A NEW TURBO EQUALIZER WITH LINEAR COMPLEXITY

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ABSTRACT

In this paper a new soft input - soft output (SISO) equalizer of linear complexity is developed. The algorithm can be used in the so-called Turbo equalization scheme as a low cost solution in place of the Maximum A-Posteriori (MAP) equalization algorithm which has a prohibitive complexity for most real world applications. The proposed equalizer consists of two parts, namely, a Soft Interference Canceller (SIC) and a pre-processing part which is a new Variable-Threshold Decision Feedback Equalizer (VTDFE). The role of the second part is to increase the amount of a-priori information supplied to the SIC. Simulation results have shown that the proposed turbo equalizer exhibits a superior performance as compared to the turbo equalization scheme based on the conventional SIC as well as other linear complexity SISO equalizers.

1. INTRODUCTION

Turbo Equalization [1] was motivated by the breakthrough of Turbo Codes [2], and has emerged as a promising technique for drastical reduction of the intersymbol interference effects in frequency selective wireless channels. A *Turbo Equalization* procedure, in its generic form, exhibits the following two traits [3]: a) the decoder and the equalizer exchange *soft information* between each other, with this soft information being interpreted as a-priori probability information, and b) the decoder and the equalizer exchange *extrinsic information*, which is possible if their output at time instant n does not directly rely on their soft input for the same time index but only on information gained by using the soft information about symbols at adjacent (past and future) time instants.

Unfortunately, the trellis-based turbo equalizer of [1] can be a heavy computational burden for wireless systems with limited processing power, especially in cases the wireless channel has long delay spread. For such reasons, a number of low complexity alternative equalization methods that can be properly incorporated in the generic Turbo Equalization scheme have been proposed, offering good complexity/performance trade-offs.

In this context, it was proposed [4] to replace the trellis-based equalizer by an adaptive SIC with linear complexity. In [5], an improved extension of the algorithm of [4] was presented. In [3] an MMSE-optimal equalizer based on linear filters was derived and it was proved that several other algorithms (such as the one in [4]) could be viewed as approximations of this one.

The SIC of [4] and its fixed (i.e. non-adaptive) version studied later in [3], turn out to be good choices for easy to

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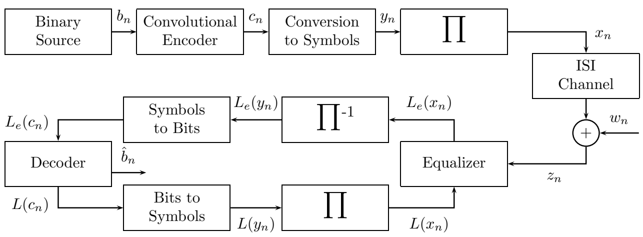

Figure 1: The Model of Transmission

medium difficulty channels at relatively high SNR. The aim of this work is mainly to improve the above fixed SIC by means of a suitable pre-processor so that it may by applicable for hostile channels with severe ISI and low SNR as well. The proposed pre-processor is a Variable-Threshold DFE (VTDFE) of linear complexity whose decisions thresholds are varying by using a Bayesian classification rule that incorporates a-priory probabilities coming from the decoder. The overall turbo equalization scheme based on the so-called VTDFE-SIC equalizer exhibits a superior performance as compared to the turbo equalization scheme based on the conventional SIC as well as other linear complexity SISO equalizers.

The rest of this paper is organized as follows: In section 2, the communication system model is formulated. In section 3, the MMSE Soft Interference Canceller, which is a constituent part of the new scheme, is briefly reviewed. In section 4, the new VTDFE-SIC equalizer is derived. Finally, in section 5, simulation results verifying the performance of the proposed equalizer are provided.

2. TRANSMISSION MODEL

Let us consider the communication system depicted in Figure 1. The system transmits blocks of data, with each block containing S information bits. A discrete memoryless source generates binary data $b_n, n = 1 \dots S$. These data, in blocks of length S, enter a convolutional encoder of rate R, so that new blocks of S/R bits are created, where S/R is assumed integer. The output of the convolutional encoder is then grouped into groups of q bits each (with $\frac{S}{Rq}$ also assumed integer) and each group is mapped into a 2^q-ary symbol from the alphabet $A = \{\ _1,\ _2, \dots,\ _{2^q}\}$. The resulting symbols y_n are then permuted by an interleaver, denoted as , and the sequence of symbols $x_n, n = 1 \dots \frac{S}{Rq}$ is finally transmitted through the channel.

We assume that the communication channel is frequency selective and constant during the packet transmission, so that the output of the channel (and input to the receiver) can be modeled as:

$$z_n = \quad {}_{i=-L_1}^{L_2} \; h_i x_{n-i} + w_n \tag{1}$$

where L_1, L_2+1 denote the lengths of the anti-causal and causal parts, respectively, of the channel impulse response. The output of the multipath channel is corrupted by Additive White Gaussian Noise (AWGN) w_n. For the rest of this paper, BPSK modulation ($q=1$) with alphabet $A=\{+1,-1\}$ is assumed.

3. THE FIXED SIC

In this section we first briefly review the conventional Soft Interference Canceller which is a constituent part of the proposed equalizer. The SIC [3], [4] consists of two filters, the matched filter

$$\mathbf{p} = [p_{-k} \cdots p_0 \cdots p_l]^T, \;\; M = k+l+1 \tag{2}$$

and the cancellation filter

$$\mathbf{q} = [\mathbf{q}_f;\ 0;\ \mathbf{q}_p] = [q_{-K} \cdots q_{-1} \quad 0 \quad q_1 \cdots q_N]^T \tag{3}$$

The input to filter $\mathbf{p}$ is the sampled output of the channel (taken here to be symbol-spaced), whereas the input to the cancellation filter consists of past and future symbols. The output s_n of the SIC is the sum of the outputs of the two filters, i.e.,

$$s_n = \mathbf{p}^T \mathbf{z}_n + \mathbf{q}_p^T \mathbf{x}_{pn} + \mathbf{q}_f^T \mathbf{x}_{fn} \tag{4}$$

where $\mathbf{z}_n = [z_{n+k} \cdots z_n \cdots z_{n-l}]^T$ and $\mathbf{x}_{pn}$, $\mathbf{x}_{fn}$ are vectors whose entries are past and future detected symbols. Minimizing the mean square error $E[|s_n - x_n|^2]$ and assuming that the cancellation filter contains correct symbols, we obtain the following expressions for the involved filters:

$$\mathbf{p} = \frac{1}{{}^2_w + E_h} \mathbf{H}\mathbf{d} \tag{5}$$

$$\mathbf{q}_p = -\mathbf{H}_B^H \mathbf{p} \tag{6}$$

and

$$\mathbf{q}_f = -\mathbf{H}_A^H \mathbf{p} \tag{7}$$

where $N = l + L_2$, $K = L_1 + k$, $E_h = \mathbf{H}\mathbf{d}\mathbf{d}^H\mathbf{H}^H$ is the energy of the channel and matrices $\mathbf{H}_A$, $\mathbf{H}_B$ contain the first K and the last N columns of the $M \times (K+N+1)$ convolution matrix $\mathbf{H}$. $\mathbf{H}$ and $\mathbf{d}$ are in turn defined as

$$\mathbf{H} = \begin{bmatrix} h_{-L_1} & \cdots & h_{L_2} & 0 & \cdots & 0 \\ 0 & \ddots & h_{L_2-1} & h_{L_2} & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_{-L_1} & \cdots & h_{L_2} \end{bmatrix} \tag{8}$$

$$\mathbf{d} = [\mathbf{0}_{1\times k+L_1} \quad 1 \quad \mathbf{0}_{1\times l+L_2}]^T \tag{9}$$

Up to this point the SIC is identical to the classical canceller. It is the incorporation of a-priori information from the decoder which mainly differentiates SIC from classical canceller. Incorporation of a-priori information can be achieved if the input data to the cancellation filter are not the detected symbols but the corresponding expected values of the symbols, which in turn depend on the constellation used and the a-priori probabilities coming from the channel decoder. For BPSK, it can be seen that $\bar{x}_n = \tanh(L(x_n)/2)$.

The SIC can produce soft outputs in the form of Log-Likelihood ratios using the assumption that its output s_n is normally distributed. To this end, the following mapping is used:

$$L_e(x_n) = \ln\left(\frac{p(s_n|x_n=+1)}{p(s_n|x_n=-1)}\right) = \frac{2 \quad s_n}{{}^2} \tag{10}$$

where $p(x|x_n = a_i)$ is the p.d.f of the soft output of the SIC given $x_n = a_i$. The parameters and ^{2} can be computed via the relations

$$= | \quad {}_i|, \qquad {}^2 = \; {}^2_w + \bar{v}(\mathbf{q}_p^H \mathbf{q}_p + \mathbf{q}_f^H \mathbf{q}_f) \tag{11}$$

where $| \quad _i|$ is the amplitude of the symbols (assumed equal for all symbols) and $\bar{v}$ is the mean value of the variances of all symbols x_n based on the a-priori information

$$\bar{v} = \frac{1}{K} \; {}_{n=1}^{K} \; E[x_n^2] - E^2[x_n] = \frac{1}{K} \; {}_{n=1}^{K} \; 1 - \tanh^2(L(x_n)/2) \tag{12}$$

The SIC equalization method as described above, is summarized in Table 1.

Input: $\mathbf{h}, L_1, L_2, \; {}^2_w, L(x_n), z_n, k, l \quad n = 1\ldots S$
Output: $L_e(x_n) \qquad n = 1\ldots S$
1. Compute $\mathbf{p}$, $\mathbf{q}_p$ and $\mathbf{q}_f$ from (5), (6) and (7)
2. $\bar{x}_n = \tanh(L(x_n)/2) \quad n = 1\ldots S$
3. $v_n = 1 - \bar{x}_n^2 \quad n = 1\ldots S$
4. $\quad = 1, \; {}^2 = \; {}^2_w + \bar{v}(\mathbf{q}_p^H \mathbf{q}_p + \mathbf{q}_f^H \mathbf{q}_f)$
5. for $n = 1\ldots S$
$\quad \bar{\mathbf{x}}_{pn} = [\bar{x}_{n-1} \cdots \bar{x}_{n-N}]^T, \bar{\mathbf{x}}_{fn} = [\bar{x}_{n+K} \cdots \bar{x}_{n+1}]^T$
$\quad s_n = \mathbf{p}^T \mathbf{z}_n + \mathbf{q}_p^T \bar{\mathbf{x}}_{pn} + \mathbf{q}_f^T \bar{\mathbf{x}}_{fn}$
$\quad L_e(x_n) = 2 \quad s_n / \; {}^2$

Table 1: Summary of the S.I.C equalization method

4. THE NEW SIC TECHNIQUE

4.1 Enhancing the a-priori information

It has been proved [3] that the SIC is the MMSE optimal soft equalizer for Turbo Equalization in the case of perfect a-priori information (i.e. $|L(x_n)| \to$, or equivalently, as assumed earlier, the cancellation filter is fed by correct symbols). On the other hand, in the presence of weak a-priori information (i.e. low SNR and/or initial iterations) the SIC becomes suboptimal and its performance deteriorates significantly. To alleviate this problem we seek a way to enhance the a-priori Log-Likelihood ratio $L(x_n)$ coming from the decoder. This can be achieved by incorporating information from sequence z_n through the use of another equalizer, and in particular a Variable Threshold Decision Feedback Equalizer (VTDFE) described in the next section. This equalizer will also be refered hereafter as a pre-processor.

The block diagram of the combined scheme, so-called VTDFE-SIC, is depicted in Figure 2. The new Log-

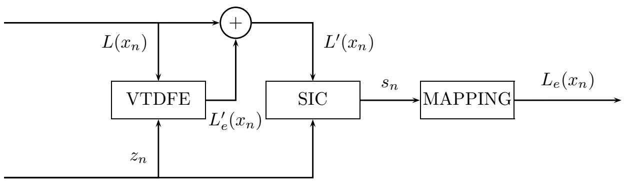

Figure 2: The proposed VTDFE-SIC equalizer

Likelihood ratios $L'(x_n)$ which are fed to the Soft Interference Canceller are now set equal to

$$L'(x_n) \;=\; \ln\left(\frac{P(x_n=+1)\;\;(s'_n|x_n=+1)}{P(x_n=-1)\;\;(s'_n|x_n=-1)}\right)$$
$$\;=\; L(x_n)+L'_e(x_n) \qquad (13)$$

where $\;(x|x_n=\;\;_i)$ is the p.d.f of the soft output of the preprocessor given $x_n=a_i$ and $P(x_n=a_i)$ stand for the a-priori probabilities coming from the decoder. For this modification to be valid we have to ensure that the output of the SIC remains extrinsic. Note that, $L'_e(x_{n+K})\cdots L'_e(x_{n+1})$ and $L'_e(x_{n-1})\cdots L'_e(x_{n-N})$ should not depend on $L(x_n)$, because otherwise these LLRs, via the cancellation filter of the SIC, will contribute to $L_e(x_n)$ at the output of the VTDFE-SIC scheme.

4.2 A Variable Threshold DFE

For the development of the DFE we will use a similar notation as in the previous section. The feedforward filter and the feedback filters are denoted as

$$\mathbf{a}=[a_{-k}\cdots a_0\cdots a_l]^T,\;\; M=k+l+1$$

and

$$\mathbf{b}=[b_1\cdots b_N]^T$$

respectively. As in standard DFE the input to filter $\mathbf{a}$ is the sampled output of the channel, while the input to filter $\mathbf{b}$ are past detected symbols. The soft output of the DFE is

$$s'_n=\mathbf{a}^T\mathbf{z}_n+\mathbf{b}^T\mathbf{x}_{pn} \qquad (14)$$

Hard decisions about the transmitted symbols are taken by passing s'_n through a decision device. The optimal filter coefficients that minimize the mean square error $E[|s'_n-x_n|^2]$, given the assumption that the feedback filter contains correct symbol estimates (optimal DFE) are given by the relations [6]

$$\mathbf{a}=\left(\mathbf{H}_1\mathbf{H}_1^H+\;^2_w\mathbf{I}\right)^{-1}\mathbf{H}_1\mathbf{d}' \qquad (15)$$

and

$$\mathbf{b}=-\mathbf{H}_2^H\mathbf{a} \qquad (16)$$

with matrix $\mathbf{H}_1$ containing the first L_1+k+1 columns and matrix $\mathbf{H}_2$ the rest L_2+l columns of $\mathbf{H}$. We also assume $N=L_2+l$. Vector $\mathbf{d}'$ is given by $\mathbf{d}'=[\mathbf{0}_{1\times(k+L_1)}\quad 1]^T$.

In the standard DFE, hard decisions $\hat{x}_n$ are made by comparing s'_n to a threshold equal to zero and subsequently are fed back to filter $\mathbf{b}$. Using such a fixed threshold, however, would deteriorate the performance of the succeeding SIC. To alleviate this problem we suggest using time-varying thresholds for the decision device by using a Bayesian classification rule which incorporates a-priory probabilities coming from the decoder. In particular the threshold t_n used for the decision $\hat{x}_n$ is found as the solution of

$$P(x_n=+1)\;\;(x|x_n=+1)=P(x_n=-1)\;\;(x|x_n=-1) \quad (17)$$

which becomes

$$t_n=-\frac{'^2L(x_n)}{2\;\;'} \qquad (18)$$

if we assume that the output of the DFE is Normally distributed, with variance $\;'^2$ and mean corresponding to symbol $+1$ equal to $\;'$. For higher order modulations ($q>1$), the equalizer must be supplied with the a-priori probabilities $P(x_n=\;\;_i), i=1\dots2^q$. Then, the decision rule consists in calculating all $P(x_n=\;\;_i)\;\;(x|x_n=\;\;_i), \forall i=1\dots2^q$ given $x=s_n$, and then deciding in favor of the symbol a_j that yields the maximum value. Clearly, in the presence of perfect a-priori information the VTDFE is identical to the optimal DFE, while for weak a-priori information it combines information from the channel and a-priori probabilities.

The soft output s'_n can be mapped to LLRs using

$$L'_e(x_n)=\ln\left(\frac{(s'_n|x_n=+1)}{(s'_n|x_n=-1)}\right)=\frac{2\;\;'s'_n}{'^2} \qquad (19)$$

The parameters $\;'$ and $\;'^2$ can be estimated via the relations

$$'=\mathbf{a}^T\mathbf{H}_1\mathbf{d}'|\;\;_i|,\qquad '^2=\;^2_w\mathbf{a}^H\mathbf{a} \qquad (20)$$

where $|\;\;_i|$ is the amplitude of the symbols (assumed equal for all symbols).

The pre-processing performed by the VTDFE reveals clearly that $L(x_{n+1}),\dots,L(x_{n+N})$ depend on the decision $\hat{x}_n$ whose computation in turn depends on $L(x_n)$ (that was used to determine the threshold). It can be seen, however, that this dependence is very weak, due to the use of hard decisions. Therefore, we deduce that, using the VTDFE as a preprocessing stage, the output of the SIC remains extrinsic. This would not be the case if another soft equalizer were used at the preprocessing stage. Tables 2 and 3 summarize the VTDFE preprocessor and the combined VTDFE-SIC schemes, respectively. Note that functions $SIC(\cdot)$ and $VTDFE(\cdot)$ appearing in Table 3 correspond to the algorithms of Tables 1 and 2, respectively.

Input: $\mathbf{h}, L_1, L_2,\;\;^2_w, L(x_n), z_n, k, l \quad n=1\dots S$ Output: $L'_e(x_n) \quad n=1\dots S$
1. Compute $\mathbf{a}$ and $\mathbf{b}$ from (15) and (16) 2. $\;'=\mathbf{a}^T\mathbf{H}_1\mathbf{d}',\quad '^2=\;^2_w\mathbf{a}^H\mathbf{a}$ 3. for $n=1\dots S$ $\quad s'_n=\mathbf{a}^T\mathbf{z}_n+\mathbf{b}^T\hat{\mathbf{x}}_n$ $\quad$ if $(s'_n\geq t_n)$ $\hat{x}_n=+1$ else $\hat{x}_n=-1$ $\quad L'_e(x_n)=2\;\;'s'_n/\;\;'^2$

Table 2: Summary of the VTDFE pre-processor

5. SIMULATION RESULTS AND CONCLUSION

To test the performance of the proposed VTDFE-SIC technique we performed some typical experiments. Information

Input: $\mathbf{h}, L_1, L_2, \ {}^2_w, L(x_n), z_n, k, l \quad n = 1 \ldots S$
Output: $L_e(x_n) \qquad n = 1 \ldots S$

1. $L'_e(x_n) = VTDFE(\mathbf{h}, L_1, L_2, \ {}^2_w, L(x_n), z_n, k, l)$
2. $L_e(x_n) = SIC(\mathbf{h}, L_1, L_2, \ {}^2_w, L(x_n) + L'_e(x_n), z_n, k, l)$

Table 3: Summary of the VTDFE-SIC equalization method

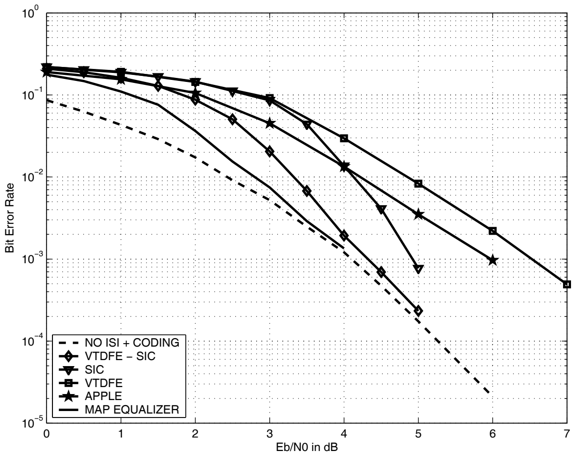

Figure 3: Performance results over the Proakis B channel

bits were generated in bursts of $S = 2048$ bits. Then an R.S.C. code with generator matrix $G(D) = [1\,\frac{1+D^2}{1+D+D^2}]$ with rate $1/2$ was applied, and the resulting bits were BPSK modulated. The 4096 symbols per burst were interleaved via the use of a random interleaver and then transmitted to a channel whose impulse response was set either $h_{-1} = 0.407, h_0 = 0.815, h_1 = 0.407$ (channel B of [7]) or $h_{-2} = 0.227, h_{-1} = 0.46, h_0 = 0.688, h_1 = 0.46, h_2 = 0.227$ (channel C of [7]).

We have compared the new VTDFE-SIC with the so-called APPLE scheme [8] which has a comparable computational complexity, and the conventional SIC [3]. The performance of the VTDFE alone (since this equalizer can also be used by its own in the Turbo Equalization framework) has been tested as well. The performance curves depicted in Figures 3 and 4 have been taken after 5 iterations and for at least 1000 symbol error events.

As shown in Figure 3 the VTDFE-SIC scheme has a superior performance compared to the other linear complexity schemes approaching the performance of the AWGN channel. The SIC performance is poor for low SNRs but we then notice a very steep slope which implies a good performance at higher SNRs. The APPLE and the VTDFE equalizers exhibit poor performance at all SNR regions.

As we can see in Figure 4 the APPLE equalizer has the best performance at low SNRs, but at higher SNRs the VTDFE-SIC scheme outperforms the APPLE. The VTDFE equalizer is about 3.5 dB worse than the VTDFE-SIC while the SIC fails to operate acceptably.

Concluding, the performance of the Soft Interference Canceller has been improved by modifying the Log-Likelihood ratio of its input via the use of a new low-complexity DFE. The so-called VTDFE equalizer seems to be a good pre-processor for this task. The resulting overall

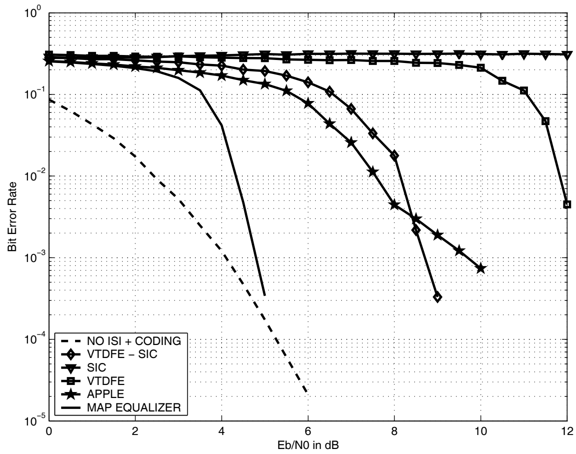

Figure 4: Performance results over the Proakis C channel

scheme has linear complexity and attains better performance than equalizers with comparable complexity. Future work will focus on unknown and time varying channels.

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