the mutual orthogonality of the codes which are used to multiplex the various users in the system. As a result, the received and code-demultiplexed user signals are subject not only to inter-symbol interference (ISI) due to channel dispersion but also to multiple access interference (MAI) due to the loss of code orthogonality.

A popular approach to suppress MAI and ISI is the minimum output power (MOE) algorithm blindly cancelling MAI and ISI terms but passing the desired user by code-constraints [1, 2]. Recovering several users at the same time exploits more knowledge of the system and has been performed blindly using a constant modulus (CM) criterion [3, 4, 5], whereby the derived algorithms either omit spreading [4] or do not take the dispersiveness of the channel into account [3, 5]. Non-blind multiuser schemes in turn are based either on the knowledge of a pilot [6, 7] or training sequences [8]. We have proposed a blind synchronous multiuser equalisation algorithm in [9], which is based on a constant modulus criterion, whereby the spreading codes inherently orthogonalise the decoded signals, making the need for additional orthogonality constraints obsolete [4, 10]. The result is a filtered-error filtered-regressor structure similar to the training/decision directed approach in [7].

In this paper, we consider the above multiuser equaliser in a DS-CDMA downlink with potential carrier offset. While standard CM algorithms are invariant to carrier frequency offset [11, 12], we analyse that the filtered-error filtered regressor-structure destroys this nice property. Instead the algorithm has to be supplied with a carrier offset estimate to work accurately, for which a solution is proposed.

The paper is organised as follows. In Sec. 2, we briefly review the various signals and system blocks in a DS-CDMA downlink. Sec. 3 first reviews the multiuser CMA algorithm before analysing its sensitivity to a carrier frequency offset, which also proposes a modified structure when operating under such conditions. Sec. 4 considers a carrier offset detection strategy, which combined with the modified multiuser CM algorithm is evaluated in Sec. 5 by simulations.

2. SIGNAL MODEL

We consider the DS-CDMA downlink system in Fig. 1 with multiple symbol-synchronous users, which for the sake of simplicity are assumed to have the same rate and fully load the system, although multiple rate users and partial load can be taken into account [13]. In a first step, the N user signals $u_l[n]$, $l = 0(1)N-1$ are code multiplexed using Walsh sequences of length N extracted from a Hadamard matrix $\mathbf{H}$. The resulting chip rate signal, running at N times the symbol rate, is further scrambled by $c[m]$ prior to transmission over a channel with dispersive impulse response $g[m]$ and corruption by additive white Gaussian noise $v[m]$, which is assumed to be independent of the transmitted signal.

The dispersive channel $g[m]$ destroys the orthogonality of the Walsh codes, such that direct decoding of the received signal $r[m]$ with descrambling by $c^*[m]$ and code-matched filtering by $\mathbf{H}^{\mathrm{T}}$ will lead to MAI and ISI corruption of the decoded user signals $\hat{u}_l[n]$, $l = 0(1)N-1$. In order to re-establish orthogonality of the codes, a chip rate equaliser $w[m]$ can be utilised [8, 7]. In the following, we are concerned with the blind updating of the equaliser coefficients $w[m]$, as well as the influence and compensation of a carrier frequency offset $\Delta\Omega$.

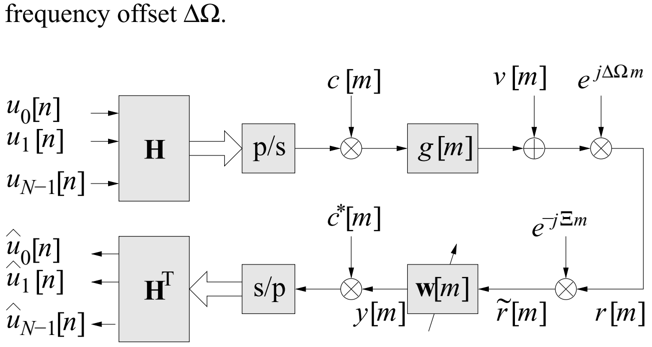

Figure 1: Flow graph for DS-CDMA downlink scenario.

The work of Markus Rupp has been funded by the Christian Doppler Pilot Laboratory for Design Methodology of Signal Processing Algorithms.

3. BLIND MULTIUSER EQUALISER

The receiver structure and the algorithm of the multiuser equaliser proposed in [9, 13] will be briefly reviewed in Secs. 3.1 and 3.2, while the influence of carrier frequency offset is considered in Sec. 3.3.

3.1 Receiver Description

The receiver structure is shown in the lower branch of Fig. 1, whereby for decoding, Walsh sequences are used as matched filters. A Walsh sequence contained in a vector $\mathbf{h}_l$ can be extracted from an $N \times N$ Hadamard matrix,

$$\mathbf{H}^{\mathrm{T}} = \left[\mathbf{h}_0 \; \mathbf{h}_1 \; \cdots \; \mathbf{h}_{N-1}\right]^{\mathrm{T}}. \tag{1}$$

and user for decoding the lth user as

$$\begin{aligned}\hat{u}_l[n] &= \mathbf{h}_l^{\mathrm{T}} \cdot \begin{bmatrix} c^*[nN] & & & \mathbf{0} \\ & c^*[nN-1] & & \\ & & \ddots & \\ \mathbf{0} & & & c^*[nN-N+1] \end{bmatrix} \cdot \begin{bmatrix} y[nN] \\ y[nN-1] \\ \vdots \\ y[nN-N+1] \end{bmatrix} \\ &= \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] \cdot \begin{bmatrix} \mathbf{w}^{\mathrm{H}} & & & \mathbf{0} \\ & \mathbf{w}^{\mathrm{H}} & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{w}^{\mathrm{H}} \end{bmatrix} \cdot \begin{bmatrix} \tilde{r}[nN] \\ \tilde{r}[nN-1] \\ \vdots \\ \tilde{r}[nN-L-N+2] \end{bmatrix}\end{aligned}$$

whereby the descrambling code $c^*[m]$ has been absorbed into a modified and now time-varying code vector $\tilde{\mathbf{h}}_l[nN]$. The vector $\mathbf{w} \in \mathbb{C}^L$ contains the equaliser's L chip-spaced complex conjugate weights. The time index highlighting that $\mathbf{w}$ can be time-varying has been omitted for simplicity. Rearranging $\mathbf{w}$ and $\tilde{\mathbf{h}}_l[nN]$ yields

$$\begin{aligned}\hat{u}_l[n] &= \mathbf{w}^{\mathrm{H}} \cdot \begin{bmatrix} \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] & & & \mathbf{0} \\ & \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] & & \\ & & \ddots & \\ \mathbf{0} & & & \tilde{\mathbf{h}}_l^{\mathrm{T}}[nN] \end{bmatrix} \cdot \begin{bmatrix} \tilde{r}[nN] \\ \tilde{r}[nN-1] \\ \vdots \\ \tilde{r}[nN-L-N+2] \end{bmatrix} \\ &= \mathbf{w}^{\mathrm{H}}\, \mathbf{H}_l[nN]\, \tilde{\mathbf{r}}_{nN},\end{aligned} \tag{2}$$

with $\mathbf{H}_l[nN] \in \mathbb{Z}^{L \times (N+L-1)}$ being a convolutional matrix comprising the lth user's modified code vector $\tilde{\mathbf{h}}^{\mathrm{T}}[n]$ and $\tilde{\mathbf{r}}_{nN} \in \mathbb{C}^{N+L-1}$.

3.2 Blind CM Multiuser Algorithm

We assume that the user signals $u_l[n]$ consist of symbols with a constant modulus γ, such as BPSK, QPSK, or 8-PSK. Therefore, the equaliser can be adapted blindly by forcing all decoded users $\hat{u}_l[n]$ onto a constant modulus. This can be formulated, similarly to [4, 14], by a suitable cost function ξ_{CM},

$$\xi_{\mathrm{CM}} = \mathscr{E}\left\{\sum_{l=0}^{N-1} (\gamma^2 - |\hat{u}_l[n]|^2)^2\right\}, \tag{3}$$

which measures deviation of each of the N users' decoded symbols from the desired modulus. The optimum equaliser coefficient vector w is therefore given by

$$\mathbf{w}_{\mathrm{opt,CM}} = \arg\min_{\mathbf{w}} \xi_{\mathrm{CM}} \quad . \tag{4}$$

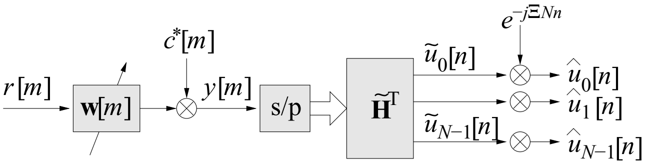

Figure 2: Flow graph of DS-CDMA receiver structure; different from Fig. 1, the carrier frequency offset compensation $e^{jN\Xi n}$ has been transfered to the receiver output.

There is no unique solution to (4), since minimising (3) is ambiguous with a manifold of solutions due to an indeterminism in phase rotation. However, any member of this manifold is a suitable solution for the equaliser $\mathbf{w}$, and can be used in combination with differential modulation schemes to recover $u_l[n]$.

A simple stochastic gradient descent update rule for $\mathbf{w}[n]$ can be found by calculating the gradient of an instantaneous cost function, i.e. omitting the expectation operator in (3), yielding

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mu \nabla \hat{\xi}_{\mathrm{CM}}(\mathbf{w}_n) \tag{5}$$

with [9] the gradient estimate

$$\nabla \hat{\xi}_{\mathrm{CM}}(\mathbf{w}_n) = -2 \sum_{l=0}^{N-1} (\gamma^2 - |\hat{u}_l[n]|^2)\, \mathbf{H}_l[nN]\, \tilde{\mathbf{r}}_{nN}\, \hat{u}_l^*[n] \tag{6}$$

This algorithm differs from the standard CM algorithm [15] or its extension in [4] in the inclusion of a code filtered term $\mathbf{H}_l[nN]\, \mathbf{r}_{nN}$ rather than just the equaliser input $r[n]$. This is structurally similar to a multiple-error filtered-X LMS algorithm [16], which has been analogously employed in equalisation [17, 8].

3.3 Carrier Frequency Offset

To investigate the influence of a carrier frequency offset on the algorithm in (5) and (6), in the following we aim to analytically move the modulator $e^{j\Delta\Omega m}$ causing the offset to the receiver output.

Let the carrier offset-free received signal be denoted as $\tilde{r}[m] = r[m] \cdot e^{-j\Xi m}$, as shown in Fig. 1. Thus, the tap delay line vector

$$\begin{aligned}\tilde{\mathbf{r}}[nN] &= e^{-jN\Xi n}\, \mathrm{diag}\left\{1,\; e^{-j\Xi},\; \ldots\; e^{-j(L-1)\Xi}\right\} \mathbf{r}[nN] \\ &= e^{-jN\Xi n} \Lambda(\Xi)\, \mathbf{r}[nN]\end{aligned} \tag{7}$$

can be substituted into (2) yielding

$$\hat{u}_l[n] = e^{-jN\Xi n}\, \mathbf{w}^{\mathrm{H}}\, \underbrace{\mathbf{H}_l[nN]\, \Lambda(\Xi)}_{\tilde{\mathbf{H}}_l[nN,\Xi]}\, \mathbf{r}_{nN}, \tag{8}$$

Analogous to [9], the gradient estimate can be derived as

$$\nabla \hat{\xi}_{\mathrm{CM}}(\mathbf{w}_n) = -2 \sum_{l=0}^{N-1} (\gamma^2 - |\tilde{u}_l[n]|^2)\, \tilde{\mathbf{H}}_l[nN,\Xi]\, \mathbf{r}_{nN}\, \tilde{u}_l^*[n] \tag{9}$$

whereby $\tilde{u}_l[n]$ are the uncompensated user signals as defined in Fig. 2.

Thus, the modulation by the carrier offset frequency affects the transfer function in the error path, and the Walsh code matrix $\mathbf{H}_l[nN]$ in (2) and (6) needs to be replaced by $\tilde{\mathbf{H}}_l[nN,\Xi]$ in (8) and (9). Therefore, the proposed blind synchronous multiuser equaliser is — unlike standard CMA algorithms [15] — sensitive to carrier frequency offset, and must be supplied with an estimate $\Xi \approx \Delta\Omega$.

4. CARRIER FREQUENCY OFFSET COMPENSATION

The aim is to estimate the carrier offset $\Delta\Omega$ from the decoded user data $\tilde{u}_l[n]$ in Fig. 2, in order to (i) supply the multiuser CM algorithm with an accurate carrier offset and (ii) compensate for the phase rotation to yield the carrier offset compensated user signals $\hat{u}_l[n]$. Assuming the transmission of QPSK user data, we investigate a carrier offset detection method raising the decoded samples to the fourth power similar to [18].

Similar to [19], the carrier frequency offset can be obtained by

$$\Xi = \frac{4}{MN^2} \sum_{l=0}^{N-1} \angle \mathscr{E}\left\{ \left(\hat{u}_l[n] \hat{u}_l^*[n-M] \right)^4 \right\} \qquad (10)$$

with a region of convergence if $|\Delta\Omega| < \pi/(2NM)$. The resulting Ξ is an estimate of $\Delta\Omega$, which however is biased if $\hat{u}_l[n] = u_l[n] + \tilde{v}[n]$ with noise $\tilde{v}[n]$ that is correlated with $u_l[n]$. If $v[m]$ in Fig. 1 is AWGN, then $\tilde{v}[n]$ is likely to be correlated due to the equaliser response $\mathbf{w}[n]$. Therefore, the bias term on Ξ in (10) can be reduced by selecting $M > L/N$, although this reduces the capability to estimate larger offset values due to the reduced region of convergence. This above carrier offset detection approach has been reported to operate reliably for channel SNRs above approximately 10dB [18].

5. SIMULATION RESULTS

For the simulations below, we apply the proposed blind multiuser equaliser combined with a carrier frequency offset correction to two types of channels characterised by their average profile shown in Fig. 3. These are the mean moduli of Rayleigh distributed coefficients of the channel impulse responses $g_1[m]$ and a more dispersive $g_2[m]$.

5.1 Convergence

In order to demonstrate the convergence behaviour of the proposed algorithm, we transmit $N = 4$ users with QPSK signals over $g_1[m]$ in the absence of channel noise, but with a carrier offset of $\Delta\Omega = 0.2\pi$. The equaliser comprises of 10 coefficients, with the second coefficient in $\mathbf{w}[0]$ set to unity.

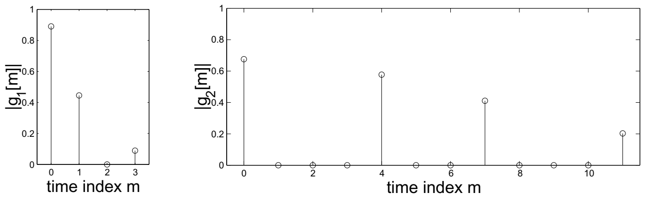

Figure 3: Moduli of complex valued channel impulse responses $g_1[m]$ (left) and $g_2[m]$ (right).

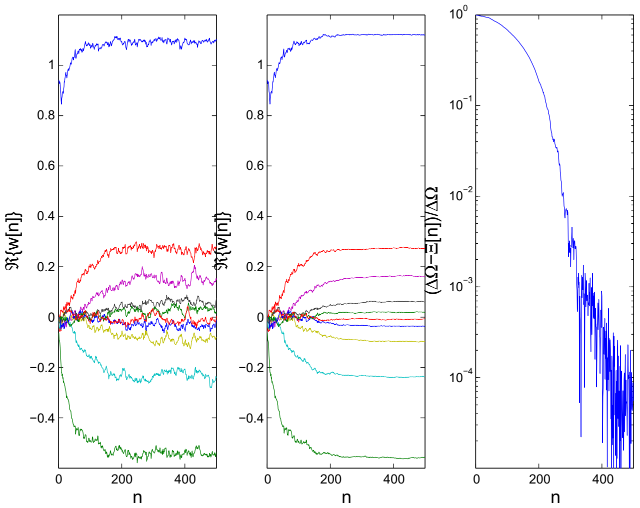

Figure 4: Real part of the equaliser coefficient trajectories during adaptation (left) without adjusting Ξ, and (middle) with adjusting Ξ together with (right) the learning curve of Ξ during adaptation.

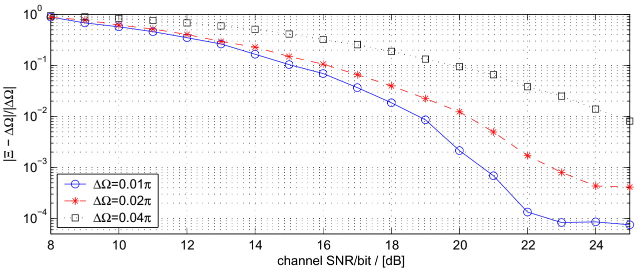

Figure 5: Relative error in estimation of carrier frequency.

With a step size $\mu = 0.05$ and an initial value $\Xi = 0$, the evolution of the filter coefficients' real part is shown in Fig. 4. For the experiment in Fig. 4(left), the carrier frequency offset estimate was set to zero during adaptation, while the trajectories in Fig. 4(left) are the ones of a blind multiuser CMA with the carrier frequency offset estimate incorporated. The estimate Ξ is calculated using a window of 200 symbol periods to evaluate (10) at each point in time. The learning curve for Ξ is given in Fig. 4(right), exhibiting good convergence to the adjusted carrier frequency offset $\Delta\Omega$.

The accuracy of the carrier frequency offset estimation in noise is detailed in Fig. 5. The scenario is the same as above, with $N = 4$ users transmitting over $g_1[m]$, and $L = 10$ coefficients in the equaliser. The system is given 500 symbol periods to converge, and uses a window length of 200 symbols for the estimation of Ξ. Thereafter, the accuracy of Ξ is monitored of the next 500 symbol periods.

5.2 Bit Error Performance

To demonstrate the achievable BER over blindly equalised dispersive channels with carrier offset, we consider two scenarios. The first scenario uses $N = 4$ users over channel $g_1[m]$ and an equaliser of $L = 20$ coefficients. The results for three different carrier offset frequencies are stated in Fig. 6, and are benchmarked against the performance of a minimum mean square error (MMSE) equaliser of same length. As seen in

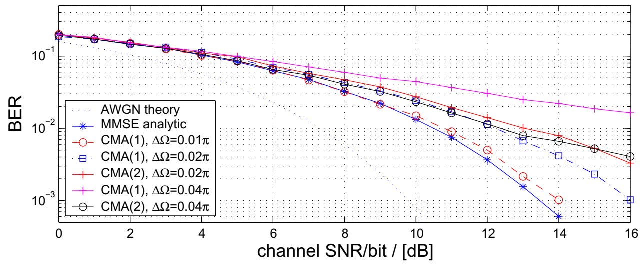

Figure 6: BER curves for transmission over $g_1[m]$; CMA(1) and(2) refer to operation with or without adjustment of Ξ.

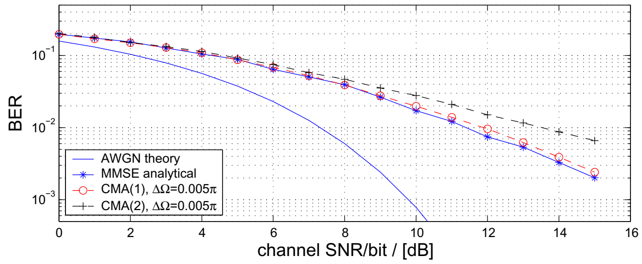

Figure 7: BER curves for transmission over $g_2[m]$; CMA(1) and(2) refer to operation with or without adjustment of Ξ.

Fig. 5, the carrier offset estimation is difficult at low SNR values and hence leads to degraded performance compared to the MMSE. However, in all cases the carrier offset estimation makes a considerable difference to the convergence of the overall system compared to the uncompensated system.

A second scenario uses a more dispersive channel $g_2[m]$ to transmit $N = 8$ users with the help of an $L = 64$ coefficient filter. The results for this case are given in Fig. 7 averaged over an ensemble of 100 realisations of $g_2[m]$, demonstrating good performance of the combined scheme.

6. CONCLUSIONS

A blind equalisation approach for a DS-CDMA downlink scenario has been presented, which aims to enforce CM conditions on the various user signals. A stochastic gradient algorithm has been reviewed, which differs from previous CM algorithms by a code-prefiltering of its input but also its sensitivity to carrier frequency offsets. The latter has been circumvented by modifications to the algorithm, necessitating the blind detection of the carrier frequency offset. A combined scheme based on the multiuser CM algorithm and a blind estimation of the carrier frequency offset has shown good results in terms of convergence speed and BER performance, and in some cases very closely approaches the MMSE solution.

REFERENCES

[1] M. Honig, U. Madhow, and S. Verdu, "Blind Adaptive Multiuser Detection," *IEEE Trans. Inf. Theory*, vol. 41, no. 4, pp. 944–960, Jul. 1995.

[2] R. Schober, W. H. Gerstacker, and L. H. J. Lampe, "Comparison of MOE and blind LMS," *IEEE Comm. Letters*, vol. 7, no. 5, pp. 204–206, May 2003.

[3] K. Fukawa and H. Suzuki, "Orthogonalizing Matched Filter (OMF) Detection for DS-CDMA Mobile Radio Systems," in *IEEE Globcom*, 1994, vol. 1, pp. 385–389.

[4] C. Papadias and A. Paulraj, "A Constant Modulus Algorithm for Multiuser Signal Separation in Presence of Delay Spread Using Antenna Arrays," *IEEE Sig. Proc. Let.*, vol. 4, no. 6, pp. 178–181, June 1997.

[5] Y. Huang and P. M. Djurić, "Multiuser Detection of Synchronous Code-Division Multiple-Access Signals by Perfect Sampling," *IEEE Trans. Sig. Proc.*, vol. 7, no. 50, pp. 1724–1734, July 2002.

[6] F. Petre, M. Moonen, M. Engels, B. Gyselinckx, and H.D. Man, "Pilot-Aided Adaptive CHip Rate Equalizer Receiver for Interference Suppression in DS-CDMA Forward Link," in *IEEE VTC*, Sept. 2000, vol. 1, pp. 303–308.

[7] P. Schniter and A. R. Margetts, "Adaptive Chip-Rate Equalization of Downlink Multirate Wideband CDMA," in *Asilomar Conf. Sig., Sys., Comp.*, Pacific Grove, CA, Nov. 2002, pp. 1228–1232.

[8] I. Stirling, D. Garcia-Alis, S. Weiss, G. Rice, and R.W. Stewart, "A Joint Adaptive MMSE Downlink Receiver," in *Asilomar Conf. Sig., Sys., Comp.*, Pacific Grove, CA, Nov. 2000.

[9] S. Weiss, M. Hadef, M. Konrad, and M. Rupp, "Blind Chip-Rate Equalisation for DS-CDMA Downlink Receiver," in *Asilomar Conf. Sig., Sys., Comp.*, Pacific Grove, CA, Nov. 2003, vol. 2, pp. 1283–1287.

[10] S. Lambotharan, J. A. Chambers, and A. G. Constantinides, "Adaptive Blind Retrieval Techniques for Multi-User DS-CDMA Signals," *IEE Electron. Let.*, vol. 35, no. 9, pp. 693–695, Apr. 1999.

[11] J. R. Treichler, M. G. Larimore, and J. C. Harp, "Practical Blind Demodulators for High-Order QAM Signals," *Proc. IEEE*, vol. 86, no. 10, pp. 1907–1926, Oct. 1998.

[12] M. Rupp, "On the Separation of Channel and Frequency Offset Estimation," in *Asilomar Conf. Sig., Sys., Comp.*, Pacific Grove, CA, Oct. 1998, vol. 2, pp. 1186–1190.

[13] M. Hadef and S. Weiss, "A Blind Synchronous Multi-User DS-CDMA Equaliser with Partial Loading," in *Int. Symp. Control, Comm., and Sig. Proc.*, Hammamet, Tunisia, Mar. 2004, vol. 1, pp. 803–806.

[14] Z. Xu and P. Liu, "Code-Constrained Blind Detection of CDMA Signals in Multipath Channels," *IEEE Sig. Proc. Let.*, vol. 9, no. 12, pp. 389–392, Dec. 2002.

[15] C. R. Johnson, P. Schniter, T. J. Endres, J. D. Behm, D. R. Brown, and R. A. Casas, "Blind Equalization Using the Constant Modulus Criterion: A Review," *Proc. IEEE*, vol. 86, no. 10, pp. 1927–1950, Oct. 1998.

[16] S.J. Elliott, I.M. Stothers, and P.A. Nelson, "A Multiple Error LMS Algorithm and its Application to the Active Control of Sound and Vibration," *IEEE Trans. Acoustics, Speech, and Sig. Proc.*, vol. 35, no. 10, pp. 1423–1434, October 1987.

[17] J. Hu and H. R. Wu, "Robust Adaptive Equalization Using the Filtered-X LMS Algorithm," in *Fifth International Symposium on Signal Processing and Its Applications*, Aug. 1999, pp. 419–422.

[18] A. Rustako, L. Greenstein, R. Roman, and A. Saleh, "Using Times Four Carrier Recovery in M-QAM Digital Radio Receivers," *IEEE J. Selected Areas of Communications*, vol. 5, no. 3, pp. 524–533, April 1987.

[19] M. Rupp, "LMS Tracking Behavior Under Periodically Changing Systems," in *Europ. Signal Processing Conf.*, Rhodes, Greece, Sept. 1998.