A TWO PARALLEL EXTENDED KALMAN FILTERING ALGORITHM FOR THE ESTIMATION OF CHIRP SIGNALS IN NON-GAUSSIAN NOISE

Mounir DJEDDI, Messaoud BENIDIR

Laboratoire des Signaux et Systèmes (L2S)
Supélec, 3 rue Joliot-Curie, Plateau de Moulon,
91190 Gif-sur-Yvette (France)
Phone: +33 1 6985 1763, fax: +33 1 6985 1765,
Email: djeddi@lss.supelec.fr

ABSTRACT

In this paper, we address the problem of the estimation of chirp signals in "ε-contaminated" impulsive noise using Kalman filtering technique. We consider an estimation method based on the exact non linear state space representation of the chirp signal. The observation noise's probability density function is assumed to be a sum of two-component Gaussians weighted by the probability of appearance of the impulsive and gaussian noises in the observations. We propose to use two extended Kalman filters (PEKF) operating in parallel as an alternative to the usual methods which generally use either clipping or freezing based algorithms. Simulation results show that the PEKF compared to the robust extended Kalman filter (REKF) based on Huber's function is less sensitive to impulsive noise and gives better estimates of the chirp parameters.

1. INTRODUCTION

Chirp signals (also called polynomial phase signals of order 2) arise in many natural phenomena like seismic and bat echolocation signals, but are mainly known in engineering applications such as in radar, sonar and telecommunication. The estimation of chirp signals is a well known problem in signal processing community and has received considerable interest in literature. Several methods have been proposed for the estimation of the parameters of chirp signals affected by additive Gaussian noise. Most of these methods rely on maximum likelihood principle (ML), LMS/RLS estimators, and time-frequency representations [1].

In signal processing, a widely used approach to estimate the signal parameters is the Kalman filter [2] which is the optimal tracking algorithm when the signal models are assumed linear and both state and observation noises are additive Gaussian [3]. When these assumptions do not hold, that is when one have to tackle non linear models, then the Extended Kalman Filter (EKF) [4] is used by considering a local linearization using first order Taylor expansion of the non-linear equations. Many algorithms based on the EKF with various configurations have been proposed. For instance, the authors in [5] proposed a state space model obtained by incorporating spatial information consisting of the corrupted signal and its delayed version using two sensors, then the chirp signal parameters were estimated using two EKFs in cascade. Still, in many situation the observation noise is non Gaussian. In [6], the authors considered the estimation of chirp signals in additive/multiplicative non-Gaussian noise using ML and LSE estimators.

In this paper, we consider the estimation of the parameters of a chirp signal corrupted by additive noise based on a state-space representation using two Kalman filters operating in parallel. In our approach, we consider the exact non linear state-space model for mono-component chirp signal derived in [3], but we assume the additive noise is impulsive with a non Gaussian distribution to obtain a non linear/ non Gaussian state space model. It is well known that impulsive noise due to natural phenomena or man-made applications [7] such as atmospheric disturbances affecting HF communication significantly degrades the performance of most frequency tracking EKF based algorithms in which noise is assumed Gaussian [8]. Many models for the impulsive noise can be considered such as α-stable distributions which do not have a general closed form distribution. A more practical model for the pdf of the impulsive noise is the sum of two weighted Gaussian density function. One way to use Kalman filtering in impulsive environment is by either clipping the observation signal or by changing the Kalman gain with respect to a defined threshold. In order to overcome the limitations due to these techniques, we propose to use parallel Kalman filtering whose principle is that each extended Kalman filter is tuned on one Gaussian component and their estimates are weighted to produce the final state estimate.

This paper is organized as follows. In section 2, we will present the exact non linear state space modelisation of mono-component chirp signal where the additive noise has a non Gaussian distribution. In section 3, we describe the PEKF algorithm for the estimation of the non linear/ non Gaussian state space model. Section 4 provides simulation results and comparison with respect to the robust extended Kalman (REKF). Finally, we give some concluding remarks and perspective work in section 5.

2. NON LINEAR/NON GAUSSIAN STATE-SPACE REPRESENTATION OF CHIRP SIGNAL

We consider the discrete signal $y(k)$ modeled by a polynomial phase signal of order 2 : $y(k)$ embedded in additive noise $v(k)$ as given below

$$y(k) = A(k)e^{\{j\phi(k)\}} + v(k) \qquad k = 0, ..., N \tag{1}$$

where $A(k)$ is the amplitude of the signal which can be constant or time varying, and ϕ is a deterministic polynomial phase given by

$$\phi(k) = \frac{\alpha}{2}k^2 + \beta k + \gamma \tag{2}$$

The parameters α, β and γ are real coefficients.

In this paper we will consider the real part of $y(k)$

$$z(k) = \Re\{y(k)\} = A(k)\cos(\phi(k)) + n(k) \tag{3}$$

and

$$n(k) = \Re\{v(k)\} \tag{4}$$

The noise $n(k)$ is modeled by

$$n(k) = r(k) + i(k) \tag{5}$$

where $r(k)$ is an additive Gaussian noise with zero mean and variance σ_1^2, and $i(k)$ is the impulsive noise with a probability of appearance equal to ε and it is assumed to be a zero mean Gaussian noise with variance σ_2^2 where $(\sigma_2^2 >> \sigma_1^2)$. Typically the ratio (σ_2^2/σ_1^2) is between 100 to 500 [9].

The probability density function (pdf) of the noise $n(k)$ is given by

$$p(n(k)) = (1-\varepsilon)\mathcal{N}(0,\sigma_1^2) + \varepsilon\mathcal{N}(0,\sigma_2^2) \tag{6}$$

Figure 1 shows the pdf of the noise with respect to ε.

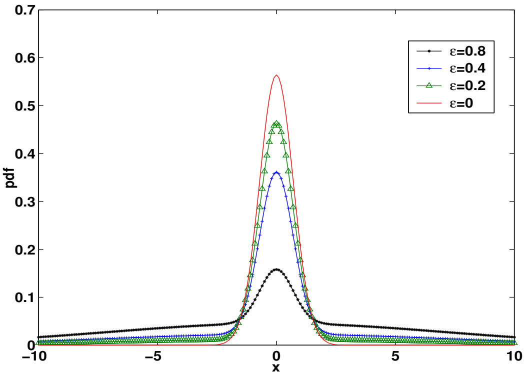

Figure 1: The pdf of the noise for different values of ε

2.1 The exact Model

Defining the state vector of dimension 4

$$\mathbf{x}(k) = \begin{bmatrix} A(k) & \phi(k) & \Delta\phi(k) & \Delta^2\phi(k) \end{bmatrix}^T \tag{7}$$

where

$$\Delta\phi(k) = \frac{1}{2}(\phi(k+1) - \phi(k-1)) \tag{8}$$

and

$$\Delta^2\phi(k) = \frac{1}{2}(\Delta\phi(k+1) - \Delta\phi(k-1)) \tag{9}$$

and the observation equation is given by

$$z(k) = A(k)\cos(\phi(k)) + n(k) \tag{10}$$

which can be rewritten as

$$z(k) = x_1(k)\ \cos(x_2(k)) + n(k) \tag{11}$$

Now assuming that the amplitude of the signal follows a random walk model

$$A(k) = A(k-1) + w(k) \tag{12}$$

Hence, the state space model associated with the chirp signal $z(k)$ can written as a linear state equation and a non-linear observation equation

$$\begin{array}{c} \mathbf{x}(k+1) = F\mathbf{x}(k) + Gw(k) \\ z(k) = h(\mathbf{x}(k)) + n(k) \end{array} \tag{13}$$

where

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } G = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T \tag{14}$$

The nonlinear function $H(\mathbf{x}(k))$ is

$$h(\mathbf{x}(k)) = x_1(k)\ \cos(x_2(k)) \tag{15}$$

In order to use the EKF, we linearize the observation function around the current vector estimate

$$H(k) = \left.\frac{\partial h(\mathbf{x}(k))}{\partial \mathbf{x}(k)}\right|_{\mathbf{x}=\hat{\mathbf{x}}} = \begin{bmatrix} \cos(\hat{x}_2(k)) & -\hat{x}_1(k)\sin(\hat{x}_2(k)) & 0 & 0 \end{bmatrix} \tag{16}$$

3. ROBUST ESTIMATION BASED ON THE PARALLEL EKF ALGORITHM

Based on [10], the author in [11] proposed a network of Kalman filters (NKF) in the case where the the observation noise is Gaussian while the state noise is non Gaussian. In [12] the authors proposed a NKF to estimate a linear/non Gaussian state space model for channel equalization problem. A modified version of this algorithm is proposed here, it is based on the computation of the a posteriori pdf of the state

$$p(\mathbf{x}(k)|Z^k) = \frac{p(\mathbf{x}(k)|Z^{k-1})p(z(k)|\mathbf{x}(k))}{p(z(k)|Z^{k-1})} \tag{17}$$

where $Z^{k-1} = [z(k-1), z(k-2), \ldots z(0)]$
The likelihood of the observation $p(z(k)|\mathbf{x}(k))$ is given by

$$\begin{aligned} p(z(k)|\mathbf{x}(k)) = (1-\varepsilon)\mathcal{N}(z(k) - H(k)\mathbf{x}(k)), \sigma_1^2) + \\ \varepsilon\mathcal{N}(z(k) - H(k)\mathbf{x}(k)), \sigma_2^2) \end{aligned} \tag{18}$$

The main idea is to approximate the densities $p(\mathbf{x}(k)|Z^k)$ and $p(\mathbf{x}(k)|Z^{k-1})$ by a weighted sums of Gaussian density functions

$$p(\mathbf{x}(k)|Z^k) = \sum_{i=1}^{\zeta_k} \alpha_{i,k}\mathcal{N}(x(k) - \hat{x}_i(k), P_i(k)) \tag{19}$$

and

$$p(\mathbf{x}(k)|Z^{k-1}) = \sum_{i=1}^{\zeta_k'} \alpha_{i,k}'\mathcal{N}(x(k) - \hat{x}_i'(k), P_i'(k)) \tag{20}$$

The predicted pdf $p(\mathbf{x}(k+1)|Z^k)$ for the next iteration is obtained using

$$p(\mathbf{x}(k+1)|Z^k) = \int p(\mathbf{x}(k+1)|\mathbf{x}(k))p(\mathbf{x}(k)|Z^k)d\mathbf{x}(k) \tag{21}$$

Since the noise $w(k)$ is assumed Gaussian, we have

$$p(\mathbf{x}(k+1)|\mathbf{x}(k)) = \mathcal{N}(\mathbf{x}(k+1) - F\mathbf{x}(k), GG^T\sigma_w^2) \tag{22}$$

Following the mathematical development in [11] (see Appendix), [12], and [13], we propose the two parallel EKF algorithm for chirp signal estimation.

Table 1: summary of the PEKF algorithm

Given the initial conditions, compute for k:1, 2, 3 ...

Prediction step
$\hat{\mathbf{x}}(k|k-1) = F\hat{\mathbf{x}}_{MMSE}(k-1)$
$e(k|k-1) = z(k) - h(\hat{\mathbf{x}}(k|k-1))$
$\hat{P}(k|k-1) = F\hat{P}_{MMSE}(k-1)F^T + G\sigma_w^2 G^T$
$\xi_j^2(k|k-1) = \sigma_j^2 + H(k)\hat{P}(k|k-1)H^T(k) \quad j = 1,2$

Filtering step
$G_j(k) = \hat{P}(k|k-1)H^T(k)/\xi_j^2(k|k-1)$
$\hat{P}_j(k|k) = (I - G(k)H(k))\hat{P}(k|k-1)$
$\hat{\mathbf{x}}_j(k|k) = \hat{\mathbf{x}}(k|k-1) + G_j(k)e(k|k-1)$

MMSE state estimation
$\beta_j(k) = \mathcal{N}(e(k|k-1), \xi_j^2(k|k-1))$
$\alpha_j(k) = \frac{\lambda_j\beta_j(k)}{\sum_{j=1}^2 \lambda_j\beta_j(k)}$ where $\lambda_1 = 1-\varepsilon$, and $\lambda_2 = \varepsilon$
$\hat{\mathbf{x}}_{MMSE}(k) = \sum_{j=1}^2 \alpha_j(k)\hat{\mathbf{x}}_j(k|k)$
$\hat{P}_{MMSE}(k) = \sum_{j=1}^2 \alpha_j(k)\left[\hat{P}_j(k|k) + (\hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}_{MMSE}(k))(\hat{\mathbf{x}}_j(k|k) - \hat{\mathbf{x}}_{MMSE}(k))^T\right]$

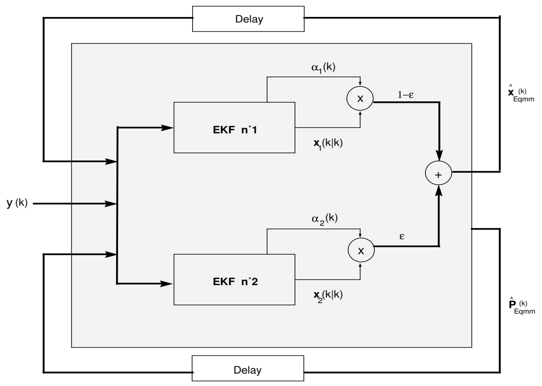

Figure 2: PEKF scheme

The algorithm given in Table 1 can be implemented as shown by the scheme in figure 2. In order to estimate the parameters of the signal given by $\theta = [\ A(k) \quad \alpha \quad \beta \quad \gamma\ ]^T$ one uses the following relation [5]

$$\hat{\theta}(k) = AF^{-k}\hat{\mathbf{x}}(k) \tag{23}$$

where the matrix A is a diagonal with elements 1, 1, 1, 0.5.

4. SIMULATION RESULTS

In this section, we give some simulation results for the estimation of the chirp signal in non Gaussian noise based on the dual EKF. We consider a signal which is 1000 samples long and the sampling period equals 1. The true values of the chirp parameters are as follows : $\alpha = 1.25\times 10^{-3}$, $\beta = 0.1$, and $\gamma = \frac{\pi}{2}$. The state noise $w(k)$ is zero mean Gaussian white noise with $\sigma_w^2 = 10^{-2}$.
The non Gaussian noise $n(k)$ is distributed as in (6), with variance $\sigma_1^2 = 0.25$, and the ratio $\sigma_2^2/\sigma_1^2 = 500$. The chirp parameters convergence plots are given in simulation figures 4 to 7 which are obtained for $\varepsilon = 0.01$ with the following initial conditions $\mathbf{x}_0 = [\ 0.5 \quad \pi/3 \quad 0 \quad 3.10^{-3}\ ]$, and
$\mathbf{P}(0) = diag[\ 1/2 \quad \pi^2/9 \quad \pi^2/9 \quad 4.3865.10^{-6}\ ]$

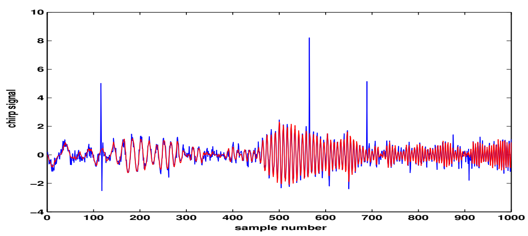

Figure 3: Chirp signal

In order to assess the gain in performance, we compare the MSE of the PEKF with the robust extended Kalman filter in [8] which is based on a modified Kalman gain expression that in terms of the modified Huber's function. We observe in figures 8 to 11 that the PEKF is more robust and gives better estimates of the chirp parameters for low SNR.

5. CONCLUSION

We have presented a method for estimating the parameters of a chirp signal described by non linear state space model where the observation noise in non Gaussian modelled by weighted-sum of two Gaussians densities having different variances. The estimation is carried out by using a parallel extended Kalman filtering algorithm without resorting to threshold computation as compared to the robust extended Kalman filter REKF. Simulation results showed that the performance of the PEKF is better in terms of robustness and estimation accuracy compared to the REKF. On the other hand, this algorithm allows the extension to multicomponent chirp signals and higher order polynomial phase signal with constant as well as variable amplitude signal which will be presented in future work.

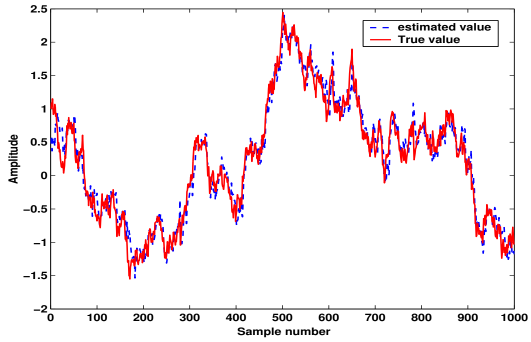

Figure 4: Amplitude estimation

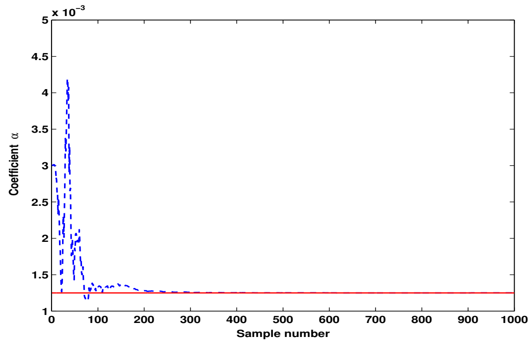

Figure 5: Estimation of the α parameter

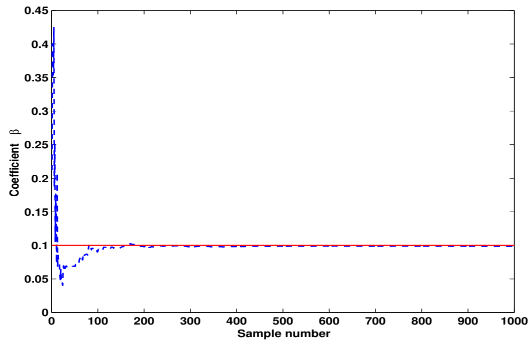

Figure 6: Estimation of the β parameter

REFERENCES

[1] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal. II. Algorithms and applications," in *Proceedings of the IEEE*, vol. 80, Issue: 4, 1992, pp. 540–568.

[2] R. E. Kalman, "A new approach to linear filtering and prediction problems," *Journal of Basic Engineering* , pp. 35–46, 1960.

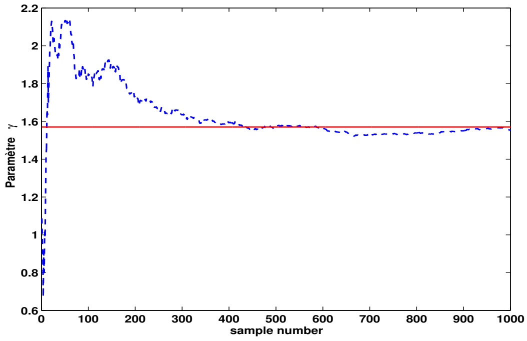

Figure 7: Estimation of the γ parameter

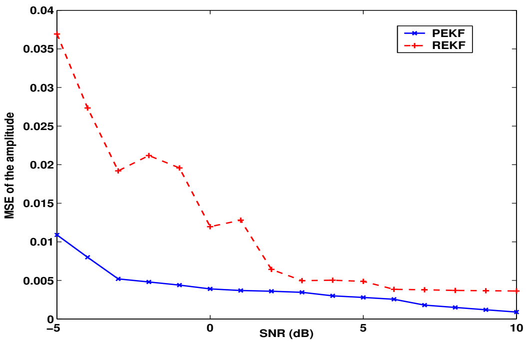

Figure 8: MSE of the amplitude versus the SNR

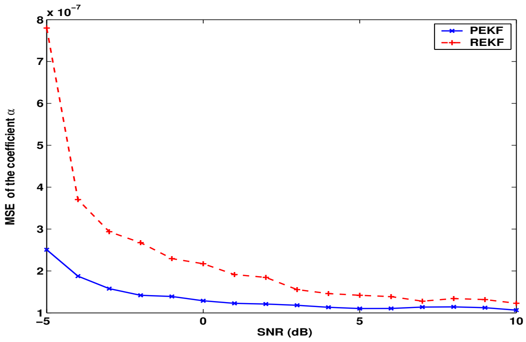

Figure 9: MSE of α versus the SNR

[3] W.E. Kaakour and M. Guglielmi and J.M. Piasco and E. Le Carpentier, “Two identification methods of chirp parameters using state space model ,” in *13th International Conference on Digital Signal Processing Proceedings*, July 1997, pp. 903–906.

[4] B. D. Anderson and J. B. Moore, *Optimal Filtering*. Prentice-Hall, New Jersey, 1979.

[5] M. Adjrad and A. Belouchrani and A. Ouldali, “Estimation of chirp signal parameters using state space modelization by incorporating spatial information ,” in *7th International Symposium on Signal Processing and Its Applications*, July 1-4 2003, pp. 531–534.

[6] S. Shamsunder and B. Giannakis, “Detection and estimation of chirp signals in non-Gaussian noise ,” in *27th Asilomar Conference on Signals, Systems and Computers* , Nov. 1993, pp. 1191–1195.

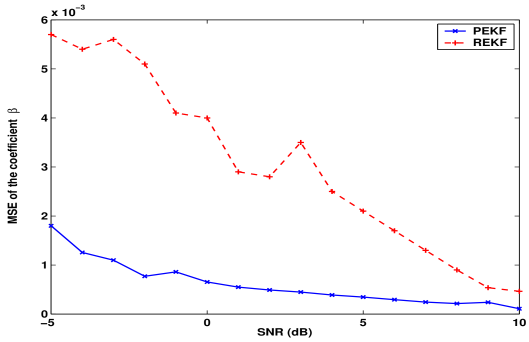

Figure 10: MSE of β versus the SNR

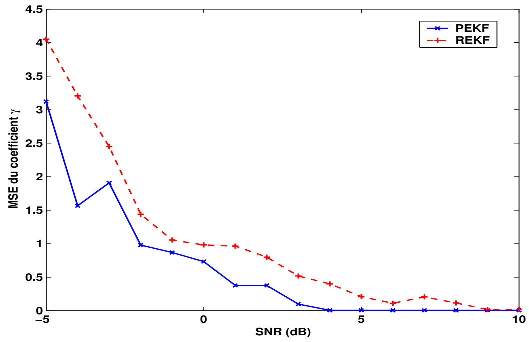

Figure 11: MSE of γ versus the SNR

[7] X. Wang, H. V. Poor, “Robust multiuser detection in non-Gaussian channels,” in *IEEE Trans. On Signal Processing*, vol. 47, No. 2, Feb. 1999, pp. 289–305.

[8] M. Djeddi and M. Benidir, “Estimation of the parameters of chirp signal in non-gaussian noise using robust extended kalman filter,” in *Physics in signal and image processing PSIP*, 2005, pp. 151–167.

[9] D. Zeghlache and S.S. Soliman, “Use of the EM algorithm in impulsive noise channels,” in *Vehicular Technology Conference*, Feb . 1988, pp. 610–615.

[10] D. Alspach and H. Sorenson, “Nonlinear Bayesian estimation using Gaussian sum approximations ,” in , 1972, pp. 439–448.

[11] S. Marcos, “A network of adaptive Kalman filters for data channel equalization,” in *IEEE Trans. On Signal Processing*, vol. 48, No. 9, Sept. 2000, pp. 2620–2627.

[12] B. Sayadi and S. Marcos, “A Robust equalization based on a network of Kalman filters in impulsive noise environments,” in *Proceedings of ICASSP*, April 2003.

[13] R. Amara and S. Marcos, “A blind network of extended Kalman filters for nonstationary channel equalization,” in *ICASSP*, vol. 4, May 2001, pp. 2117–2120.