LOW COMPLEXITY LINEAR DETECTION FOR ASYNCHRONOUS CDMA SIGNALS OVER FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

In this paper, a suboptimal low complexity structure for joint linear detection of CDMA signals is proposed. The optimal joint linear zero-forcing or minimum mean squared error detectors use a vector model of the transmission where the channel is represented by a matrix. Due to the finite memory of the channel, most of the blocks of the channel autocorrelation matrix are equal to zero. It is then possible to put this structure into profit to derive low complexity implementations for these detectors. In this paper, the optimal linear system resolution is approximated by overlapped smaller subsystems. The performance and the complexity of the resulting detector are analyzed for both downlink and uplink communications over frequency selective channels for periodic spreading codes with a period equal to the symbol period as well as for random-like codes. The trade-off between complexity and performance is flexible, which offers a great advantage to the proposed solution.

1. INTRODUCTION

Code division multiple access communications suffer from both multiple access interference and inter symbol interference. The optimal multi-user receiver has a complexity which grows exponentially with the channel memory P (measured in modulation symbol periods) and with the number of active spreading codes K. Therefore, sub-optimal receivers such as block linear receivers, interference cancellers or RAKE receivers have been proposed to handle the complexity issue.

The block linear multi-user receivers with MMSE (Minimum Mean Squared Error) or ZF (Zero Forcing) criteria give good performances [1]. They make decisions on blocks corresponding to N modulated symbols for K spreading codes. The size of the blocks is usually linked to the size of the bursts of the transmission format, which can be very large. The optimal joint linear ZF or MMSE detectors use a vector model of the transmission where the channel is represented by a matrix. The optimal solution consists in solving a linear system of size KN. Several algorithms have been proposed to minimize the complexity of the system resolution by exploiting the property of the channel autocorrelation matrix. Efficient reduction of the complexity is obtained for synchronous downlink transmission where the spreading codes are periodic with period equal to the modulation symbol period [2]. Due to the finite memory of the channel, most of the blocks of the channel autocorrelation matrix are equal to zero. Then, the truncation of the block processing window has been suggested in [3]. Finite-memory MMSE linear equalizer has been proposed in [4], performance has been evaluated for asynchronous transmission over non selective propagation channels.

In this paper, a low complexity implementation for finite-memory detectors is proposed as follows: the resolution of the whole system is replaced by the resolution of N overlapped smaller subsystems of size nK with n lower than $2P+1$. The algorithm is valid for periodic or non periodic spreading sequences, for both downlink and uplink communications over frequency selective channels. The paper is organized as follows. The CDMA transmission model and the block linear receiver are defined in section 2. The proposed algorithm is described in section 3. Complexity issues are discussed in section 4. Performance analysis is given in section 5. Numerical results are given for the TDD (Time Division Duplex) mode of UMTS (Universal Mobile Telecommunication System).

2. CDMA TRANSMISSION MODEL

In a CDMA cellular system, the spreading codes result from the product of a channelization code and a scrambling code. In the TDD mode of UMTS, for example, the spreading codes are periodic and they are furthermore orthogonal [5]. In the FDD (Frequency Division Duplex) mode, the spreading codes have long period, i.e. the signature varies from one symbol to another. Even for orthogonal codes, because of multi-path propagation and of time misalignment between different mobile stations (in the uplink), signals lose their orthogonal property at the reception. This loss of orthogonality depends on the correlation properties of the spreading waveforms and on the propagation conditions as well.

2.1 Transmission model

The received CDMA signal is given by:

$$r(t) = \sum_{k=1}^{K} \beta^{(k)} \sum_{i=1}^{N} u_i^{(k)} \sum_{n=1}^{M_c} c_n^{(k)} p_i^{(k)} (t - iT - \tau_n^{(k)}) + \nu(t)$$

where N is the number of the modulation symbols in the burst, K is the number of active codes (or users), $u_i^{(k)}$ is the i^{th} data symbol of user k, $p_i^{(k)}$ is the spreading waveform of user k during symbol i. M_c is the number of coefficients in the channel impulse response model, $c_n^{(k)}$ and $\tau_n^{(k)}$ are respectively the n^{th} complex coefficient and delay of the channel for user k. $\beta^{(k)}$ includes power amplification, path-loss and shadowing attenuations. The channel is assumed to be invariant during the burst. $\nu(t)$ is the white additive Gaussian noise with one side Power Spectral Density equal to N_0. The spreading waveform has the following expression:

$$p_i^{(k)}(t) = \sum_{n=0}^{Q} v_{iQ+n}^{(k)} g(t - nT_c)$$

where Q is the spreading factor, $v_{iQ+n}^{(k)}$ is the n^{th} chip of the spreading code during symbol i of user k, $g(t)$ is chip waveform, which is usually a square root raised cosine filter.
The received signal can also be written as:

$$r(t) = \sum_{k=1}^{K} \sum_{i=1}^{N} u_i^{(k)} h_i^{(k)} (t - iT) + \nu(t)$$

$h_i^{(k)}(t)$ is the global channel response including the signature and the channel impulse response for user k during symbol i.

2.2 Equivalent discrete-time channel model

At the receiver side, the signal is filtered and sampled at the rate $1/T_e = J/T_c$ (with $J \geq 2$).

$$z(lT_e) = \sum_{k=1}^{K_c} \sum_{i} u_i^{(k)} h_i^{(k)} (lT_e - iT) + w(lT_e)$$

Let Z be the vector of the received signal for the entire block of N symbols, its length is $N_z = JN + L$, where L is the maximum length of the sampled global responses (measured in T_c / J intervals). Using a matrix-vector notation, it can be written as:

$$Z = AU + W$$

W is the sampled filtered noise vector. U is the vector of the data symbols ordered as follows:

$$U = \left[u_1^{(1)} .. u_1^{(K)} \quad u_2^{(1)} .. u_2^{(K)} \quad \ldots \quad u_N^{(1)} . \; u_N^{(K)} \right]^{t}$$

The matrix A is built of N sub-blocks H_i. Each column vector of H_i represents the sampled global channel impulse response of user k as shown below.

$$A = \begin{bmatrix} \cdots & & . \\ h_0^{(1)}[0] \ldots h_0^{(K)}[0] & h_1^{(1)}[-JQ] \ldots h_1^{(K)}[-JQ] & \ldots\ldots\ldots \\ h_0^{(1)}[1] \ldots h_0^{(K)}[1] & & . \\ \cdots & & . \end{bmatrix}$$

2.3 Optimal joint block linear equalizer

The optimal joint block linear equalizer (BLE) makes decisions on the whole data vector Z through a linear transformation given by: $U' = \Lambda Z$ where Λ is a matrix of size NKx N_z, utilising the ZF or MMSE criterion applied to U' – U. These criteria lead a solution given by:

$$U' = M^{-1} Y \text{ , where } Y = A^h Z$$

The matrix M equals $A^h A + \sigma_w^2 I$ for the MMSE criterion and equals $A^h A$ for the ZF criterion. The matrix M is an autocorrelation matrix, having hermitian symmetry and block structure. The Cholesky algorithm can then be used to solve the system MU' = Y.
Due to the finite memory of the autocorrelation of the channel $A^h A$, most of the blocks the matrix M are equal to zero.
For the particular case of periodic codes with period equal to the symbol period, the matrix is furthermore block Toeplitz with $2P$+1 block B_k of size KxK (*cf.* figure 1). Reduced complexity algorithms have been proposed in that case in [2] to solve the whole system.

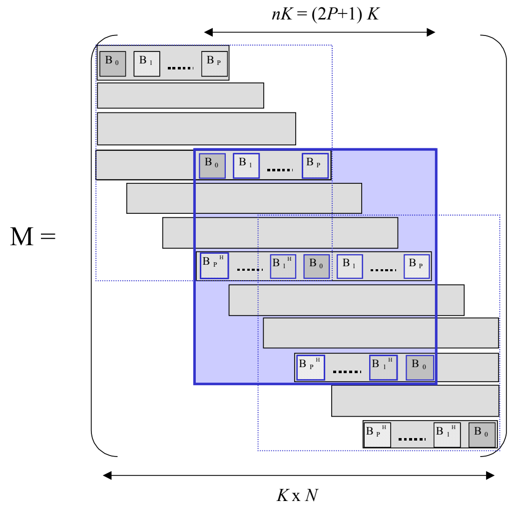

Figure 1: Block Toeplitz structure of the matrix M for symbol-period codes .

3. PROPOSED SUB-OPTIMAL LINEAR EQUALIZER

Due to the finite memory of the autocorrelation of the channel $A^h A$, most of the blocks are null. This structure is put into profit to derive low complexity implementations for the detector. The proposed sub-optimal detector consists in considering smaller block matrices of size nK with n equal to $2P$+1 (*cf.* figure 1). For complexity reasons, the size can be educed toa lower value $n = 2D$+1.
Let M_i and Y_i denote the portion of matrix M and vector Y corresponding to the subsystem involving the finite-memory vector $U_i' = [u_{i-D}'^{(1)} .. u_{i-D}'^{(K)} \ldots u_i'^{(1)} .. u_i'^{(K)} \ldots u_{i+D}'^{(1)} .. u_{i+D}'^{(K)}]^t$ with $n = 2D + 1$:

$$U'_i = (M_i)^{-1} Y_i \qquad (1)$$

In order to avoid border effects, the decisions are made only on the i^{th} centred data symbols of the K users $U'_{i,t} = [u_i'^{(1)}..u_i'^{(K)}]$, which leads to the truncated solution:

$$U'_{i,t} = [\,M_i^{-1}\,]_t\, Y_i \qquad (2)$$

$[M_i^{-1}]_t$ denote the K central lines of the matrix M_i^{-1}. Therefore, there is an overlapping between subsystems, and the size of the truncated matrix is equal to KxnK For a channel length inferior to one symbol duration (P = 1), the matrix M_i is a matrix of size $3K$x$3K$ only. It should also be pointed out that for transmission with periodic codes with period equal to the symbol duration, the matrix M_i is always the same for each symbol.

4. COMPLEXITY EVALUATION

Assuming that the channel impulse responses are estimated with the help of training sequences, the blocks of the matrix A^hA can be computed.

Then the size of subsystems n is chosen so as to obtain the best trade-off between complexity and performance. Typically, n equals ($2P$+1). However, n can be chosen lower than $2P$+1 if required by complexity constraints.

Note that for most channels of practical interest for the TDD mode of UMTS, P equals 1 with a spreading factor of 16, except for the vehicular B channel model, where P equals 5.

4.1 Subsystem resolution with block–Levinson algorithm for periodic codes

The complexity of the proposed receiver can be very low for periodic codes with period equal to the symbol period. This property is realized in practice for the TDD mode of UMTS in the downlink with spreading factor equal to 16. The matrices M_i are all identical, and they are moreover block Toeplitz. In that particular case, it becomes interesting to invert the matrix once per burst and then to compute the N sample vectors $U'_{i,t}$ according to equation (2). This solution is computationally efficient because the computation of the matrix inversion M_i^{-1} can be done by means of the block-Levinson algorithm having a computation complexity order given by $O(n^2K^3)$ [6]. Then, the computation of decision symbols generates K^2nN multiplications and $K^2(n-1)N$ additions.

4.2 Subsystem resolution with Cholesky algorithm

When the matrix is not block Toeplitz, the inversion of the matrix is more consuming than the Cholesky algorithm. The algorithm is two steps: decomposition of the matrix M_i into a product $L_iL_i^h$ where L_i is a low triangular matrix and iterative resolution of two successive systems $L_iX'_i=Y_i$ and $L_i^hU'_i=X_i$.

The first step generates a computational complexity which is proportional to $nK(nK+1)(nK-1)/6 \approx n^3K^3/6$.

For a periodic code with period equal to mQ, this decomposition is made m/Q times per burst. For random-like codes, it is realised N times since the equivalent channel is varying from one symbol to another.

The second step leads to $(N-2D)(DK+1)(3DK+2)$ operations for n equal to $2D$+1.

4.3 Numerical results

Some numerical results are here given to illustrate complexity issues of the proposed algorithm.

A first example is given for periodic codes: Q = 16, m = 1, K = 8, N = 61.

For small channel memory (P = 1) and therefore for small subsystems (n = 3) both proposed algorithms are equivalent in terms of complexity. The algorithm described in 4.1 leads to 408 + 11 712 = 16 120 computations (MAC, multiplications, additions and comparisons). The algorithm described in 4.2 gives 2 300 + 13 806 = 16 306 computations.

For long channel memory, the first proposed algorithm becomes much more efficient than the second one. For P equal to 4 and n equal to 9, the complexity of the first algorithm leads to 41 472 + 35 136 = 76 608 computations instead of 62 196 + 172 402 = 233 598 for the second one.

A second numerical example is given for varying spreading factors between 4 and 16, the codes are periodic with a period equal to 16 chip periods (like in the uplink of TDD UMTS mode), and for different channel memory lengths given in chip periods pT_c. Table I gives the complexity for a constant throughput given by NK = 488 data symbols per burst. It is worthwhile to note that computation limitation may influence the choice of the spreading factor according to the channel length (*cf.* Table I).

Q	K	N	p	P	n	1st step	2nd step	total
16	8	61	57	4	9	62196	171402	233598
8	4	122	57	8	17	104788	342804	447592
4	2	244	57	15	31	158844	610328	769172
16	8	61	16	1	3	2300	13806	16106
8	4	122	16	2	5	2660	27612	30272
4	2	244	16	4	9	3876	55224	59100
16	8	61	8	1	3	2300	13806	16106
8	4	122	8	1	3	572	8400	8972
4	2	244	8	2	5	660	16800	17460
16	8	61	4	1	3	2300	13806	16106
8	4	122	4	1	3	572	8400	8972
4	2	244	4	1	3	140	5808	5948

Table I. Complexity evaluation of the 4.2 algorithm for different channel lengths and spreading factors with a fixed throughput.

5. PERFORMANCE ANALYSIS

5.1 Performance evaluation

Decisions are made on the sample vectors $U'_{i,t}$ which have the following expressions:

$$U'_{i,t} = [\,M_i^{-1}]_t\, A_i^h\, AU + [\,M_i^{-1}]_t\, A_i^h\, W$$

We can easily derive the signal to noise ratio at the output of the receiver on the decision samples, which can be expressed by:

$$SNR_{i,k} = \frac{1}{\lambda_{i,k}} \left(\frac{P_{i,k}}{\frac{N_0}{T} + \sigma_{i,k}^2} \right)$$

where $P_{i,k}$ denotes the received power of user k for symbol i, $\sigma^2_{i,k}$ is the variance of the residual multiple access and inter-symbol interference, $\lambda_{i,k}$ represents the noise amplification at the out put of the receiver. They can be expressed a function of the matrices A^hA, M_i^{-1} and A_i^h. The bit-error-rate evaluation is approximated, for QPSK modulation, by:

$$BER_{i,k} \approx Q\left(\sqrt{SNR_{i,k}}\right)$$

5.2 Performance results

In this subsection, we present some performance results. The evaluation has been carried out with UMTS TDD transmission parameters [5]: the chip rate is equal to 3,84 Mchip/s and the roll-off of the chip waveform is equal to 0,22, the spreading factor is 16.

When n is taken equal to $2P$+1, the performance of the proposed algorithm is very close to the performance of the ideal block linear detector for any number of users and for any channel profile.

In order to manage complexity issues, the size of subsystems can be reduced to a fixed arbitrary value n lower than $2P$+1. Figures 2 and 3 evaluate the degradation due to truncation when the channel memory is long (P = 5). It is worthwhile to note that the degradation increases with the number of codes when n is smaller than $2P$+1.

Figure 2 shows the performance of the proposed algorithm with the lowest value for n with 8 users. The obtained performance is compared to the ideal joint detector performance and to the RAKE receiver performance. The channel impulse response is constant with amplitude in dB equal to −2,5 0 −12,8 −10 −25,2 and delays expressed in nanosecond given by 0 300 8900 12900 17100.

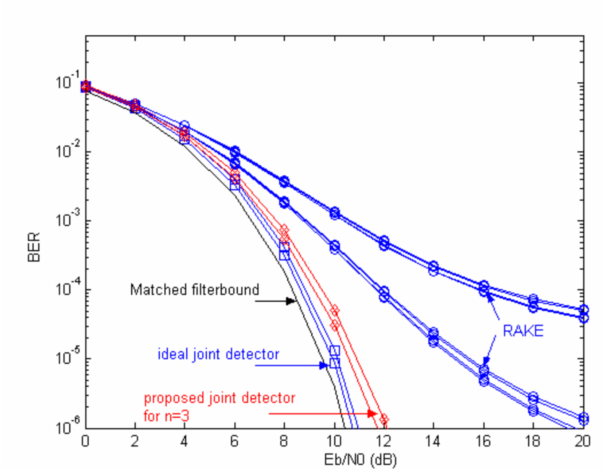

Figure 2. Comparison of the truncated detector with other receivers on a fixed channel: Q=16, K=8, P=5, n=3.

Figure 3 evaluates the effect of truncation over a random channel for the downlink (P = 5). It gives the mean bit error rate (averaged over users, symbols and channel responses) as a function of the mean energy per bit E_b over N_0. This shows the trade-off between complexity and performance.

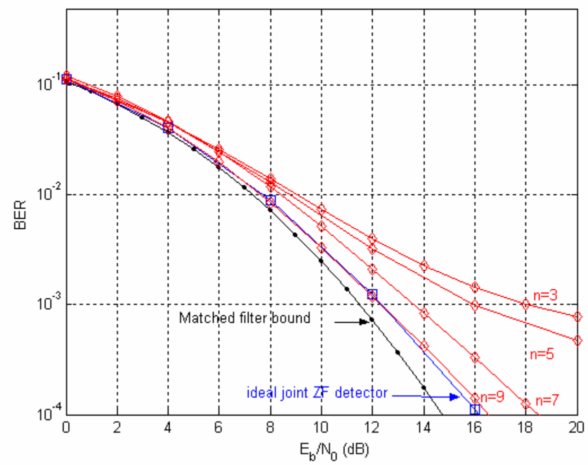

Figure 3. Influence of the truncation order n over the random ITU Vehicular B channel: K=8, Q=16, P = 5.

6. CONCLUSION

A low complexity joint detector has been proposed and performance analysis shows that it can be a good candidate for CDMA communications using either short periodic codes or long codes. Its complexity can be constrained for long memory channels, and the performance degradation is not severe, which gives an alternate solution between the ideal block linear detector and the rake receiver.

ACKNOWLEDGMENT

This research work was carried out in the framework of the French RNRT project PETRUS in partnership with Mitsubishi Electric ITE, Bouygues Telecom, Supelec, CEA-LETI and ENST-Bretagne.

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