PLM SEQUENCES FOR THE PERFORMANCE OPTIMIZATION OF LINEAR MULTIUSER DETECTORS

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ABSTRACT

This paper deals with the general problem of the design/choice of spreading sequences for asynchronous DS-CDMA communication systems, employing a linear multiuser detection front-end. After a brief problem formulation, we propose a new near-optimal spreading sequences design, based on piece wise linear chaotic Markovian sequences (PLM sequences). The performance evaluation of this spreading sequences design, when the system is considered in the Gaussian approximation settings is shown to outperform the conventional Gold based spreading and enhances the system capacity, evaluated in terms of available users for a target bit error rate (BER) system performance.

1. INTRODUCTION

Linear multiuser detectors have been widely studied as sub- optimal low complexity alternatives to more complex globally optimum (maximum likelihood, maximum a posteriori, etc) multiuser detectors [9, 18, 21]. In [16], R.Lupas and S.Verdù, have proposed the first implementation of such reduced complexity linear multiuser schemes and evaluated its performance and stability under a quite general spreading sequences and users power profiles. In [13, 20] the authors have introduced the minimum mean square (MMSE) linear multiuser detector and proposed different implementations of the MMSE multiuser detection strategy. In the abovementioned work, only the performance and complexity evaluation of the linear multiuser detectors was considered [8, 21]. However, the performance of this class of multiuser detectors depends also on the spreading sequences correlations. So, a carful attention must be given to the design/selection of the spreading sequences of the system. In this paper, we propose a new and general spreading sequences design procedure for the optimization of the performance in the linear multiuser detection context, based on chaotic piece wise linear Markovian sequences. The proposed design relies on two main ideas. Firstly, an approximation of the relevant performance measures of the system employing linear multiuser detection by quantities related to the multiple access interference at the output of the matched filter bank [19]. The spreading sequences design procedure is then carried out on the basis of the minimization of the second order statistics of the multiple access interference (MAI) at the output of the multiuser linear transformation i.e. the standard Gaussian approximation settings. This standard Gaussian approximation of the error probability of the system is proved to be a reasonable approximation of the overall system performance [3, 4]. The outline of the paper is as follows. In section (2), we introduce the system model and performance measures and discusses the spreading sequences design problem. In section (3) we propose the PLM sequences formalism for the implementation of optimal spreading sequences. The section (4) is devoted to the presentation of our simulation results, where we compare the performance of the sequences with classical Gold based spreading. In section (5) we give some conclusions and future work outlines.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Let us consider a K active users DS-CDMA communication system employing a centralized linear multiuser detection strategy [9, 18]. In such a system the BPSK data symbol sequence of each user $b_k(t)$ with symbol period T_s is spread by a BPSK spreading sequence $a_k(t)$ with period T_c.

The overall received signal, when an additive Gaussian channel is considered can be modeled as

$$r(t) = S(t) + n(t) \tag{1}$$

where $\{n(t)\}$ is an additive white Gaussian noise process with power spectral density σ_n^2 and $S(t)$ is the multiple access signal which is given as a function of the symbol sequence and the spreading codes as :

$$S(t) = \sum_{k=1}^{K} A_k \sum_{i=-M}^{M} b_k(i) a_k(t - iT_s - \tau_k) \tag{2}$$

where $A_k = \sqrt{2P_k}$ is the user k signal amplitude, $2M+1$ is the frame length of the $k-$th user signal, $b_k(i)$ is the $i-$ th symbol of user k and τ_k is the user k access delay.

It is assumed that the spreading sequences $a_k(t)$ are normalized signaling waveforms, supported by the signaling interval $[0, T_s]$ and given as

$$a_k(t) = \sum_{n=0}^{N-1} a_{k,n} P_{T_c}(t - nT_c) \tag{3}$$

$a_{k,n}$ is the $n-$th chip of the user k, $P_{T_c}(t)$ is a rectangular pulse shaping function of length T_c and $N = T_s/T_c$ is the system's spreading factor. The main subject of this paper is to investigate the optimal spreading sequences design for asynchronous DS-CDMA communication system with linear multiuser detection front end. For that purpose, let us consider without loss of generality a simplified but yet relevant system and detector model. The DS-CDMA system is considered as synchronous, with $(2M+1)K$ virtual users [18] and in a perfectly power controlled situation, i.e. $\{A_k\}_{k=1}^{K} = A_0 = 1$. The decision statistic associated with the detection of the symbol $b_k(i)$ can be given as

$$y_k(i) = \int_{iT_s}^{(i+1)T_s} a_k(t - iT_s) r(t) dt \tag{4}$$

Statistically, this detection problem is invariant with the choice of the symbol interval i, so without loss of generality, let us consider the case where $i = 0$ and suppress the dependence on the index i. The sufficient statistic vector y of size $(2M+1)K$ can be written as

$$y = Rb + n \tag{5}$$

with R being the normalized correlation matrix of the system, b is the system's symbol vector and n is a centered Gaussian noise vector with covariance matrix $\sigma_n^2 R$. The correlation matrix R depends on the continuous aperiodic left and right correlation functions [17, 18]. Those correlations are defined respectively as

$$\begin{aligned} \rho_{k,j}(\tau) &= \int_{\tau}^{T_s} a_k(t) a_j(t-\tau) dt \\ \rho_{j,k}(\tau) &= \int_{0}^{\tau} a_k(t) a_j(t+T_s-\tau) dt \end{aligned} \quad (6)$$

Let us consider a particular class of multiuser detectors which apply a linear transformation L to the vector of the sufficient statistics y. The post- linear transformation decision statistics, known in the literature as the *leakage coefficients*, are the relevant performance measure when the error probability of the system is considered. The post -linear transformation decision statistic for the detection of the first symbol of user 1 can be given as follows

$$z_1 = b_1 + \sum_{l=2}^{(2M+1)K} \beta_l b_l + \frac{\sigma_n^2}{B_1} \tilde{n}_1 \quad (7)$$

where the leakage coefficients are given by $\beta_l = B_l / B_1$ and the parameters B_l, $l = 1, \cdots, (2M+1)K$ are given by

$$B_l = (LR)_{1,l} \quad (8)$$

The post- linear transformation noise $\tilde{n}_1$ is a centered Gaussian noise process with second order statistic given by $(LRL)_{1,1}$.

Recently, many studies have pointed- out the Gaussian nature of the interference and noise terms in the relation (7) when linear multiuser detection in asynchronous DS-CDMA communication is considered [3, 4, 15].

In this paper we will consider the Gaussian approximation of the average error probability of the asynchronous DS-CDMA system with linear multiuser detection front -end as a main system performance measure. The Gaussian approximation of the average error probability of the first user can then be evaluated as

$$P_1 \approx Q\left(\sqrt{SNIR}\right) \quad (9)$$

where $Q(x)$ is the Gaussian Q- function defined by $Q(x) = \frac{1}{\sqrt{(2\pi)}} \int_x^\infty \exp \frac{-y^2}{2} dy$ and $SINR$ is the signal to interference ratio. The SINR can be written as

$$SINR = \frac{1}{\dfrac{\sigma_n^2 (LRL)_{1,1}}{(LR)_{1,1}} + \sigma_I^2} \quad (10)$$

where σ_I^2 is the second order statistic of the multiple access interference process defined by $I = \sum_{l=2}^{(2M+1)K} \beta_l b_l$. Let us consider, for analysis purposes two typical linear multiuser detectors, the linear decorrelating detector [16] and the linear minimum mean square error detector (LMMSE) [20].

We will use in this paper a recently proposed approximation of the SINR given in the relation (10) [19]. This approximation permits us to express the SINR when the linear decorrelating detector is considered as

$$SINR_{\text{Dec}} \approx \frac{1 - \sum_{k=2}^{K} \rho_{j,k}(\tau_k)^2 + \rho_{k,j}(\tau_k)^2}{\sigma_n^2} \quad (11)$$

where the LMMSE detector is considered the residual multiple access is not fully eliminated by the linear transformation $L = \left(R + \sigma_n^2 I\right)^{-1}$.

However, we can use for the evaluation of the SINR , associated with the LMMSE multiuser detector, the same approximation that the one reported earlier which leads to a quite similar expression of the SINR. This SINR is given by the following relation

$$SINR_{\text{LMMSE}} \approx \frac{1 - \sum_{k=2}^{K} \rho_{j,k}(\tau_k)^2 + \rho_{k,j}(\tau_k)^2}{1 + \sigma_n^2} \quad (12)$$

Let us suppose that the spreading sequences are picked randomly from a spreading sequences family $\Xi = \{a_k(t)\}_{k=1}^K$. We can formulate the sequences design problem for asynchronous DS-CDMA communications with linear multiuser detection as

Design Ξ to optimize the average SINR of the system

Relations (11), (12) are showing that optimization of the average SINR is equivalent to design Ξ to minimize the overall power of the interference $\iota = E\left\{\sum_{k=2}^{K} \rho_{j,k}(\tau_k)^2 + \rho_{k,j}(\tau_k)^2\right\}$ before the linear transformation L. The minimization of the parameter ι is equivalent to the design of the spreading sequences family that leads to a minimum average interference parameter AIP in the system. This minimum AIP based spreading sequences design is relatively well known in the literature [10–12] but was applied exclusively for the optimization of the performance of the conventional single user detector. The solution to that problem is a sequences family Ξ_{opt} with an aperiodic auto- correlation profile given by

$$C_{\text{opt}}(n) = (N - |n|)(-r)^n \quad (13)$$

with $r = 2 - \sqrt{3}$ and $C_{\text{opt}}(.)$ is the mean Pursley's auto- correlation [11] of the spreading sequences in Ξ_{opt}. In this paper we claim that those minimum AIP spreading sequences are near- optimum in the sense of maximize the average SINR and so minimize the mean error probability of the asynchronous DS-CDMA communication system with a linear multiuser detection front- end.

In section (3) we propose a very simple implementation of these optimal sequences by means of quantized chaotic piece- wise linear Markov sequences.

3. PLM SPREADING SEQUENCES DESIGN

In this section we propose an implementation of the optimal spreading sequences defined in section (2) by means of chaotic piece-wise linear Markov sequences. The general generation procedure of such sequences can be described by the following state transition and observation equations

$$\begin{aligned} x_{n+1} &= f(x_n) + \varepsilon\, w_k \\ \tilde{a}_n &= Q(x_n) \end{aligned} \quad (14)$$

with $n \in \{0, \cdots, N-1\}$, x_0 is the initial condition of the sequence, $f(.)$ is the chaotical mono-dimensional sequence mapping, w_n is a state noise to the evolution of the chaotical sequence states, $Q(.)$ is the usual binary quantification function and ε is an arbitrary small real number. We consider a very simple chaotical mapping, where $f(x)$ is a piece wise linear affine Markov function [5, 6, 17], defined on a partition $\mathfrak{C}\left(N_p\right)$ of a real interval I and of cardinality N_p. The state noise distribution, $f_{w_n}(w_n)$ is the invariant distribution $\bar{f}(x)$ of the chaotic system [1, 7]. In our case the invariant distribution is the uniform distribution on the interval I [1]. The binary sequence $\{\tilde{a}_n\}_{n=0}^{N-1}$ are BPSK modulated to construct the BPSK Markovian sequence $\{a_n\}_{n=0}^{N-1}$ for all the users of the system. The aperiodic correlation properties of the sequences can be evaluated by using the transition matrix $\mathscr{K}$ of the chaotic system. The entries of the transition matrix, represent the amount of a given element of the partition $\mathfrak{C}$, mapped to a distinct and disjoint element of the partition. For example, consider the two sub-intervals I_i and I_j being in the partition $\mathfrak{C}$ of the interval $I = [0, 1]$. We have $I_i \subset I$ and $I_j \subset I$, with $I_i \bigcap I_j = \varnothing$. The elements of the transition matrix can be defined as :

$$\mathscr{K}_{i,j} = \frac{\mu\left(I_i \bigcap f^{-1}\left(I_j\right)\right)}{\mu\left(I_i\right)} \qquad i, j \in \left\{0, \cdots N_p - 1\right\} \quad (15)$$

The measure $\mu(.)$ can be defined as the standard Lebesgue measure for the interval I (i.e. $\mu(I_i) = \int_{I_i} \bar{f}(x)dx$)). In our case, this measure can be interpreted as the length of the interval I_i. The mean aperiodic auto -correlation properties of the sequences are then given by the following quadratic form.

$$C(n) = \frac{1}{N_p} \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} Q(I_i) \mathscr{K}_{i,j}^{(n)} Q(I_j) \tag{16}$$

where, N_p is the partition size, $\mathscr{K}^{(n)}$ is the $n-$th power of the transition matrix $\mathscr{K}$ with the convention that $\mathscr{K}^{(0)}$ is the $N_p \times N_p$ identity matrix. By adding a "tail" of t_p shifts to the original N_p- Markovian sequences we can derive a new sequences family with exponential decay of the aperiodic auto-correlations. A typical example of the chaotic system map $f(x)$ is shown in figure (1). By using

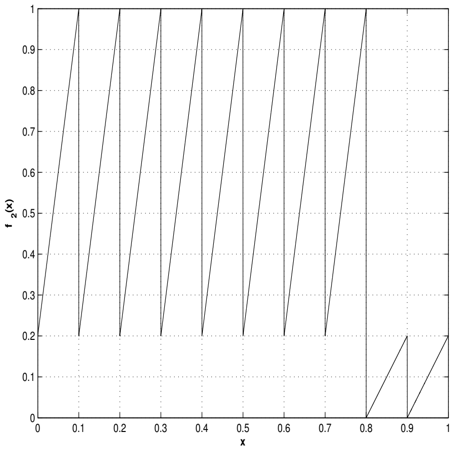

Figure 1: $(10,2)-$way PLM mapping function $f(x)$ on $I = [0,1]$

relations (15) and (16) the aperiodic auto- correlation of the general $(N_p, t_p)-$Markovian sequences can be written as :

$$C(n) = \begin{cases} N & \text{if } \tau = 0 \\ (N-|n|)h^n & \text{otherwise} \end{cases} \tag{17}$$

The parameter h is given as $h = \frac{-t_p}{N_p - t_p}$. By a proper choice of the parameters t_p and N_p we can generate various BPSK chaos-based sequences with exponentially decaying aperiodic auto- correlations. A particular choice of $N_p = 10$ and $t_p = 2$ gives us a sequence family with $h = 0.25 \approx 2 - \sqrt{3}$ we propose to use this sequence family for the optimization of the performance of the asynchronous DS-CDMA communication system employing linear multiuser detection strategy. Section (4) is devoted to the evaluation of the performance of the proposed sequences.

4. SIMULATION RESULTS

In this section we will investigate the validity of the proposed $(10,2)-$Markov sequences design when an asynchronous DS-CDMA communication system is considered with a decorrelator/ LMMSE detector front- end. The DS-CDMA communication system considered in these simulations is a system employing BPSK symbols, BPSK sequences and a rectangular pulse shaping function. The system is considered in a perfect power control situation. We fix the spreading factor of the system to $N = 31$ and we compare the performance of the $(10,2)-$Markovian sequences with the performance of the conventional Gold sequences, randomly picked from the Gold sequences family. We have considered in the simulation the trunked L-decorrelator and LMMSE detectors using the truncation procedure reported in [14]. Figure(2) represents the capacity plot of the decorrelator, evaluated in terms of users per available BER. We plotted in the figure, the exact performance of the decorrelator with the approximate performance predicted by the relation (11). The signal to noise ratio is fixed to 9dB. Figure(2) shows that

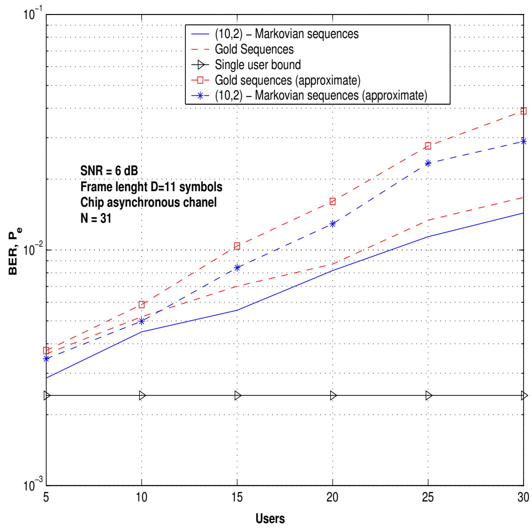

Figure 2: performance of the $(10,2)-$Markov sequences with linear decorrelating detector

the $(10,2)-$Markovian sequences exhibit better error performance, evaluated in terms of bit error rate (BER) as the number of users increases than the classical sequences. The overall error performance improvement is of about 4 users when compared to the performance of the Gold sequences based design, for a target BER of 5×10^{-3}. The performance of the $(10,2)-$Markovian sequences are of 2 users better than the performance of the Gold sequences for a target performance BER of 6×10^{-3}. The approximate evaluation of the signal to interference ratio over- estimate the performance of the system when the overall system charge increases i.e. $l = K/N > 0.5$. In figure (3), we present our simulation results under the same conditions as previously, when the linear MMSE multiuser detector is considered. Figure(3) shows that the $(10,2)-$Markovian sequences

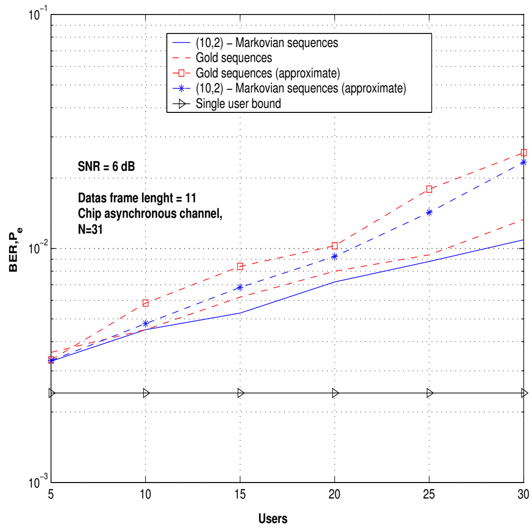

Figure 3: performance of the $(10,2)-$Markov sequences with linear MMSE detector

outperform the performance of the Gold sequences when a LMMSE multiuser detector is considered in a chip asynchronous DS-CDMA communication system. The overall capacity gain is, at best, of 3 users for a target system BER of 5×10^{-3} for the exact simulations and 5 users for a target BER of 8×10^{-3} when approximation (12) of the SINR is considered. We can also note that the approximate evaluation of the SINR over- estimates the error performance of the system where a full loaded situation is considered, i.e. the system load factor $K/N > 0.5$.

5. CONCLUSION

In this paper we have proposed a new and original spreading sequences design, based on chaotic piece wise linear sequences for the performance optimization of an asynchronous DS-CDMA communication system employing a linear multiuser detection strategy. This optimization procedure is based on the minimization of the second order statistics of the multiple access interference (MAI) by a sequences design based on the shaping of the aperiodic auto- correlation of the sequences. The proposed sequences design outperforms the Gold sequences quite clearly when the linear decorrelating detector is used, however in the case of the LMMSE the performance improvement is not really important. This fact illustrates that the second order interferences statistics minimization, proposed in this paper is not appropriate for the LMMSE detectors optimization and that further studies, perhaps based on higher order statistics [2], are interesting to look at.

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