

# 3D PASSIVE LOCALIZATION IN THE PRESENCE OF LARGE BEARING NOISE

*Kutluyıl Doğançay and Gokhan Ibal*

School of Electrical & Information Engineering  
University of South Australia  
Mawson Lakes, SA 5095, Australia  
Email: Kutluyil.Dogancay@unisa.edu.au

Air Operations Division  
Defence Science & Technology Organisation  
Fishermans Bend, VIC 3207, Australia  
Email: Gokhan.Ibal@dsto.defence.gov.au

## ABSTRACT

This paper derives three-dimensional passive bearings-only localization algorithms and examines their performance when the sensor measurements are corrupted by large additive noise. Among the algorithms studied, the maximum likelihood (ML) estimator is shown to have the best localization performance. The ML estimate is computed using the iterative Gauss-Newton (GN) algorithm with the initial guess obtained from a pseudolinear estimator. Bearing measurements are averaged over finite-length non-overlapping windows in order to reduce the computational complexity of the GN algorithm when the number of bearing measurements is large. Simulation studies are provided to illustrate the superior performance of the ML estimator in a radar localization application.

## 1. INTRODUCTION

Bearings-only emitter localization is a passive localization technique that employs bearing (direction-of-arrival) measurements of received signals originating from an emitter. In bearings-only localization, the emitter location is obtained from the point of intersection of bearing lines emanating from different observer positions. This estimation process, which is referred to as triangulation, yields a unique intersection point for bearing lines in the absence of measurement errors. However, the noise present in bearing and observer position measurements necessitates the formulation of an optimal solution in a statistical framework by making use of the noisy measurements, as well as any *a priori* information about the noise statistics.

Passive emitter localization has been an active research area for several decades. The pioneering work in this area is that of Stansfield [1]. Most of the current emitter localization algorithms are based on Stansfield's algorithm. The Stansfield estimator is a weighted least squares (WLS) estimator that can be viewed as a small bearing noise approximation of the maximum likelihood (ML) estimator for independent Gaussian bearing noise and no observer position error [2]. It also assumes the prior knowledge of the emitter range from the observer positions to calculate a weighting matrix. This strong assumption can be dispensed with by resorting to the method of orthogonal vectors which results in a pseudolinear estimator [3]. The passive emitter localization problem can be recast as a nonlinear LS problem by using the ML solution. Linearization of the nonlinear LS

problem was proposed in [4] by way of Taylor series expansion, resulting in an iterative estimation algorithm.

Despite its simplicity and low complexity, a major drawback of the pseudolinear estimator is the large estimation bias due to the correlation between the measurement matrix and the bearing noise. The bias of the pseudolinear and Stansfield estimators has been studied in the tracking and emitter localization literature (see e.g. [5, 3, 2, 6]). To overcome the bias of the pseudolinear estimator, a modified instrumental variable (MIV) estimation algorithm was proposed in [3]. The MIV estimator is a batch iterative algorithm that is obtained from the Gauss-Newton iterations for the ML estimator by a linear approximation of the estimation error. In [7], an iterative instrumental variable (IV) estimator was developed for emitter tracking. This estimator uses the predictions of the present parameters from past measurements to achieve uncorrelation between the measurements and the present bearing noise under the assumption of independent bearing noise. Neither of these IV algorithms have a closed-form solution because they both rely on an iterative calculation of the instrumental variable matrix. The convergence of iterative estimators is known to be sensitive to initialization and the stepsize parameter [8].

In the work presented here we report three-dimensional (3D) extensions of the pseudolinear and ML estimators for 3D geolocation of emitters when the sensor noise is large. First a 3D orthogonal vector estimator is proposed. Then, a novel 3D PLE is developed based on 2D projection and separate estimation of the  $z$ -coordinate of the emitter location. Finally the optimal ML estimator is derived. While the first two estimators are closed-form, the ML estimator requires iterative numerical search. The estimation performance of the algorithms is demonstrated by way of computer simulations.

## 2. 3D LOCALIZATION USING BEARING MEASUREMENTS

A 3D localization scenario for an emitter at  $\mathbf{p} = [p_x, p_y, p_z]^T$  in Cartesian coordinates is depicted in Fig. 1 where a moving receiver collects bearing measurements  $(\theta_k, \phi_k)$  at locations  $\mathbf{r}_k = [r_x(k), r_y(k), r_z(k)]^T$ . Each bearing measurement consists of an azimuth angle  $\theta_k$  and an elevation angle  $\phi_k$  in spherical coordinates.

The relationship between  $\mathbf{p}$  and  $\mathbf{r}_k$  is given by

$$\mathbf{p} = \mathbf{r}_k + \mathbf{s}_k \quad (1)$$

where  $\mathbf{s}_k$  is the noise-free bearing vector connecting

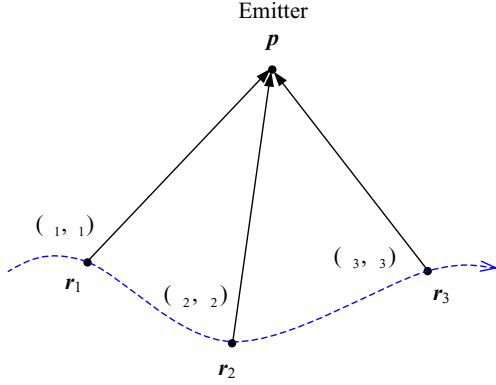


Figure 1: Geometry for 3D bearings-only localization.

$\mathbf{r}_k$  to  $\mathbf{p}$ . Premultiplying (1) with  $\mathbf{a}_k^T$  that satisfies (i)  $\mathbf{a}_k^T \mathbf{s}_k = 0$  (i.e.,  $\mathbf{a}_k$  is orthogonal to  $\mathbf{s}_k$ ) and (ii)  $\|\mathbf{a}_k\| = 1$  gives

$$\mathbf{a}_k^T \mathbf{p} = \mathbf{a}_k^T \mathbf{r}_k \quad (2)$$

where knowledge of  $\mathbf{s}_k$  is no longer necessary. The vector  $\mathbf{s}_k$  can be written as

$$\mathbf{s}_k = \|\mathbf{s}_k\| \begin{bmatrix} \cos \phi_k \cos \theta_k \\ \cos \phi_k \sin \theta_k \\ \sin \phi_k \end{bmatrix}. \quad (3)$$

Increasing  $\phi_k$  by  $\pi/2$  radians in the normalized noise-free bearing vector  $\mathbf{s}_k/\|\mathbf{s}_k\|$  gives

$$\mathbf{a}_k = \begin{bmatrix} \cos(\phi_k + \pi/2) \cos \theta_k \\ \cos(\phi_k + \pi/2) \sin \theta_k \\ \sin(\phi_k + \pi/2) \end{bmatrix} = \begin{bmatrix} -\sin \phi_k \cos \theta_k \\ -\sin \phi_k \sin \theta_k \\ \cos \phi_k \end{bmatrix} \quad (4)$$

which satisfies the aforementioned orthogonal unit vector requirements.

To calculate the bearing angles  $(\theta_k, \phi_k)$  from  $\mathbf{s}_k = [s_x(k), s_y(k), s_z(k)]^T$ , we use

$$\begin{aligned} \theta_k &= \tan^{-1} \frac{s_y(k)}{s_x(k)} \\ \phi_k &= \sin^{-1} \frac{s_z(k)}{\|\mathbf{s}_k\|}. \end{aligned} \quad (5)$$

In practice, the noise-corrupted versions of the bearing angles  $(\tilde{\theta}_k, \tilde{\phi}_k)$  are available for locating the emitter:

$$\begin{aligned} \tilde{\theta}_k &= \theta_k + w_k \\ \tilde{\phi}_k &= \phi_k + n_k \end{aligned} \quad (6)$$

where  $w_k, n_k \sim \mathcal{N}(0, \sigma^2)$ , i.e., white Gaussian noise with zero mean and variance  $\sigma^2$ .

### 3. 3D LOCATION ESTIMATORS

#### 3.1 Orthogonal Vector Estimator

Define the orthogonal unit vectors for noisy bearing angles:

$$\tilde{\mathbf{a}}_k = \begin{bmatrix} -\sin \tilde{\phi}_k \cos \tilde{\theta}_k \\ -\sin \tilde{\phi}_k \sin \tilde{\theta}_k \\ \cos \tilde{\phi}_k \end{bmatrix}. \quad (7)$$

Then for  $N$  consecutive bearing measurements we can write

$$\underbrace{\begin{bmatrix} \tilde{\mathbf{a}}_1^T \\ \vdots \\ \tilde{\mathbf{a}}_N^T \end{bmatrix}}_{\mathbf{A}} \mathbf{p} = \underbrace{\begin{bmatrix} \tilde{\mathbf{a}}_1^T \mathbf{r}_1 \\ \vdots \\ \tilde{\mathbf{a}}_N^T \mathbf{r}_N \end{bmatrix}}_{\mathbf{b}} + \underbrace{\begin{bmatrix} \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}}_{\boldsymbol{\eta}} \quad (8)$$

where the  $\eta_k$  are nonlinear functions of the bearing noise  $w_k$  and  $n_k$ :

$$\begin{aligned} \eta_k &= \frac{\|\mathbf{s}_k\|}{4} (\sin(w_k - n_k) - \sin(n_k + w_k) \\ &\quad - \sin(2\phi_k + n_k + w_k) - \sin(2\phi_k + n_k - w_k)) \\ &\quad + 2 \sin(2\phi_k + n_k) - 2 \sin n_k. \end{aligned} \quad (9)$$

A least squares (LS) solution of  $\mathbf{A}\mathbf{p} \approx \mathbf{b}$  is given by

$$\hat{\mathbf{p}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b} \quad (10)$$

which is the closed-form 3D orthogonal vector estimator (OVE). In the 2D case the OVE is identical to the well-known pseudolinear estimator [3]. Because  $\mathbf{A}$  and  $\boldsymbol{\eta}$  are correlated, the OVE exhibits bias [6]. The bias problem becomes worse with increased bearing noise variance and increased range-to-baseline ratio.

#### 3.2 3D Pseudolinear Estimator

The projection of the bearing lines onto the  $xy$ -plane reduces the original 3D localization problem to a 2D localization problem. The 2D localization problem can be solved by the 2D pseudolinear estimator (PLE) [5, 6]. The  $z$ -coordinate of the emitter location can be subsequently obtained from (5). We will refer to this location estimator as the closed-form 3D PLE.

#### 3.3 ML Estimator

The optimum maximum likelihood (ML) location estimator is obtained from maximization of the joint probability density function of the bearing measurements  $(\tilde{\theta}_k, \tilde{\phi}_k)$ ,  $k = 1, \dots, N$ . The ML cost function to be minimized is given by

$$J_{\text{ML}}(\mathbf{p}) = \mathbf{e}^T(\mathbf{p}) \mathbf{K}^{-1} \mathbf{e}(\mathbf{p}) \quad (11)$$

where  $\mathbf{K} = \sigma^2 \mathbf{I}$  is the  $2N \times 2N$  covariance matrix of the bearing noise and  $\mathbf{e}(\mathbf{p})$  is the  $2N \times 1$  error vector

$$\mathbf{e}(\mathbf{p}) = [\tilde{\theta}_1 - \theta_1(\mathbf{p}), \dots, \tilde{\theta}_N - \theta_N(\mathbf{p}), \tilde{\phi}_1 - \phi_1(\mathbf{p}), \dots, \tilde{\phi}_N - \phi_N(\mathbf{p})]^T \quad (12)$$

with  $\theta_k(\mathbf{p})$  and  $\phi_k(\mathbf{p})$  denoting the azimuth and elevation angle of  $\mathbf{p} - \mathbf{r}_k$ , respectively.

The ML estimate (MLE) is

$$\hat{\mathbf{p}}_{\text{ML}} = \arg \min_{\mathbf{p} \in \mathbb{R}^3} J_{\text{ML}}(\mathbf{p}). \quad (13)$$

The MLE does not have a closed-form solution and requires the use of a numerical search algorithm.

The Gauss-Newton (GN) algorithm, which is a batch iterative minimization technique, is often employed to calculate the MLE. The GN algorithm consists of

$$\hat{\mathbf{p}}_{i+1} = \hat{\mathbf{p}}_i - (\mathbf{J}_i^T \mathbf{K}^{-1} \mathbf{J}_i)^{-1} \mathbf{J}_i^T \mathbf{K}^{-1} \mathbf{e}(\hat{\mathbf{p}}_i), \quad i = 0, 1, \dots \quad (14)$$

where  $\mathbf{J}_i$  is the  $2N \times 3$  Jacobian of  $\mathbf{e}(\mathbf{p})$  with respect to  $\mathbf{p}$  evaluated at  $\mathbf{p} = \hat{\mathbf{p}}_i$ :

$$\mathbf{J}_i = \begin{bmatrix} \frac{\sin \theta_1(\hat{\mathbf{p}}_i)}{\hat{d}_{i1}} & \frac{-\cos \theta_1(\hat{\mathbf{p}}_i)}{\hat{d}_{i1}} & 0 \\ \vdots & \vdots & \vdots \\ \frac{\sin \theta_N(\hat{\mathbf{p}}_i)}{\hat{d}_{iN}} & \frac{-\cos \theta_N(\hat{\mathbf{p}}_i)}{\hat{d}_{iN}} & 0 \\ \frac{sc_1(\hat{\mathbf{p}}_i)}{\|\hat{\mathbf{p}}_i - \mathbf{r}_1\|} & \frac{ss_1(\hat{\mathbf{p}}_i)}{\|\hat{\mathbf{p}}_i - \mathbf{r}_1\|} & \frac{-\cos^2 \phi_1(\hat{\mathbf{p}}_i)}{\hat{d}_{i1}} \\ \vdots & \vdots & \vdots \\ \frac{sc_N(\hat{\mathbf{p}}_i)}{\|\hat{\mathbf{p}}_i - \mathbf{r}_N\|} & \frac{ss_N(\hat{\mathbf{p}}_i)}{\|\hat{\mathbf{p}}_i - \mathbf{r}_N\|} & \frac{-\cos^2 \phi_N(\hat{\mathbf{p}}_i)}{\hat{d}_{iN}} \end{bmatrix}.$$

In the above expression

$$\begin{aligned} \hat{d}_{ik} &= \|\hat{\mathbf{p}}_i(1:2) - \mathbf{r}_k(1:2)\| \\ sc_k(\mathbf{x}) &\triangleq \sin \phi_k(\mathbf{x}) \cos \theta_k(\mathbf{x}) \\ ss_k(\mathbf{x}) &\triangleq \sin \phi_k(\mathbf{x}) \sin \theta_k(\mathbf{x}) \end{aligned}$$

and  $\hat{\mathbf{p}}_i(1:2)$  is a  $2 \times 1$  vector containing the first two entries of  $\hat{\mathbf{p}}_i$ .

The initialization for the GN algorithm  $\hat{\mathbf{p}}_0$  must be chosen sufficiently close to the final estimate in order to avoid divergence. This usually requires the use of another closed-form estimator such as the 3D PLE. An alternative approach is to select a point on one of the measured bearing lines, say, the first one given by  $(\tilde{\theta}_1, \tilde{\phi}_1)$ , a certain distance away from the relevant receiver, and use it as an initial guess:

$$\hat{\mathbf{p}}_0 = \mathbf{r}_1 + d_0 \begin{bmatrix} \cos \tilde{\phi}_1 \cos \tilde{\theta}_1 \\ \cos \tilde{\phi}_1 \sin \tilde{\theta}_1 \\ \sin \tilde{\phi}_1 \end{bmatrix}. \quad (15)$$

Here  $d_0$  is the distance of  $\hat{\mathbf{p}}_0$  from  $\mathbf{r}_1$  and must not be too large. The GN iterations are stopped once the update term becomes sufficiently small, i.e.,

$$\|\hat{\mathbf{p}}_{i+1} - \hat{\mathbf{p}}_i\|^2 < \gamma^2 \quad (16)$$

where  $\gamma$  is a threshold.

#### 4. AVERAGING OF BEARING MEASUREMENTS

In certain applications the sensor on the moving receiver may produce very crude bearing angle estimates with large noise. In such cases the location estimators will require a large number of bearing measurements to reduce the estimation variance. The computational complexity of the estimators increases with  $N$ . In particular, the complexity of the GN algorithm can become prohibitively large for large  $N$ . Therefore, the selection of  $N$  often involves a trade-off between complexity and estimation performance.

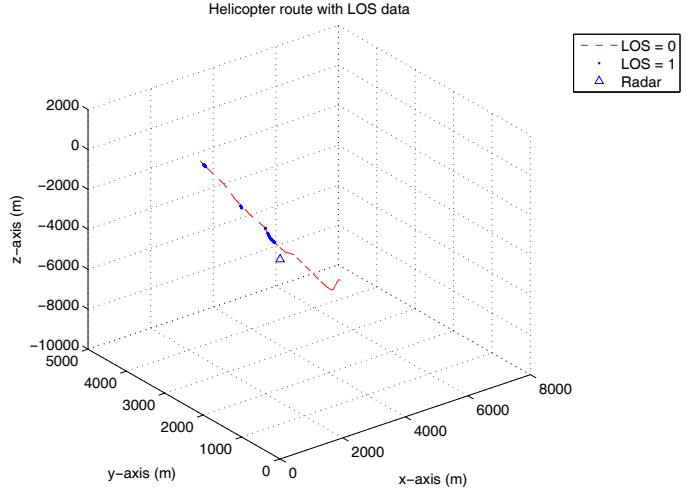


Figure 2: Simulated 3D localization geometry with line-of-sight information in local coordinates. The radar is at the origin.

For a given  $N$ , the complexity can be reduced significantly by means of averaging with negligible performance degradation. The assumption made here is that the difference between bearing angles for a range of bearing measurements is very small. This means that for  $L$  successive bearing measurements we have

$$\begin{aligned} \theta_k &\approx \theta_{k+1} \approx \dots \approx \theta_{k+L-1} \\ \phi_k &\approx \phi_{k+1} \approx \dots \approx \phi_{k+L-1}. \end{aligned} \quad (17)$$

Segmenting  $N$  bearing measurements into non-overlapping blocks of length  $L$  and averaging each block gives

$$\begin{aligned} \bar{\theta}_i &= \frac{1}{L} \sum_{k=(i-1)L+1}^{iL} \tilde{\theta}_k \\ \bar{\phi}_i &= \frac{1}{L} \sum_{k=(i-1)L+1}^{iL} \tilde{\phi}_k \end{aligned} \quad (18)$$

where  $i = 1, 2, \dots, N/L$ . This averaging has the effect of reducing the variance  $\sigma^2$  by a factor  $L$ , i.e.,

$$E\{\bar{\theta}_i^2\} = E\{\bar{\phi}_i^2\} = \frac{\sigma^2}{L}. \quad (19)$$

It also reduces the number of measurements by a factor  $L$ , thereby easing the computational burden on the localization algorithms.

#### 5. COMPARATIVE SIMULATIONS

In this section we evaluate the performance of the 3D location estimators for a simulated helicopter route and line-of-sight information between the helicopter and the radar. The emitter is assumed to be a stationary pulsed radar with pulse repetition interval (PRI) of 1 ms. The bearing angles are measured by a radar warning receiver (RWR) on-board the helicopter. The RWR

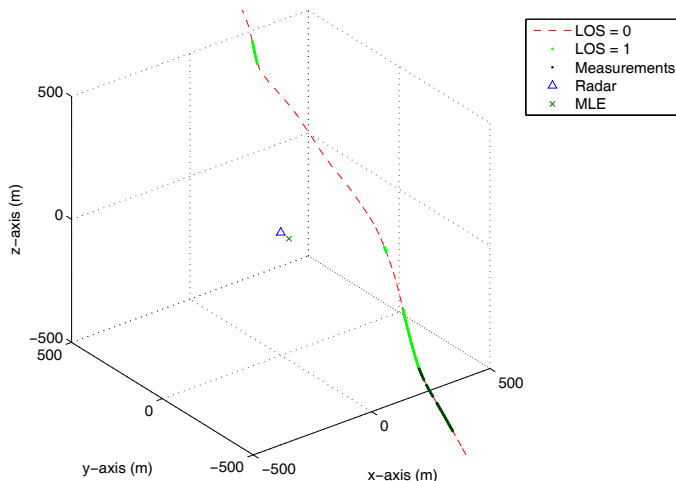


Figure 3: ML estimate of the radar location.

$T$ (s)	Bias Norm (m)		
	MLE	PLE	OVE
5	291	3332	3285
6	104	3004	3124
7	61	2685	2995
8	31	2399	2943
9	25	2147	2899
10	18	1973	2879
15	6	1524	2606
20	2	1310	2285

Table 1: Estimated bias.

produces very crude bearing measurements of received radar pulses with root mean square error (RMSE) of  $7^\circ$ . The average speed of the helicopter is approx. 30 m/s. The bearing measurements are averaged using  $L = 100$ , which means that the RMSE of averaged bearing measurements ( $\hat{\theta}_k, \hat{\phi}_k$ ) is  $0.7^\circ$ . The time separation between averaged bearing measurements is  $L \times \text{PRI} = 100$  ms.

The simulated geometry in local Cartesian coordinates is shown in Fig. 2. The range-to-baseline ratio is very large for parts of the helicopter route with line of sight (LOS=1). This makes the localization task quite challenging. Fig. 3 shows the MLE for one realization of the bearing angle measurements over an observation period of 10 s. The part of the helicopter route where angle measurements were used for localization purposes is also marked in Fig. 3 along with line-of-sight data. The GN initial guess was obtained from the 3D PLE. The stopping threshold was set to  $\gamma = 10^{-5}$ . The bias and RMSE estimates of the localization algorithms obtained from 5,000 Monte Carlo simulations are listed in Tables 1 and 2 for several observation periods  $T$ . The MLE has by far the best localization performance. The severe bias problem with the PLE and OVE is evident. The PLE appears to perform better than the OVE especially for large baselines.

## 6. CONCLUSION

Three bearings-only localization algorithms were developed for 3D emitter location. An averaging approach

$T$ (s)	RMSE (m)		
	MLE	PLE	OVE
5	1321	3336	3285
6	688	3009	3124
7	491	2690	3031
8	365	2404	2978
9	289	2152	2943
10	245	1978	2907
15	137	1527	2648
20	93	1313	2297

Table 2: Estimated RMSE.

was adopted to reduce the computational complexity of the estimation algorithms. By way of simulation examples the PLE and OVE were shown to suffer from severe bias problems in the presence of large bearing noise and in large range-to-baseline ratio situations. The PLE was observed to perform better than the OVE, thereby making it a good candidate for initializing the GN algorithm. The MLE implemented using the GN algorithm was shown to have the best location performance.

## REFERENCES

- [1] R. G. Stansfield, "Statistical theory of DF fixing," *Journal of IEE*, vol. 94, no. 15, pp. 762–770, December 1947.
- [2] M. Gavish and A. J. Weiss, "Performance analysis of bearing-only target location algorithms," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 28, no. 3, pp. 817–828, 1992.
- [3] S. C. Nardone, A. G. Lindgren, and K. F. Gong, "Fundamental properties and performance of conventional bearings-only target motion analysis," *IEEE Trans. on Automatic Control*, vol. 29, no. 9, pp. 775–787, September 1984.
- [4] W. H. Foy, "Position-location solutions by Taylor-series estimation," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 12, no. 2, pp. 187–194, March 1976.
- [5] V. J. Aidala and S. C. Nardone, "Biased estimation properties of the pseudo-linear tracking filter," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 18, pp. 432–441, July 1982.
- [6] K. Doğançay, "On the bias of linear least squares algorithms for passive target localization," *Signal Processing*, vol. 84, no. 3, pp. 475–486, March 2004.
- [7] Y. T. Chan and S. W. Rudnicki, "Bearings-only and Doppler-bearing tracking using instrumental variables," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 28, no. 4, pp. 1076–1083, October 1992.
- [8] J.-P. Le Cadre and C. Jauffret, "On the convergence of iterative methods for bearings-only tracking," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 35, no. 3, pp. 801–818, July 1999.