A NEW MULTISTAGE LATTICE VQ (MLVQ) TECHNIQUE FOR IMAGE COMPRESSION

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ABSTRACT

Lattice vector quantization (LVQ) reduces coding complexity and computation due to its regular structure. In this paper a new multistage LVQ (MLVQ) technique is presented and applied to image compression. The technique concentrates on reducing the quantization error of quantized vectors. Experimental results for standard images using the new MLVQ technique are shown to be better than JPEG2000 and Man's LVQ based codec. At 0.1 bpp the new MLVQ is 0.17dB higher for image "lena".

1. INTRODUCTION

Image compression is normally achieved using appropriate quantization and subsequent coding of quantized coefficients in the transform or pixel domain [1]. In last ten years there have been significant efforts in producing efficient image coding algorithms based on wavelet transform and vector quantization [2-8]. In [2-4] subband image coding VQ based techniques with the LBG codebook construction algorithm [9] were described. Lattice vector quantization (LVQ) techniques were utilized and presented in [5-8].

Adaptive vector quantization with quadtree model has been used as an initial compression in [2] and [8] of each subband. The significant data in the subband are searched by comparing them with a threshold, and the location are represented in binary sequence in the form of quadtree structure (MAP sequence) and are coded using a variable length coding. A second level of compression is achieved via quantizing the significant vectors. In [8] multistage residual vector quantization is used along with LVQ that produced results that are comparable to JPEG2000 [10] at low bit rate.

Multistage quantization is an effective way to reduce quantization error. It is achieved by having a few quantizers in series to quantize the vectors as described in [11]. The first quantizer output represents the quantized vectors and the remaining quantizers deal with the quantization errors. The more quantizer stages available for quantization, the better refinement will be of the quantized vectors. In this paper a new multistage lattice VQ (MLVQ) is presented that concentrates on reducing the quantization error. The resulting MLVQ is applied to image compression.

This paper is organized as follows. Section 2 gives a review of lattice VQ. Section 3 presents the new multistage LVQ (MLVQ) algorithm. This section describes the significant differences between the new MLVQ and Man's [8] method. The advantage of the new MLVQ is discussed. The performance of the MLVQ algorithm for image compression is presented in Section 4. MLVQ is shown to be significantly superior to both Man's [8] method and JPEG2000 [10]. Section 5 concludes the paper.

2. LATTICE VQ

Lattice is a regular arrangement of points in *k*-space that includes the origin or the zero-vector [1]. Hence, subtracting one lattice point from another generates another lattice point in *k*-space. A lattice is defined as a set of linearly independent vectors [1], [13]:

$$\Lambda = \{X : X = a_1 u_1 + a_2 u_2 + \dots\dots + a_n u_n\} \qquad (1)$$

where $\Lambda \in \Re^k$, $n \le k$ a_i are integers, and u_i for $i = 1,2,\dots.,n$ are the basis vectors. Usually it is convenient to express the basis vectors as a generating matrix $U = [u_1, u_2, \dots\dots, u_n]$. The reciprocal or dual lattice Λ^* consists of all points Y in the subspace of $\Re^k$ spanned by $u_1, u_2, \dots\dots, u_n$ such that the inner product $X.Y = x_1 y_1 + \dots. + x_k y_k$ is an integer for all $x \in \Lambda$ [12]. When the lattices are contained in their duals, there exist the cossets representatives $r_0, \dots\dots, r_{d-1}$ such that

$$\Lambda^* = \bigcup_{i=0}^{d-1} (r_i + \Lambda) \qquad (2)$$

where d is the *determinant* of Λ [12]. In a different approach, the dual lattice is obtained by taking the transpose of the inverse generating matrix given by $(U^{-1})^t$ once the generating matrix is known [13]. The most common lattice type for vector quantization are the root lattices such as A_n, D_n, E_n, Z^n and their dual as described in [12-13].

Conway and Sloane have developed an algorithm for finding the closest point of the n–dimensional integer lattice Z^n as reported in [14]. The Z^n or cubic lattice is the simplest form of a lattice structure. This suggests that finding the closest point in the Z^n lattice to the arbitrary point or input vectors in space $x \in \Re^n$ is simple. In [14] the fast quantizing algorithm for other root lattices such as A_n, D_n, E_n, and their dual were also developed.

Lattice quantizer codebook is designed by truncating the lattice into a spherical or pyramid shapes as described in [5-7]. In spherical codebook the following theta function is used [12], [15];

$$\Theta_\Lambda(z) = \sum_{\Lambda} q^{x.x} = \sum_{m=0}^{\infty} N_m q^m \tag{3}$$

where N_m is the number of lattice points lying on the surface of energy m (L_2 norm-squared or Euclidean distance squared) of the codebook from the origin. In pyramid codebooks the "Nu" function is used [6];

$$\upsilon_\Lambda(z) = \sum_{Y \in \Lambda} z^{\|Y\|_1} = \sum_{m=0}^{+\infty} \left[z^m\right] \left\{Y \in \Lambda / \|Y\|_1 = m\right\} \tag{4}$$

where the coefficient of z^m denotes the number of lattice points lying on the pyramid with energy m. Table of N_m versus m can be found in [6], [15] for Z^n and D_n spherical and pyramid codebooks.

3. THE NEW MULTISTAGE LVQ (MLVQ)

Figure 1 illustrates the new Multistage LVQ (MLVQ) algorithm that is applied to image compression. A wavelet transform is used to transform an image into a number of levels. Adaptive vector quantization with a quadtree model is used to identify the significant coefficients in each subband and code the location information defined as a MAP sequence. This is followed by multistage LVQ to quantize the significant coefficients.

The encoding procedure for each stage LVQ of the significant coefficients uses the modification of the work presented in [6-7]. As a result of these modifications we can set the significant or input vectors to reside only in a granular region for multistage greater than one. If at a single stage LVQ produces N codewords and multistage is M, the resulting codebook size is MxN as shown in figure 1. At each LVQ stage a spherical Z^n quantizer with codebook radius (m=3) is used for vector dimension of 4. Hence, there are 64 lattice points (codewords) available with 3 layers codebook [6]. Each codeword is represented by 6 bits.

At every LVQ stage the quantization error vectors are obtained and "blown out" by multiplying them with the significant vectors scale (first loop iteration) or the input vectors scale. They are then used as the input vectors for the subsequent LVQ stage, and this process repeats up to stage *M*.

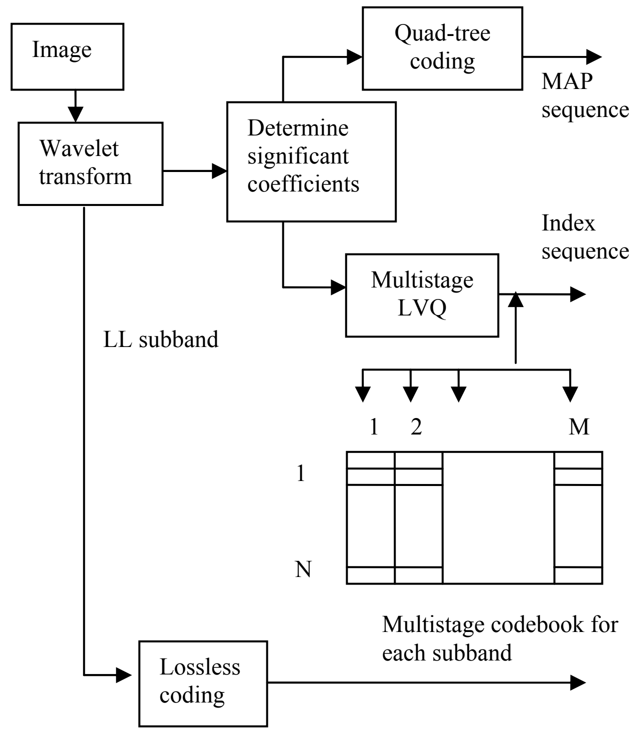

Figure 1: Proposed image coder scheme

The MLVQ algorithm is described as follows;

1. Determine the multistage level M.
2. For i =1,…M do the following;
   - Scale the significant vectors (i=1) or input vectors, and save into a scale record.
   - Vector quantize the scaled vectors, and save into a quantized vectors record
   - Quantization error vectors = (scaled vectors – quantized vectors) x significant vectors scale (i=1) or input vectors scale.
   - Input vector = quantization error vectors
3. Repeat steps 1 and 2 for every subband.
4. If allocated bits are still available, continue further refinement of the quantized vectors from

the lower frequency subband towards the higher frequency.

The threshold setting is an important entity in searching for the significant vectors in the subband. The significant vectors are represented with "1"s in the MAP sequence while the insignificant vectors as "0"s. The insignificant vectors eventually will be replaced with zero vectors in the reconstruction process. Image subbands at different levels of decomposition carry different weight of information. The lower frequency subbands carry more significant data as compared to the higher one [16]. Therefore we need to save more significant coefficients in the lower frequency subband by lowering the threshold. In this work we do approximation with a threshold function. The threshold function is summarized as follows:

Approximate threshold function:

Define a value of x;

x = 1, for $(0 < \text{bit rate} \leq 0.05)$,

x = 2, for $(0.05 < \text{bit rate} \leq 0.1)$,

x = 3, for $(0.1 < \text{bit rate} \leq 0.15)$,

..

x = 10, for $(0.45 < \text{bit rate} \leq 0.5)$.

1. Calculate the average energy per coefficient of the subband and divide by x.
2. At every DWT level
   Level 1 DWT, threshold = average energy.
   Level 2 DWT, threshold = average energy /2.
   Level 3 DWT, threshold = average energy /4.
   Level 4 DWT, threshold = average energy /8.

The MLVQ differs from Man's method [8] in two major aspects. First, in [8] the MAP and QUAN processes are always carried out in every pass of image coding, whereas in the MLVQ the MAP process is executed only once, and multistage LVQ carries the QUAN operation to refine the quantized vectors. Secondly, in [8] the quantization errors are quantized until the error converge to zero. In the MLVQ, the quantization errors are "blown out" by multiplying them with the significant vectors scale (first loop iteration) or input vectors scale at every LVQ stage, thus producing an extra set of input vectors to be quantized. The advantage of "blowing out" the quantization error vectors is that they can be mapped to many more lattice points during the subsequent LVQ stages. In [8] many quantization error vectors are mapped to one lattice point. Thus the MLVQ aims to reduce quantization error while producing better image quality.

4. RESULTS

The standard gray image "lena" of size 512x512 is used in the simulation. Other images such as "baboon" and "boat" were also evaluated for comparison with JPEG2000 [10]. The image coding scheme uses four WT levels. The lowest resolution subband is encoded with lossless compression. Figure 2 compares the results of the new MLVQ to that of Man's method in [8] and JPEG2000 [10]. This illustrates that the MLVQ has better performance at 0.1 bpp and higher. Figure 3 shows the experiment results on "baboon" and "boat" images of the proposed codec compared to that of the JPEG2000. Figure 4 and 5 show the reconstructed "boat" image at 0.1 bpp using JPEG2000 and the MLVQ.

5. CONCLUSION

Multistage LVQ technique in this paper refines the quantized vectors, and reduces quantization errors. The invented threshold function also serves as good approximation at different level of WT. Thus the image compression scheme has out performed both JPEG2000 and Man's method in the range of 0.1bpp and above for image "lena". The multistage LVQ also performs better on "baboon" and "boat" images than JPEG2000.

6. ACKNOWLEDGEMENT

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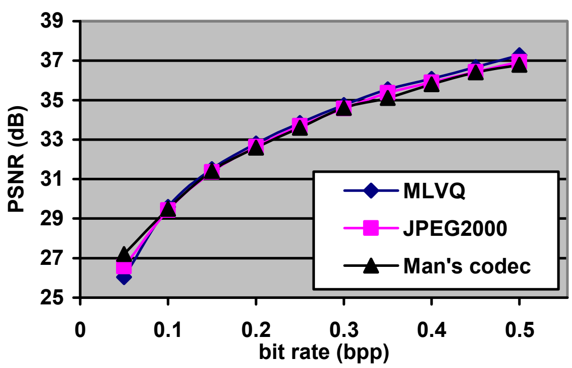

Figure 2: Performance for standard "lena" image.

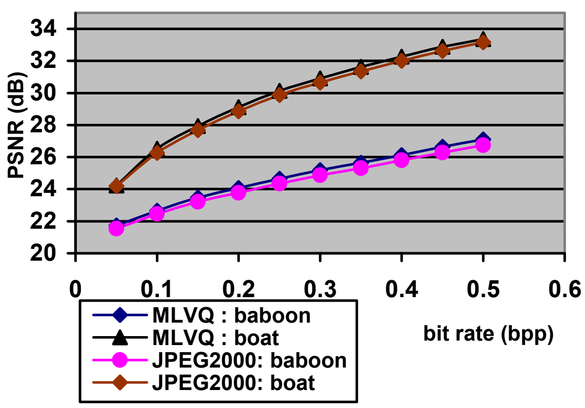

Figure 3: Performance for "baboon" and "boat" images.

Figure 4: "boat" at 0.1 bpp with JPEG2000 (PSNR=26.27dB)

Figure 5: "boat" at 0.1 bpp with MLVQ (PSNR=26.52dB)