RECENT ADVANCES IN PARTIAL UPDATE AND SPARSE ADAPTIVE FILTERS

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ABSTRACT

This paper presents an overview of recent advances in partial update and sparse adaptive filters. Following an introduction to the notion of partial updating, several application examples are presented in the areas of echo cancellation, blind equalization and multiuser detection. These examples demonstrate the capability of partial-update algorithms to maintain a performance close to their computationally more demanding full-update counterparts. The paper also discusses recently proposed techniques for sparse system identification, as well as the concept of joint partial and sparse updating for adaptive identification of sparse systems.

1. INTRODUCTION

Identification of sparse and/or long discrete-time systems has always been a challenging problem. In many applications, including acoustic/network echo cancellation and channel equalization, the system to be identified can be characterized as sparse and/or long. Partial update adaptive filtering algorithms were proposed to address the large computational complexity associated with long adaptive filters. However, the initial partial update algorithms had to incur performance losses, such as slow convergence, compared with full-update algorithms because of the absence of clever updating approaches. More recently, better partial update techniques have been developed that are capable of minimizing the performance loss. In certain applications, these partial update techniques have even been observed to produce improved convergence performance with respect to a full-update algorithm. The potential performance gain that can be achieved by partial-update algorithms is an important feature that was not recognized earlier.

Sparse system identification is a vital requirement for fast converging adaptive filters in, for example, certain specific deployments of echo cancellation. Recent advances based on proportionate update schemes have been used to good effect in network echo cancellation for VoIP gateways (to take account of unpredictable bulk delays in IP network propagation), and acoustic echo cancellation (to handle the unknown propagation delay of the direct acoustic path). Several new techniques are emerging in the current literature. This paper aims to summarize the recent advances in partial-update and sparse adaptive filtering algorithms. Within the limits of the available space, some key algorithms and their applications are reviewed.

2. PARTIAL UPDATE ADAPTIVE FILTERS

In adaptive filtering algorithms the complexity increases with the number of coefficients that have to be updated every time a new input sample comes in. The complexity of adaptation is mainly due to multiplications in the update process. A straightforward approach to complexity reduction is to update a small number of filter coefficients rather than the entire filter at every iteration. Usually a compromise has to be made between affordable complexity and desired convergence speed.

2.1 Time-Domain Selective Partial Update Adaptive Filters

Consider a stochastic-gradient-descent adaptive filtering algorithm

$$\underbrace{\begin{bmatrix} \boldsymbol{h}_1(n+1) \\ \vdots \\ \boldsymbol{h}_P(n+1) \end{bmatrix}}_{\boldsymbol{h}(n+1)} = \underbrace{\begin{bmatrix} \boldsymbol{h}_1(n) \\ \vdots \\ \boldsymbol{h}_P(n) \end{bmatrix}}_{\boldsymbol{h}(n)} + \mu e(n) \underbrace{\begin{bmatrix} \boldsymbol{x}_1(n) \\ \vdots \\ \boldsymbol{x}_P(n) \end{bmatrix}}_{\boldsymbol{x}(n)} \quad (1)$$

where $\boldsymbol{h}(n)$ is the $N \times 1$ filter coefficient vector at time n, μ is the stepsize, $e(n)$ is the error signal, and $\boldsymbol{x}(n)$ is the $N \times 1$ filter input regressor vector. In partial updating, the adaptation algorithm (1) is replaced by $\boldsymbol{h}_i(n+1) = \boldsymbol{h}_i(n) + \mu e(n)\boldsymbol{x}_i(n), \quad i \in \{1, \ldots, P\}$ where i needs to be selected at every iteration n. The chief advantage of partial updating is reduced complexity. Its main disadvantage is the potential reduction in convergence speed, the extent of which depends on how i is chosen.

Several partial update algorithms have been proposed in the literature. A summary of the key algorithms is provided below:

- **Max-NLMS [1]:** The NLMS algorithm derived from a constrained optimization problem by using the ℓ_∞ norm rather than ℓ_2 in the optimization criterion.
- **Sequential and periodic partial-update LMS [2]:** No selection criterion is used; the block index i is changed sequentially or periodically at every iteration.
- **M-max NLMS [3]:** A selection criterion is obtained from the minimization of a modified *a posteriori* error expression. Filter coefficients corresponding to the filter inputs with the largest squared ℓ_2 norm are updated.
- **Selective-block-update NLMS [4]:** Extension of the M-max NLMS algorithm to coefficient blocks to reduce memory requirements.
- **Selective-partial-update NLMS [5]:** A selection criterion is obtained from the solution of a constrained optimization problem. The resulting algorithm has a different update term to the M-max NLMS algorithm, but uses the same selection criterion.
- **Data-selective partial-update NLMS [6]:** Set-membership filtering fused with selective partial updating.

2.2 Transform-Domain Selective Partial Update LMS

The selective-partial-update transform-domain-LMS (SPU-TD-LMS) algorithm updating B coefficient blocks out of P at every iteration is given by [7]

$$\boldsymbol{w}_{\mathcal{I}}(n+1) = \boldsymbol{w}_{\mathcal{I}}(n) + \mu e(n) \boldsymbol{\Lambda}_{\mathcal{I}}^{-2} \boldsymbol{v}_{\mathcal{I}}^*(n),$$
$$\mathcal{I} = \arg\max_{\mathcal{J}} \boldsymbol{v}_{\mathcal{J}}^H(n) \boldsymbol{\Lambda}_{\mathcal{J}}^{-2} \boldsymbol{v}_{\mathcal{J}}(n). \quad (2)$$

The selection criterion that yields $\mathcal{I}$ where $\mathcal{I} = \{i_1, i_2, \ldots, i_B\}$ is a B-subset (subset with B members) of the set $\{1, 2, \ldots, P\}$ ensures that the best convergence performance is achieved under the constraint of partial updating. In (2), the coefficient vector $\boldsymbol{w}(n)$, the power matrix $\boldsymbol{\Lambda}^2$ and the transformed regressor vector $\boldsymbol{v}(n) = \boldsymbol{T}\boldsymbol{x}(n)$ are partitioned into P blocks with $\boldsymbol{w}_{\mathcal{I}}(n)$, $\boldsymbol{\Lambda}_{\mathcal{I}}^2$ and

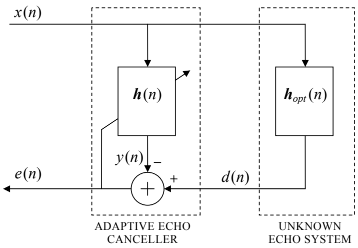

Figure 1: Adaptive echo cancellation structure.

$\boldsymbol{v}_{\mathcal{I}}(n)$ augmented accordingly. Complexity reduction for generalized subband decomposition adaptive filters is also possible by way of selective partial updating [7].

3. APPLICATIONS

3.1 Acoustic Echo Cancellation

In acoustic echo cancellation problems the acoustic echo path is often modelled as an FIR filter $d(n) = \boldsymbol{h}_{\text{opt}}^T(n)\boldsymbol{x}(n) + \nu(n)$ (see Fig. 1) where ^{T} denotes matrix transpose, $d(n)$ is the echo signal, $\boldsymbol{h}_{\text{opt}}(n)$ is the acoustic echo path, $\boldsymbol{x}(n)$ is the loudspeaker signal and $\nu(n)$ is the additive noise (not shown in the figure). In the echo cancellation examples presented in this section, $x(n)$ is a signal with speech-like spectrum and the acoustic echo path is a measured car echo impulse response of length 256. The signal-to-noise ratio of the echo signal is 30 dB.

The TD-LMS was implemented using a 225-point discrete cosine transform ($N = 225$). The selective partial update parameters were set to $P = 225$ and $B = 45$, i.e., $1/5$th of the filter coefficients are updated per iteration. Fig. 2 shows the time-averaged convergence curves for TD-LMS, SPU-TD-LMS and SB-TD-LMS (sequential-block TD-LMS). Although both SPU-TD-LMS and SB-TD-LMS update $1/5$th of the coefficients, the convergence speed of SB-TD-LMS is very slow compared to that of SPU-TD-LMS.

3.2 Blind Channel Equalization

The constant modulus algorithm for blind channel equalization aims to estimate the equalizer parameter vector $\boldsymbol{\theta}(n) = [\theta_0(n), \theta_1(n), \cdots, \theta_{N-1}(n)]^T$ such that, for a given equalizer input vector $\boldsymbol{x}(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T$, the equalizer output signal $y(n) = \boldsymbol{\theta}^T(n)\boldsymbol{x}(n)$ satisfies the equality

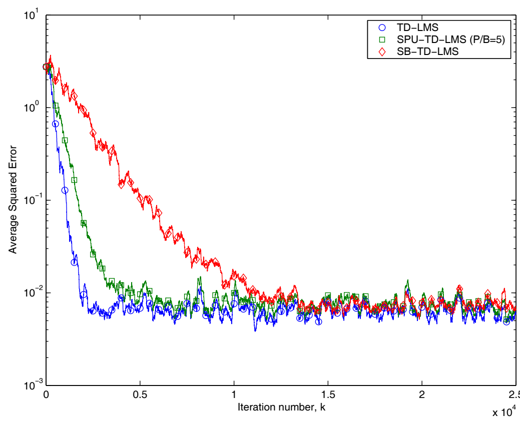

Figure 2: Converge comparison for TD-LMS and SPU-TD-LMS in acoustic echo cancellation example.

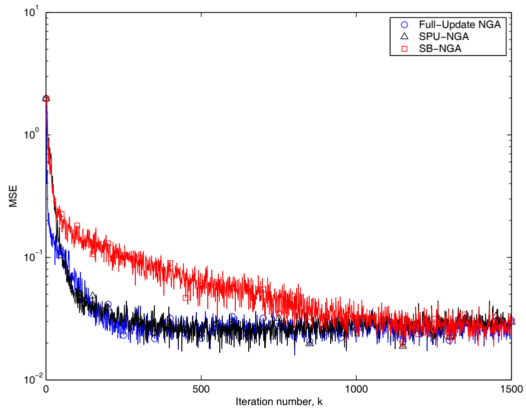

Figure 3: Converge comparison for blind channel equalization algorithms NGA and SPU-NGA.

$|y(n)|^2 = R^2 \ \forall n$ where R is a constellation-dependent dispersion factor.

Partition the regressor vector and the equalizer parameter vector into P blocks:

$$\boldsymbol{x}(n) = \begin{bmatrix}\boldsymbol{x}_1^T(n) & \cdots & \boldsymbol{x}_P^T(n)\end{bmatrix}^T$$
$$\boldsymbol{\theta}(n) = \begin{bmatrix}\boldsymbol{\theta}_1^T(n) & \cdots & \boldsymbol{\theta}_P^T(n)\end{bmatrix}^T.$$

Consider the following constrained minimization problem

$$\min_{1\le i\le P} \min_{\boldsymbol{\theta}_i(n+1)} \|\boldsymbol{\theta}_i(n+1) - \boldsymbol{\theta}_i(n)\|_2^2 \tag{3a}$$

$$\text{subject to } \boldsymbol{\theta}^T(n+1)\boldsymbol{x}(n) = y(n)(R^2 - |y(n)|^2 + 1) \tag{3b}$$

where the constraint on the adapted equalizer parameters $\boldsymbol{\theta}(n+1)$ is a "soft" constraint in that it does not force the equalizer output to strictly satisfy the constant modulus criterion.

The recursive algorithm that solves (3) is given by

$$\boldsymbol{\theta}_i(n+1) = \boldsymbol{\theta}_i(n) + \mu_{\text{NGA}} \frac{y(n)(R^2 - |y(n)|^2)}{\|\boldsymbol{x}_i(n)\|_2^2}\boldsymbol{x}_i(n),$$
$$i = \arg\max_{1\le j\le M} \|\boldsymbol{x}_j(n)\|_2^2. \tag{4}$$

which we refer to as the selective-partial-update normalized Godard algorithm (SPU-NGA). In $T/2$-spaced equalization, the communication channel is modelled as two subchannels with outputs $x_1(n)$ and $x_2(n)$. The subchannel outputs are applied to subequalizers with $N/2 \times 1$ parameter vectors $\boldsymbol{\theta}_1(n)$ and $\boldsymbol{\theta}_2(n)$. Expressing the regressor vector as $\boldsymbol{x}(n) = [\boldsymbol{x}_1^T(n), \boldsymbol{x}_2^T(n)]^T$ and the equalizer parameter vector as $\boldsymbol{\theta}(n) = [\boldsymbol{\theta}_1^T(n), \boldsymbol{\theta}_2^T(n)]^T$, which are both N-dimensional vectors, SPU-NGA given by (4) can be used for fractionally-spaced equalization with no modification.

We have simulated the full-update NGA, SPU-NGA and sequential-block NGA (SB-NGA) algorithm for the following $T/2$-spaced channel with $N = 4$ and $P = N$:

$$\boldsymbol{h} = [-0.2, -0.3, 0.4, 0.1, -0.35, -0.15, -0.005, -0.002]^T.$$

The mean-square error (MSE) values, averaged over 100 simulations, are shown in Fig. 3. Note that the SPU-NGA has a comparable convergence rate to the full-update NGA while the SB-NGA exhibits much slower convergence.

3.3 Blind Multiuser Detection

The synchronous DS-CDMA signal model for a K-user system using binary phase shift keying (BPSK) modulation is

$$\boldsymbol{r}(i) = \sum_{k=1}^{K} A_k b_k(i)\boldsymbol{s}_k + \boldsymbol{n}(i) \tag{5}$$

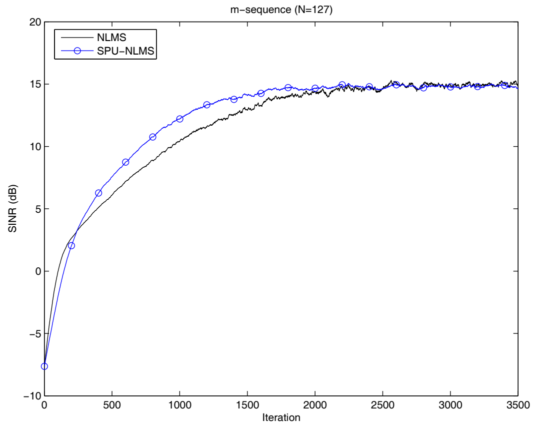

Figure 4: NLMS vs SPU-NLMS in blind MUD ($N = 127$).

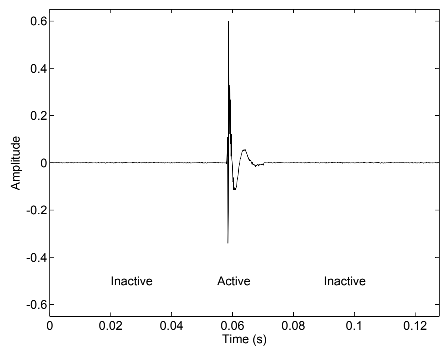

Figure 5: An example of a sparse impulse response.

where $\boldsymbol{r}(i)$ is the received signal vector at the ith symbol interval, A_k is the received signal amplitude for user k, $b_k(i) = \pm 1$ is the ith symbol of user k, $\boldsymbol{s}_k$ is the normalized signature (scrambling code) of user k N with period N, and $\boldsymbol{n}(i) \sim \mathcal{N}_c(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_N)$ is the additive complex Gaussian channel noise.

The minimum output energy (MOE) blind linear multiuser detector for user 1 is given by [8]:

$$\boldsymbol{m} = \boldsymbol{s}_1 + \boldsymbol{P} \arg\min_{\boldsymbol{x} \in \mathbb{C}^N} E\{\|(\boldsymbol{s}_1 + \boldsymbol{P}\boldsymbol{x})^H \boldsymbol{r}(i)\|^2\} \tag{6}$$

where $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{s}\boldsymbol{s}^H$ and ^{H} denotes Hermitian. Partition $\boldsymbol{x}(i)$ and $\boldsymbol{P}$ into P blocks:

$$\boldsymbol{x}(i) = \left[\boldsymbol{x}_1^T(i) \quad \cdots \quad \boldsymbol{x}_P^T(i)\right]^T, \quad \boldsymbol{P} = [\boldsymbol{P}_1 \quad \cdots \quad \boldsymbol{P}_P]. \tag{7}$$

For the sake of simplicity, assume that A_k and $\boldsymbol{n}(i)$ are real. Then the solution of

$$\min_{1 \le j \le P} \min_{(\boldsymbol{s}_1 + \boldsymbol{P}\boldsymbol{x}(i+1))^T \boldsymbol{r}(i) = 0} \|\boldsymbol{x}_j(i+1) - \boldsymbol{x}_j(i)\|^2 \tag{8}$$

gives the SPU-NLMS algorithm for blind multiuser detection

$$\begin{aligned} \boldsymbol{x}_j(i+1) &= \boldsymbol{x}_j(i) - \mu_{\text{SPU}} \frac{(\boldsymbol{s}_1 + \boldsymbol{P}_1 \boldsymbol{x}_1(i))^T \boldsymbol{r}(i) \boldsymbol{P}_j^T \boldsymbol{r}(i)}{\|\boldsymbol{P}\boldsymbol{r}(i)\|^2}, \\ j &= \arg\max_{1 \le m \le B} \|\boldsymbol{P}_m^T \boldsymbol{r}(i)\|^2. \end{aligned} \tag{9}$$

The SPU-NLMS was simulated for a system with $K = 10$ users, six 10 dB multiple access interferences (MAIs), three 20 dB MAIs, and the desired signal to ambient noise ratio of 20 dB. The signature sequences are m-sequences with period $N = 127$. We set $P = N$, i.e., only one coefficient out of N is updated at each iteration. It was observed that the SPU-NLMS algorithm not only performed as well as the full-update NLMS algorithm, but it also outperformed the full-update NLMS algorithm in some cases. The convergence curves for the NLMS and the SPU-NLMS are shown in Fig. 4.

4. ADAPTIVE FILTERS FOR SPARSE SYSTEM IDENTIFICATION

An impulse response (or a signal) can be considered "sparse" if a large fraction of its energy is concentrated in a small fraction of its duration. Adaptive system identification is a particularly challenging problem for sparse systems.

An application of sparse system identification which is of current interest is packet-switched network echo cancellation. The increasing popularity of packet-switched telephony has led to a need for the integration of older analog systems with, for example, IP or ATM networks. Network gateways enable the interconnection of such networks and provide echo cancellation. In such systems, the hybrid echo response is delayed by an unknown bulk delay due to propagation through the network. The overall effect is therefore that an "active" region associated with the true hybrid echo response occurs with an unknown delay within an overall response window that has to be sufficiently long to accommodate the worst case bulk delay. The example in Fig. 5 shows a sparse system with an overall response window of 128 ms duration with an active region containing a hybrid response of 12 ms duration. It may be necessary in practical systems for the echo cancellation algorithm to identify and cancel the effect of, for example, up to three independently delayed hybrid echo responses, each of up to 16 ms duration, within an overall window of, say, 128 ms. Such multi-hybrid responses occur in conference calls.

It has been shown [9] that direct application of NLMS [10] to sparse system identification in the context of, for example, echo cancellation gives unsatisfactory performance when the echo response is sparse. This is because the adaptive algorithm has to operate on a relatively long filter, and because of the coefficient noise that will occur during adaptation for the near-zero-valued coefficients in the inactive regions.

To address this problem, the concept of proportionate NLMS (PNLMS) was introduced in the context of echo cancellation by Duttweiler [11] and Benesty *et al* [12]. In PNLMS each coefficient is adapted with step-size proportional to its magnitude as given by

$$e(n) = d(n) - \sum_{l=0}^{L-1} h_l(n) x(n-l) \tag{10a}$$

$$\gamma_{\min}(n) = \rho \max\{\delta_p, |h_0(n)|, |h_1(n)|, \ldots, |h_{L-1}(n)|\} \tag{10b}$$

$$\gamma_l(n) = \max\{\gamma_{\min}(n), |h_l(n)|\}, \quad 0 \le l < L \tag{10c}$$

$$g_l(n) = \frac{\gamma_l(n)}{\frac{1}{L}\sum_{i=0}^{L-1} \gamma_i(n)}, \quad 0 \le l < L \tag{10d}$$

$$\boldsymbol{G}(n) = \text{diag}\{g_0(n), \ldots, g_{L-1}(n)\} \tag{10e}$$

$$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \frac{\mu \, \boldsymbol{G}(n)}{\boldsymbol{x}^T(n)\boldsymbol{G}(n)\boldsymbol{x}(n) + \delta_{\text{PNLMS}}} \boldsymbol{x}(n) e(n). \tag{10f}$$

where δ_{PNLMS} is the regularization constant and L is the length of the adaptive filter. Typical values for the algorithm constants are given as $\delta_p = 0.01$ and $\rho = 5/L$. At initialization $\boldsymbol{h}(n) = \boldsymbol{0}$, the errors in the large coefficients are dominant. In subsequent iterations, PNLMS uses proportionately high step-sizes for these coefficients, obtaining fast initial reduction in error. Proportionately small step-sizes are employed for the small coefficients in the inactive regions. Significant performance improvements have been reported using PNLMS for sparse system identification. The performance for non-sparse system can however be relatively poor. In addition, PNLMS performance can be poor when the unknown system is time-varying such that coefficients cross zero. Close to the zero-crossing, inappropriately small step-sizes are employed by PNLMS and tracking performance suffers.

Improvements to the original proportionate scheme have been recently developed. In [13] the PNLMS++ scheme switches between PNLMS and NLMS updates, for example, for alternate iterations. The Improved PNLMS algorithm (IPNLMS) is presented in [14] as

$$k_l(n) = \frac{1-\alpha}{2L} + (1+\alpha)\frac{|h_l(n)|}{2\|\boldsymbol{h}(n)\|_1+\varepsilon} \quad l = 0,\ 1,\ \ldots,\ L-1 \tag{11a}$$

$$\boldsymbol{K}(n) = \mathrm{diag}\{k_0(n),\ \ldots,\ k_{L-1}(n)\} \tag{11b}$$

$$\boldsymbol{h}(n+1) = \boldsymbol{h}(n) + \frac{\mu\,\boldsymbol{K}(n)}{\boldsymbol{x}^T(n)\boldsymbol{K}(n)\boldsymbol{x}(n)+\delta_{\mathrm{IPNLMS}}}\,\boldsymbol{x}(n)e(n) \tag{11c}$$

where δ_{IPNLMS} is the regularization parameter and ε is a small positive constant to avoid division by zero. Here, a mix of PNLMS and NLMS updating is employed at every iteration with the amount of each type of update controlled by α. IPNLMS has the advantage of consistently high performance even if the unknown system is not strongly sparse. It has been shown in [15] that the IPNLMS algorithm is an approximation of the exponentiated gradient algorithm EG$\pm$ [16]. The IIPNLMS algorithm [17] extends the concept of IPNLMS to enable the amount of proportionate and non-proportionate updating to be controlled independently for each coefficient. A block frequency domain version of IPNLMS, known as IPMDF, has also been proposed [18] and is based on multi-delay filtering [19].

In [20], the Sparse Partial Update NLMS algorithm (SPNLMS) is developed in which coefficients h_l are only updated if $|x_l(n)h_l(n)| \in M$ maxima of $|\boldsymbol{x}(n).*\boldsymbol{h}(n)|$ for $l = 0,\ 1,\ \ldots,\ L-1$ where $.*$ represents the element-by-element vector product. Although the SPNLMS algorithm does not directly employ proportionate updating, it exploits sparseness in both the echo response and the input signal since updating of a particular coefficient will be avoided if either the tap-input sample or the coefficient are sufficiently small. The same authors in [21] consider how to specify the relationship between coefficient magnitude and step-size for optimal convergence rate. In PNLMS, linear proportionality is used. It is shown that optimal convergence is achieved when all coefficients attain the vicinity of their optimal value in the same number of iterations following initialization and to achieve this a non-linear relationship is required. The μ-law PNLMS algorithm (MPNLMS) is subsequently formulated and fast convergence demonstrated.

5. CONCLUSION

We have reviewed the latest developments in partial update and sparse adaptive filter algorithms. These two topics were considered jointly because several of the concepts are common to both; indeed, joint algorithms are starting to be proposed in the literature. The chief application areas of partial update and sparse adaptive filters are acoustic/network echo cancellation and channel equalization. New application areas have also been emerging such as CDMA multiuser detection. The paper illustrated the potential performance improvement that can be achieved by selective partial updates and sparse updates.

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