INVESTIGATION OF THE CHANNEL ESTIMATION ERROR ON MIMO SYSTEM PERFORMANCE

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ABSTRACT

Multi-Input Multi-Output (MIMO) wireless communication systems provide large capacity allowing high data rates transmission. However, this huge increase of the capacity requires perfect channel knowledge at the receiver. In this paper, we analyze the effect of imperfect channel knowledge on the achievable rates. More precisely, we express a lower bound on the capacity as a function of the channel Cramer-Rao bound (CRB). In previous works, we made the calculation and comparison of CRB for different channel estimation schemes [1] and [2]. This allows us to make comparison of channel achievable rates for different context under various data and pilot design assumptions.

1. INTRODUCTION

MIMO channel capacity was widely investigated under different assumptions on channel state information (CSI) which corresponds to the channel response estimate. First results on MIMO channels capacity with perfect CSI at the receiver (CSIR) or both at the transmitter (CSIT) and the receiver was provided by Telatar [3]. In [4], Medard derived bounds on mutual information with imperfect CSIR and no CSIT for time-varying communication in a SISO channel model. Caire and Shamai, in [5], studied some channels with imperfect CSIT and perfect CSIR. The CSI can be driven by blind, semi-blind or training sequence based techniques. The effects of training sequence based estimation of MIMO channels on achievable data rates was already analyzed in [6], [7].

Semi-blind techniques were proposed to overcome waste of channel throughput by training symbols and to improve channel estimation. Channel estimation performance of semi-blind methods was investigated in [8] and [1] by considering (CRB) for different pilot symbol design.

In this paper, we would like to compare semi-blind and training sequence based estimation methods regarding achievable rates. We investigate the following questions:

- Relation of channel achievable rate to the channel CRB.
- Comparison of semi-blind and training sequence based channel estimation errors effect on channel achievable rate.

2. CHANNEL MODEL

We consider a MIMO system equipped with K transmit and M receive antennae. The MIMO channel is assumed to follow the Rayleigh block fading model. This means that the channel matrix has i.i.d coefficients with Rayleigh distribution that are constant for T transmitted consecutive symbols and changing independently to a new realization in the next interval. The complex baseband representation of the received signal is given by

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{N} \tag{1}$$

where $\mathbf{S} \in \mathbb{C}^{K\times T}$ is the matrix of the transmitted signal block during T time samples, i.e during a channel realization, $\mathbf{Y} \in \mathbb{C}^{M\times T}$ is the observed matrix during T time samples and $\mathbf{H} \in \mathbb{C}^{M\times K}$ is the channel gain matrix.

$$\mathbf{S} = \begin{bmatrix} s_1(1) & \dots & s_1(T) \\ \vdots & \ddots & \vdots \\ s_K(1) & \dots & s_K(T) \end{bmatrix};\ \mathbf{Y} = \begin{bmatrix} y_1(1) & \dots & y_1(T) \\ \vdots & \ddots & \vdots \\ y_M(1) & \dots & y_M(T) \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} h_{11} & \dots & h_{1K} \\ \vdots & \ddots & \vdots \\ h_{M1} & \dots & h_{MK} \end{bmatrix};\ \mathbf{N} = \begin{bmatrix} n_1(1) & \dots & n_1(T) \\ \vdots & \ddots & \vdots \\ n_M(1) & \dots & n_M(T) \end{bmatrix}$$

The channel is Rayleigh i.e, the channel gains from the k^{th} transmit antenna to the m^{th} receive antenna, h_{mk} with $m = 1,\dots,M$ and $k = 1,\dots,K$ are assumed to be i.i.d complex Gaussian variables. $\mathbf{N}$ is the noise matrix which is assumed to be with i.i.d circular complex Gaussian zero mean entries with variance $\ _n^2$. At each time instant, the received vector is

$$\mathbf{y}(t) = \mathbf{H}\mathbf{s}(t) + \mathbf{n}(t)$$

In the following, the time index t will be omitted.

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \bar{\mathbf{S}}\mathbf{h} + \mathbf{n} \tag{2}$$

with $\bar{\mathbf{S}} = \mathbf{s}^T \otimes \mathbf{I}$ where $\otimes$ is the Kronecker symbol $\mathbf{s} = [s_1 \dots s_K]^T$ and $\mathbf{h} = \mathrm{vec}(\mathbf{H}) = [\mathbf{h}_1^T \dots \mathbf{h}_K^T]^T$ the vector in which the columns $\mathbf{h}_k$ with $k = 1 \dots K$ of $\mathbf{H}$ are stacked.

2.1 Pilot Design

Semi-blind and training sequence estimation techniques require the insertion of known symbols. In training sequence estimation, known symbols are placed at the beginning of the frame and are used for the channel estimation. In semi-blind estimation, known symbols can be time-multiplexed with data symbols (see Fig.1) or embedded (see Fig.2) to them. The transmitted power is divided between data and pilot symbols. Let us consider the model in (2), where the k^{th} transmitted symbol s_k is decomposed into data and pilot parts:

$$s_k = s_{d,k} + s_{p,k} \tag{3}$$

Here $s_{d,k}$ are either zero or i.i.d data symbols with zero mean and variance $\ _d^2$ and $s_{p,k}$ are pilot symbols with allocated power ^2_p. This model applies to both time-multiplexed pilot scheme and embedded pilot scheme. For the time-multiplexed-pilot scheme, $s_{p,k} = 0$ (resp. $s_{d,k} = 0$) when a data (resp. pilot) symbol is transmitted. Whatever the pilot design, the channel input/output relationship described by (2) can be rewritten as follows (using similar notation as above):

$$\mathbf{y} = \mathbf{H}(\mathbf{s}_d + \mathbf{s}_p) + \mathbf{n} = (\bar{\mathbf{S}}_d + \bar{\mathbf{S}}_p)\mathbf{h} + \mathbf{n}$$

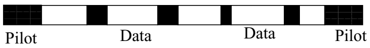

Figure 1: Time-multiplexed pilot scheme

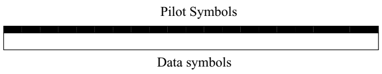

Figure 2: Embedded pilot scheme

3. CHANNEL ESTIMATION

In this paper, we do not consider a specific channel estimation error due to particular estimation method (ML, MMSE). In presence of estimation error one can write, $\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$, where $\hat{\mathbf{H}}$ is the channel estimate and $\tilde{\mathbf{H}}$ is the channel estimation error. Therefore, equation (2) becomes:

$$\mathbf{y} = \hat{\mathbf{H}}\mathbf{s} + \mathbf{v} \quad \text{with} \quad \mathbf{v} = \tilde{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

We make the following assumptions on the channel estimate and the channel estimation error.

- (A1) The coefficients of the channel estimate matrix $\hat{h}_{ij}$ are complex Gaussian i.i.d. It follows that coefficients of the channel estimation error matrix $\tilde{h}_{ij}$ are also complex Gaussian.
- (A2) The channel estimation error is zero mean, i.e we consider only unbiased estimators (or asymptotically unbiased estimators).
- (A3) Channel estimation error $\tilde{\mathbf{H}}$ is uncorrelated with the data vector $\mathbf{s}$.

The later assumption, is an asymptotic (i.e for large sample size T) approximation as shown in [9]. Afterward, we will study the effective noise $\mathbf{v}$ statistics. Assumptions (A2) and (A3) implies that $\mathbf{v}$ is zero mean. Its covariance matrix $\mathbf{R_v}$ is given by $\mathbf{R_v} = E[\tilde{\mathbf{H}}\mathbf{s}\mathbf{s}^H\tilde{\mathbf{H}}^H] + {}^2_n\mathbf{I}$ with

$$\mathbf{R_v}(i,j) = E[\mathbf{v}_i\mathbf{v}_j^\star] = \sum_{k=1}^{K}\sum_{k'=1}^{K} E[\tilde{h}_{ik}s_k s_{k'}^\star \tilde{h}_{jk'}^\star] + E[n_i n_j^\star]$$

The channel estimation error $\tilde{\mathbf{H}}$ and data symbols are uncorrelated.

$$E[\tilde{h}_{ik}s_k s_{k'}^\star \tilde{h}_{jk'}^\star] = E[s_k s_{k'}^\star]E[\tilde{h}_{ik}\tilde{h}_{jk'}^\star]$$

The data symbols being i.i.d, it follows that:

$$E[s_k s_k^\star] = \begin{cases} {}^2_d + {}^2_p & \text{for embedded pilot scheme} \\ {}^2_d & \text{for time-multiplexed scheme} \end{cases}$$

Let us denote ${}^2_s = E[s_k s_k^\star]$

$$\mathbf{R_v}(i,j) = \begin{cases} {}^2_s \sum_{k=1}^{K} E[\tilde{h}_{ik}\tilde{h}_{jk}^\star] & i \neq j \\ {}^2_s \sum_{k=1}^{K} E[|\tilde{h}_{ik}|^2] + {}^2_n & j = i \end{cases}$$

Then, the expression of the covariance matrix of the effective noise, whatever the considered estimation technique, is

$$\mathbf{R_v} = {}^2_s E[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H] + {}^2_n\mathbf{I} \tag{4}$$

4. CHANNEL CRAMER-RAO BOUND

In this section, we seek to relate the effective noise covariance matrix to the CRB. The CRB is defined as a lower bound on the mean square error of an unbiased estimator $\hat{}$ of a parameter .

$$E[(\hat{} -)(\hat{} -)^H] \geq CRB$$

We take $= \mathbf{h}$. CRB_k denotes the CRB on the complex parameter vector $\mathbf{h}_k$. Consequently,

$$\begin{aligned} E[\tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^H] &\geq CRB_k \quad \forall\ k = 1\ldots K \\ \sum_{k=1}^{K} E[\tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^H] &\geq \sum_{k=1}^{K} CRB_k \end{aligned}$$

We notice that $E[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H] = \sum_{k=1}^{K} E[\tilde{\mathbf{h}}_k\tilde{\mathbf{h}}_k^H]$. Then, we deduce the underneath lower bound on the effective noise covariance matrix:

$$\mathbf{R_v} \geq {}^2_s \sum_{k=1}^{K} CRB_k + {}^2_n\mathbf{I} \tag{5}$$

5. MUTUAL INFORMATION LOWER BOUND LINK TO CHANNEL CRB

5.1 Mutual information lower bound

Mutual information (MI) lower bound for imprecise CSIR was already derived for SISO channel in [4] and for MMSE MIMO channel estimator based on training sequence in [7] and [6]. In this section, we derive MI lower bound for imprecise CSIR independently of the MIMO channel estimation technique under the assumptions listed in the previous section. We, therefore, relate it to the channel CRB. To make this paper fully independent, we include most of calculation steps. The ergodic capacity of the fading channel with an estimated channel $\hat{\mathbf{H}}$ at the receiver is given by:

$$\mathbf{C} = \max_{p(\mathbf{s}_d)} E_{\hat{\mathbf{H}}}\left[\mathbf{I}(\mathbf{s}_d;\mathbf{y}|\hat{\mathbf{H}})\right] \tag{6}$$

where $\mathbf{I}(.;.)$ denotes the MI. We begin by expanding the conditional MI:

$$\mathbf{I}(\mathbf{s}_d;\mathbf{y}|\hat{\mathbf{H}}) = (\mathbf{s}_d|\hat{\mathbf{H}}) - (\mathbf{s}_d|\mathbf{y},\hat{\mathbf{H}})$$

where represents the entropy function. The first term can be upper bounded as in [4].

$$(\mathbf{s}_d|\mathbf{y},\hat{\mathbf{H}}) \leq log| \; e\mathbf{\Sigma}_{\mathbf{s}_d-\mathbf{Ay}}| \tag{7}$$

$\mathbf{\Sigma}_{\mathbf{s}_d-\mathbf{Ay}}$ is the covariance matrix of $\mathbf{s}_d - \mathbf{Ay}$ where $\mathbf{A}$ is a $K \times M$ matrix picked such that $\mathbf{\Sigma}_{\mathbf{s}_d-\mathbf{Ay}}$ is minimized. This is the case when $\mathbf{Ay}$ is the MMSE estimate of $\mathbf{s}_d$, in which case

$$\mathbf{\Sigma}_{\mathbf{s}_d-\mathbf{Ay}} = \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d} - \mathbf{R}_{\mathbf{s}_d\mathbf{y}}\mathbf{R}_{\mathbf{yy}}^{-1}\mathbf{R}_{\mathbf{ys}_d}$$

where $\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}$ is the covariance matrix of the data signal vector $\mathbf{s}_d$. We assume that the channel estimation error, the Gaussian noise and the transmitted signal $\mathbf{s}_d$ are all uncorrelated and of zero mean. This assumption leads to the following expression of $\mathbf{R}_{\mathbf{s}_d\mathbf{y}}$, $\mathbf{R}_{\mathbf{yy}}$ and consequently $\mathbf{\Sigma}_{\mathbf{s}_d-\mathbf{Ay}}$

$$\mathbf{R}_{\mathbf{s}_d\mathbf{y}} = \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H, \qquad \mathbf{R}_{\mathbf{yy}} = \hat{\mathbf{H}}\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H + \mathbf{R}_{\mathbf{v}}$$

$$\begin{aligned}\mathbf{\Sigma}_{\mathbf{s}_d-\mathbf{Ay}} &= (\mathbf{I} - \mathbf{R}_{\mathbf{s}_d\mathbf{s}}\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H + \mathbf{R}_{\mathbf{v}})^{-1}\hat{\mathbf{H}})\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d} \\ &= (\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H\mathbf{R}_{\mathbf{v}}^{-1}\hat{\mathbf{H}})^{-1}\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\end{aligned}$$

Then, inequality (7) becomes:

$$(\mathbf{s}_d|\mathbf{y},\hat{\mathbf{H}}) \leq \log| \quad e(\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H\mathbf{R}_{\mathbf{v}}^{-1}\hat{\mathbf{H}})^{-1}\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}|$$

Consequently, a lower bound on the MI for a channel estimator $\hat{\mathbf{H}}$ is given by:

$$\mathbf{I}(\mathbf{s}_d;\mathbf{y}|\hat{\mathbf{H}}) \geq \quad (\mathbf{s}_d|\hat{\mathbf{H}}) - \log| \quad e(\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H\mathbf{R}_{\mathbf{v}}^{-1}\hat{\mathbf{H}})^{-1}\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}|$$

The Gaussian distribution is the MI maximizing distribution in AWGN Rayleigh flat fading channel with perfect channel knowledge at both receive and transmit sides or only perfect knowledge at the receiver and absence of channel knowledge at the transmitter, see [3]. Under this assumption on the data distribution, $(\mathbf{s}_d|\hat{\mathbf{H}}) = log| \quad e\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}|$. Then, we consider Gaussian distribution to simplify the lower bound calculation.

$$\mathbf{I}(\mathbf{s}_d;\mathbf{y}|\hat{\mathbf{H}}) \geq \log|\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H\mathbf{R}_{\mathbf{v}}^{-1}\hat{\mathbf{H}}| \qquad (8)$$

The bound found in [6] is a particular case of (8) because it corresponds to the training sequence based MMSE channel estimation. However, this bound is true for all channel unbiased estimation techniques. The difference between two channel estimators resides into the expression of $\mathbf{R}_{\mathbf{v}}$ that includes the channel estimation error covariance matrix. The $\log|.|$ function is an increasing function on the cone of positive definite Hermitian matrices. Therefore, a tight lower bound on the MI is:

$$\mathbf{I}(\mathbf{s}_d;\mathbf{y}|\hat{\mathbf{H}}) \geq \log|\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H(\ {}_s^2 \sum_{k=1}^{K} CRB_k + \ {}_n^2\mathbf{I})^{-1}\hat{\mathbf{H}}|$$

5.2 Ergodic capacity lower bound

The bound on the mutual MI induces a lower bound on the ergodic capacity of the channel. In the time-multiplexed scheme, we assume that within a frame of length T, T symbols are pilot. Consequently,

$$\mathbf{C} \geq E_{\hat{\mathbf{H}}}\frac{T-T}{T}\log|\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H(\ {}_s^2 \sum_{k=1}^{K} CRB_k + \ {}_n^2\mathbf{I})^{-1}\hat{\mathbf{H}}| \qquad (9)$$

Otherwise, if pilot symbols are embedded with data symbols the lower bound on the ergodic capacity is

$$\mathbf{C} \geq E_{\hat{\mathbf{H}}}\log|\mathbf{I} + \mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\hat{\mathbf{H}}^H(\ {}_d^2 \sum_{k=1}^{K} CRB_k + \ {}_n^2\mathbf{I})^{-1}\hat{\mathbf{H}}| \qquad (10)$$

Note that the CRB in (9) is different from that in (10) since the pilot design is not the same.

5.3 Semi-blind CRB and training sequence CRB

In this subsection, we will recall the CRBs expressions of the semi-blind and training sequence based estimation techniques. As the data symbols are Gaussian we consider the channel estimation error is the Gaussian CRB derived in [2] for MIMO channels. We remind that $CRB = \mathbf{J}_{\mathbf{hh}}^{-1}$ where $\mathbf{J}_{\mathbf{hh}}$ is the Fisher information matrix (FIM).

- FIM of semi-blind estimation

$$\begin{aligned}\mathbf{J}_{\mathbf{hh}}(i,j) &= (\bar{\mathbf{S}}_p^H\mathbf{R}_{\mathbf{yy}}^{-1}\bar{\mathbf{S}}_p)(i,j) \\ &+ trace\left\{\mathbf{R}_{\mathbf{yy}}^{-1}\left(\frac{\mathbf{R}_{\mathbf{yy}}}{\mathbf{h}_i^\star}\right)\mathbf{R}_{\mathbf{yy}}^{-1}\left(\frac{\mathbf{R}_{\mathbf{yy}}}{\mathbf{h}_j^\star}\right)^H\right\}\end{aligned} \qquad (11)$$

with $\mathbf{R}_{\mathbf{yy}} = \mathbf{H}\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d}\mathbf{H}^H + \ {}_n^2\mathbf{I}$.

- Gaussian CRB of training sequence estimation

$$\mathbf{J}_{\mathbf{hh}} = \frac{1}{\ {}_n^2}\bar{\mathbf{S}}_p^H\bar{\mathbf{S}}_p \quad \text{then} \quad CRB = \ {}_n^2(\bar{\mathbf{S}}_p^H\bar{\mathbf{S}}_p)^{-1}$$

6. ERGODIC CAPACITY IN CORRELATED FADING CHANNEL

In this section, we make use of results on ergodic capacity of correlated Rayleigh flat fading channels to have an explicit analytic expression of the bounds in (9) and (10). The correlated MIMO channel can be described by the matrix product $\mathbf{H} = \mathbf{A_r}^H\mathbf{H_w}$ where $\mathbf{H_w}$ is a $M \times K$ matrix of complex i.i.d Gaussian entries of unit variance and $\mathbf{A_r}\mathbf{A_r}^H = \mathbf{R_{rx}}$ where $\mathbf{R_{rx}}$ is the receive correlation matrix. The ergodic capacity of a Rayleigh flat fading correlated MIMO channel in Gaussian noise (when time-multiplexed pilot scheme is used) was derived in [10].

$$\mathbf{C} = E_{\hat{\mathbf{H}}}\frac{T-T}{T}\log|\mathbf{I} + \mathbf{R}_{\mathbf{s_d s_d}}\mathbf{H}^H\mathbf{R}_{\mathbf{nn}}^{-1}\mathbf{H}| \qquad (12)$$

$\mathbf{R_{nn}}$ is the Gaussian noise covariance matrix. Let $=$ $\text{eig}(\mathbf{R}_{\mathbf{nn}}^{-1}\mathbf{R_{rx}}) = \text{diag}(\ {}_1,\ldots, \ {}_R)$ and $= \text{eig}(\mathbf{R}_{\mathbf{s}_d\mathbf{s}_d})$ where $\text{eig}(A)$ represents the diagonal matrix of the eigenvalues of A.

$$\mathbf{C} = E_{\mathbf{H}_w}\frac{T-T}{T}\log|\mathbf{I} + \mathbf{\Sigma}\mathbf{H}_w^H\mathbf{\Omega}\mathbf{H}_w| \qquad (13)$$

The explicit expression of (13) is given by a theorem in [10].

Theorem 1 *The ergodic capacity of a receive correlated MIMO system with K input and M output antennae* $(M \geq K)$

$$\mathbf{C} = \frac{(-1)^{\frac{M(M-1)}{2}}(-1)^{-(M-K)K}| \quad |^{M-K-1}}{ln2 \quad {}_M(\ ) \quad {}_K(K)} \sum_{l=1}^{K} \left| \begin{matrix}\Xi_M(l) \\ \mathbf{\Psi}_M\end{matrix} \right|$$

where $\Xi_M(l)$ *with* $l = 1,\ldots K$ *are* $K \times M$ *matrices*

$$\Xi_M(l)(i,j) = \begin{bmatrix} \mathbf{\Lambda}_1(\frac{1}{\ {}_j},1,i) & i = l \\ (i) \ {}_j^{\ i} & i \neq l \end{bmatrix}$$

and the $(M-K) \times M$ *matrix* $\mathbf{\Psi}_M$ *are given by*

$$\mathbf{\Psi}_M(i,j) = (\frac{1}{\ {}_j})^{M-K-i}$$

$$\mathbf{\Lambda}_1(\frac{1}{\ {}_j},1,i) = \quad (i)e^{(1/ \ {}_j)}. \sum_{k=1}^{i-1} \quad (-i+k, \ {}_j) \ {}_j^{\ k}$$

$$(-i, \frac{1}{j}) = \frac{(-1)^i}{i!}\left[E_l(\frac{1}{j}) - e^{-\frac{1}{j}} {}_{j=0}^{i-1} \frac{(-1)^j j!}{\frac{1}{j}^{j+1}}\right]$$

with $E_l(\frac{1}{j}) = \int_{\frac{1}{j}}^{+} \frac{e^{-t}}{t}dt$, ${}_M(\boldsymbol{\Omega}) = {}_{i<j}({}_i - {}_j)$ and ${}_K(K) = {}_{i=1}^{K} (M-i+1)$, (x) standard Gamma function.

The explicit expressions of (9) and (10) come directly from this theorem by replacing $\mathbf{R}_{\mathbf{nn}}^{-1}\mathbf{R}_{\mathbf{rx}}$ by $\mathbf{R_v}^{-1}$. Then $= \mathrm{eig}(\mathbf{R_v}^{-1})$. The case of $K \geq M$ can be reduced to an equivalent problem by switching $\boldsymbol{\Sigma}$ and $\boldsymbol{\Omega}$.

$$\mathbf{C} = E_{\mathbf{H}_w} \frac{T-T}{T} \log|\mathbf{I} + \boldsymbol{\Omega}\mathbf{H}_w\boldsymbol{\Sigma}\mathbf{H}_w^H|$$

7. EXPERIMENTAL RESULTS

In Fig. 3, we compare the trace of the Gaussian CRBs for different channel estimation techniques. We consider training sequence estimation and semi-blind estimation with the two possible pilot designs time-multiplexed and embedded pilot. We consider MIMO systems with $K = 2, 6$ and $M = 2$. For the time-multiplexed scheme, we consider packets of length $T = 100$ in which T symbols are allocated to pilots, with equal data and pilot power ${}^2_d = {}^2_p = 1$. However, for the embedded pilot scheme, the same packet length is considered but in this case 100 data symbols are transmitted in addition with 100 pilot symbols. We consider a power allocation scheme where the power allocated to T superimposed (data+pilot) symbols is the same as that allocated to time-multiplexed $T - T$ data and T pilot symbols. Therefore, in the embedded pilot case, the number of pilot symbols is unchanged ($T = 100$) but their power changes according to the previous rule. In Fig.4, we compare the achievable rates for

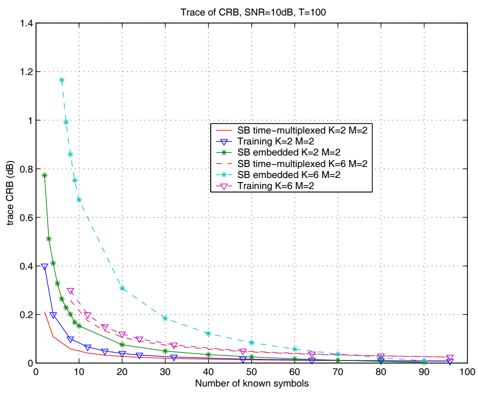

Figure 3: CRBs with semi-blind and training sequence estimation

these different estimation techniques and pilot designs. We noticed that:

- The semi-blind technique with time-multiplexed pilot symbols outperforms the training sequence based one particularly when a few number of pilot symbols is used.
- The semi-blind technique with embedded pilot outperforms the two former techniques at a given threshold on the number of pilot symbols. Even if the embedded pilot scheme allows higher achievable rates this doesn't mean that the embedded pilot scheme would be preferred to the time-multiplexed one since we should not only compare the achievable rates but also the BER (or FER).

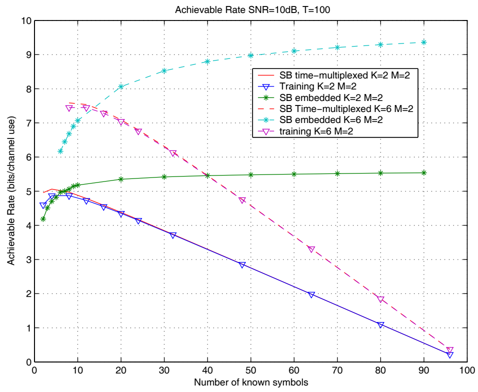

Figure 4: Rates with semi-blind and training sequence estimation

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