IMPROVED FREQUENCY DOMAIN SUPER-RESOLUTION ALGORITHM WITH CONJUGATE GRADIENT – NUFFT METHOD AS ITS RECONSTRUCTION CORE

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ABSTRACT

In the paper frequency domain Super-Resolution algorithm with enhanced reconstruction stage is presented. The Fourier transform properties of relocated images are used to easy estimation of rotation and translation, as well as artifacts caused by subsampling. Previously, the bicubic interpolation has been employed in the reconstruction phase, in this paper it is replaced by iterative conjugate-gradient method with inverse nonuniform fast Fourier transform at its core. The new algorithm indeed gives improved results, if compared to those of the previous ones.

1. INTRODUCTION

The spatial resolution is the major perception factor of image quality. Namely, the sensor manufacturing limitations are the principle aspects that determine the number of details of captured scene. However economical and technical reasons may disqualify rebuilding of existing imaging system. One of existing possibilities allowing to overcome such kind of system limitations is digital signal processing approach which is called Super-Resolution (SR). The Super Resolution has been proposed in 1984 [3] and has been applied to multiframe image restoration from a set of bandlimited images. The Super-Resolution is feasible when capturing of a set of Low-Resolution (LR) images of the same scene is possible and, what is particularly important, sub-pixel shifts between LR images exist. If fractional shifts between LR images were not present, all images would have contain the same information. In such situations a better solution may be simple image interpolation for image enhancement. Additionally, interpolated images usually suffer from blur which makes analysis of details difficult.

The most influential part of SR algorithms is precision of its first stage, motion (i.e. displacement) estimation between low resolution images. The next step of SR algorithm is a reconstruction of High Resolution (HR) image from nonuniformly sampled grid, which has been obtained from appropriately positioned motion estimated captured images. In such situation Strohmer [1,2] suggested employing complex exponentials and proposed iterative algorithm to get set of samples on Cartesian (i.e. regular) grid. Differences between SR methods mainly concern the motion estimation part. The technique based on Taylor series motion estimation has been proposed by Keren et. al. [4]. Another method based on mean square error criterion applied to minimization of error between reference and arbitrary chosen images can be found in [5]. This paper presents an improved version of frequency domain SR algorithm presented in [3]. The principle advantage of frequency domain approach is the fact, that degradations due to physical limitations are much easier to describe there than in the spatial domain. We have applied in its reconstruction stage an extension of the concept from [6]. The algorithm has been compared to three other ones [3,4,5], and the efficiency of our approach has been shown.

This paper is organized as follows: section I presents an introduction, sections II and III describe a motion estimation step, and a reconstruction step. Finally, sections IV and V present experiment, results and conclusion.

2. MOTION ESTIMATION IN THE POLAR COORDINATES SYSTEM [3]

The motion between reference image and arbitrarily chosen another one from the set of the low resolution images is assumed to be translation and rotation motion on the flat space. We consider static scenes as in such case global motion sufficiently describes differences between acquired LR images.

Considering such kind of displacement between two images the motion may be simply described by three parameters: horizontal and vertical shifts: appropriately Δx_1, Δx_2 and planar rotation factor representing by angle φ. In the Fourier transform domain relation between two mutually shifted and rotated images can be expressed as follows:

$$\begin{aligned} F_{second}(\omega) &= \iint_x f_{second}(x) e^{-j2\pi\omega^T x} dx = \\ &= \iint_x f_{reference}(R(x+\Delta x)) e^{-j2\pi\omega^T x} dx = \\ &= e^{-j2\pi\omega^T \Delta x} \iint_{x'} f_{reference}(R(x')) e^{-j2\pi\omega^T x'} dx' \end{aligned}, \quad (1)$$

where R means rotation angle expressed in planar coordinates:

$$R = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}, \quad (2)$$

and x and x' represent the coordinates of the reference and shifted images, respectively.
The displacement in the spatial domain results in changed phase values in the Fourier domain (F_{second} and $F_{reference}$ do not depend on the shift Δx). The rotation angle φ may be computed from the amplitude spectrum of images:

$$\left|F_{second}(\omega)\right| = \left|F_{reference}(R\omega)\right| ,$$

where F_{second} and $F_{reference}$ are Fourier transforms of the rotated and the reference images, respectively.

2.1 Rotation factor estimation

Rotation estimation is based on a simple premise that for the appropriately rotated version of the f_{secind} the correlation factor between f_{secind} and $f_{reference}$ is maximum. Unfortunately, such an approach needs computation of correlation for every assumed rotation angle, which is not computationally efficient. Transforming spectrums into polar coordinates limits the number of operations to circular shifts by φ. This may be estimated by computing phase difference between two spatial domain signals. Transforming into regular polar coordinates from regular Cartesian grid usually leads to interpolation and is a relatively complex operation.

2.2 Shifts estimating

If the rotation factor has been estimated, the shift may be computed as follows:

$$\begin{aligned} F_{second}(\omega) &= \iint_x f_{second}(x) e^{-j2\pi\omega^T x} dx = \\ &= \iint_x f_{reference}(x+\Delta x) e^{-j2\pi\omega^T x} dx = \\ &= e^{-j2\pi\omega^T \Delta x} \iint_{x'} f_{reference}(x') e^{-j2\pi\omega^T x'} dx' = \\ &= e^{-j2\pi\omega^T \Delta x} F_{reference}(\omega) \end{aligned} \quad . \qquad (3)$$

Shifts values may be evaluate by computing $\angle \dfrac{F_{second}}{F_{reference}}$.

3. RECONSTRUCTION STEP

Assuming that motion estimation problem has been solved the reconstruction step may be applied. The previous part of the algorithm results in motion estimators. Then, registration of motion estimated samples on previously computed irregular positions in the space domain are done. After mapping onto High Resolution grid, the next step needs computation of values of samples on Cartesian spatial domain grid. To solve this problem we have employed the Conjugate Gradient method with the Nonuniform Fast Fourier Transform at its core. This iterative algorithm results in high quality images and is computationally efficient.

3.1 Conjugate gradient method with INUFFT at its core

The image transformation to the spatial domain may be expressed as follows:

$$z(k(t)) = \int X(\omega) e^{j2\pi(\omega k(t))} dt , \qquad (4)$$

where t means the location of a pixel of the SR-reconstructed image in the space domain, $k(t)$ is the k-th irregular sample of the image in spatial domain, and $X(\omega)$ denotes the (regularly sampled) Fourier Transform of the image. Therefore, after the registration of the motion corrected noisy image its samples can be expressed in the following form:

$$f_i = z(t_i) + \varepsilon_i , \qquad (5)$$

where ε_i denotes complex Gaussian noise. Since the dominant noise in the spatial domain is assumed to be white Gaussian one [6], we estimate image on the regular grid in the Fourier domain by minimizing the following penalized least-squares cost function:

$$\psi(x) = \frac{1}{2}\left\|f - Ax\right\|^2 + \delta R(x), \qquad (6)$$

It means that we are looking for:

$$\hat{x} = \arg\min_x \psi(x) . \qquad (7)$$

where f denotes noisy samples of the image, and *R(x)* means regularization function, that penalizes the roughness of the estimated image. This regularization can speed up convergence. We choose the parameter δ according to the point spread function of the reconstructed image [6]. The space domain needs to be segmented into sub-spaces where sample density is approximately constant. Now we are able to directly compute Ax via Inverse Nonuniform Fast Fourier Transform (INUFFT).
The conjugate gradient algorithm has been used to estimate the regular sampled image by minimization of the cost function (6). The principle step of the CG algorithm consists in computing the gradient of $\psi(x)$, which may be expressed as follows:

$$\nabla\psi(x) = -A'W(f - Ax) + \nabla R(x), \qquad (8)$$

The most computationally complex operation here is to compute the matrix-vector multiplication Ax and its transpose $A'v$, without storing A or A'. Fortunately, we have here efficient and accurate algorithms for computing these matrix-vector multiplications, it may be done by using nonuniform *fast Fourier transform* (*INUFFT*).
The algorithm finishes with the inverse fast Fourier transform to get the reconstructed image in the spatial domain.

3.2 NUFFT [6]

In our paper the roles of space and frequency domains of the NUFFT are interchanged. In the following text, and that from section 3.3, the unmodified notation from [6] and [7] is used.

Consider regularly-spaced signal samples x_n, *n=0,...,N-1*, having Fourier transform:

$$X(\omega) = \sum_{n=0}^{N-1} x_n e^{-i\omega n} \, . \tag{9}$$

Our goal is to compute the Fourier transform on a set of (irregularly spaced) frequency locations $\{\omega_m\}$:

$$X_k = X(\omega_m) = \sum_{n=0}^{N-1} x_n e^{-i\omega_m n} \, , \; m=1,...,M. \tag{10}$$

The ω_m can be arbitrary real numbers. This form has been called the non-uniform discrete Fourier transform (NDFT). Directly evaluating would require *O(MN)* operations, which would be undesirably slow. Fast computation of (10) is called NUFFT. The first step of most NUFFT algorithms is to choose a convenient value $K \geq N$ and compute a weighted K-point FFT of $\{x_n\}$:

$$Y_k = \sum_{n=o}^{N-1} s_n x_n e^{-i\xi k n} \, , \; k=0,...,K\text{-}1 \tag{11},$$

where $\xi = 2\pi / K$ is the fundamental frequency of the *K-point* DFT. The nonzero s_n values are algorithm design variables that have been called "scaling factors", we call $s = (s_1,...,s_N)$ the *scaling vector*. The first step requires *O(KlogN)* operations.

The next step of NUFFT algorithms is to approximate each X_m by interpolating the Y_k using X_m for some of the neighboring ω_m in the frequency set $\Omega_K = \{\xi k : k = 0,...,K-1\}$. The general form of linear interpolator is:

$$\hat{X}(\omega_m) = \sum_{k=0}^{K-1} v_{mk}^{*} Y_k = \langle Y, v_m \rangle , \; m=1,...,M, \tag{12}$$

where v_{mk}'s denote interpolation coefficients, "*" denotes comples conjugate, and $v_m = (v_{m1},...,v_{mK})$. Choosing the scaling vector s and the interpolators $\{v_m\}$ are the main problems of designing NUFFT algorithm.

3.3 Inverse nonuniform fast Fourier transform (INUFFT, section 3.2) [7]

We compute an oversampled IFFT of the given 2D signal, and then we interpolate it optimally onto the desired nonuniform spatial locations using small local neighborhoods in the spatial domain.

Figure 1: Original image.

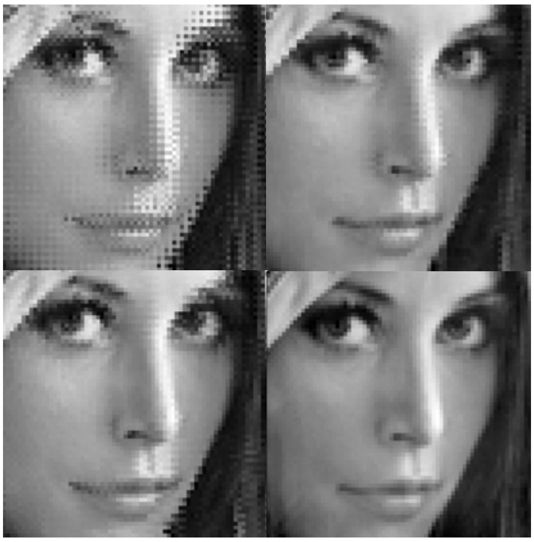

Figure 2. Images after SR restoration: upper left: MSE motion estimation, upper right: frequency domain algorithm [3], lower left: algorithm by Keren with Taylor series motion estimation, lower right: our algorithm.

4. EXPERIMENT

In the experiment a cropped test image 'Lena' has been used, figure 1. The image has been two times subsampled, rotated and shifted. Four such prepared images have been combined into one super - resolution image, having two times higher sampling density. Our algorithm has been compared to three another methods [3], see figure 2.

In the paper [3] it is claimed, that the frequency domain method presented there usually gives better results than two other compared here ones: MSE algorithms and algorithm by Keren. Our technique has improved reconstruction stage and if frequency based motion estimation is done correctly then our method should be even better, which is reflected in figure 2. The improvement is due to replacement of straight bicubic interpolation by iterative reconstruction algorithm, the described above CG-INUFFT method. However some other results show that the frequency domain motion estimator need not be always the best. In fact the motion estimation problems seem to be much more important, than those linked with reconstruction step. We then think that the principle challenge of future research is the improvement of the motion estimation phase.

5. CONCLUSION

A new improved scheme of Super-Resolution restoration has been described in this paper. Motion estimation of the method is done in the Fourier transform domain which is time and memory effective. The reconstruction step is based on iterative CG-NUFFT method which is the new feature of this algorithm. It is shown that for correctly estimated motion parameters the new technique outperforms a similar one described in [3], as well as those from [4, 5]. Nevertheless the performance of the compared methods strongly depends on the success of the estimation phase, hence, its optimization seems to be a challenging problem for future research.

6. REFERENCES

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