INTERPOLATION-BASED MULTI-MODE PRECODING FOR MIMO-OFDM SYSTEMS

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ABSTRACT

Spatial multiplexing with multi-mode precoding provides a means to achieve both high capacity and high reliability in multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems. Multi-mode precoding uses linear transmit precoding that adapts the number of spatial multiplexing data streams or modes, according to the transmit channel state information (CSI). To do so, it typically requires complete knowledge of the transmit precoding matrices for each subcarrier at the transmitter. In this paper, we propose to reduce the transmit CSI requirements to only the precoding matrices on a fraction of the subcarriers. We use interpolation to recover the missing precoders followed by mode selection to enforce the optimal spatial multiplexing mode on all subcarriers. Three interpolation solutions are presented, which reduce the transmit CSI requirements of multi-mode precoding and yet still outperform fixed spatial multiplexing for various channels and number of pilots.

1. INTRODUCTION

Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM), allows high spectral efficiency and improved link reliability over space- and frequency-selective wireless channels. By converting a frequency-selective MIMO channel into a number of parallel flat fading MIMO channels, MIMO-OFDM enables MIMO processing on a per subcarrier basis. Robustness to rank deficiencies in the MIMO channel can be improved by combining spatial multiplexing with linear precoding [1, 2]. Additional throughput and reliability can be realized through multi-mode optimization, which adapts the number of spatial multiplexing streams according to the transmit CSI [10, 11]. Unfortunately, to realize its full gains, multi-mode spatial multiplexing with linear precoding requires full channel state information (CSI) at the transmitter; an assumption, which does not hold for practical systems.

In this contribution, we consider a reduced transmit CSI scenario where the transmitter only knows the precoders on a few pilot subcarriers. This is reasonable in a limited-feedback system where the feedback link can only accomodate feedback of the precoders on a fraction of the subcarriers. The precoders on the remaining subcarriers then need to be reconstructed at the transmitter. We propose three different interpolations that exploit known pilot precoders to recover the precoders on the remaining subcarriers. Interpolation is reasonable because the frequency correlation exhibited by the MIMO channels across subcarriers was shown to hold for the linear precoders on the subcarriers [3]. Given the full precoding matrices for every subcarrier, we then use our previously proposed mode selection [10, 11] to enforce the optimal spatial multiplexing mode on every subcarrier.

Notation: Normal letters designate scalar quantities, boldface lower case letters indicate vectors and boldface capitals represent matrices. $\mathbf{I}_p$ is the $p \times p$ identity matrix. Moreover, $trace(\mathbf{M})$, $[\mathbf{M}]_{i,j}$, $[\mathbf{M}]_{.,j}$, $[\mathbf{M}]_{.,1:j}$ respectively stand for the trace, the $(i,j)^{th}$ entry, the j^{th} column and the j first columns of matrix $\mathbf{M}$. Finally, $()^H$, $(.)^\dagger$ and $(.)^{-1}$ denote the conjugate transpose, the pseudo-inverse and the inverse of a matrix, respectively.

2. SYSTEM MODEL

We consider a spatial-multiplexing MIMO-OFDM wireless communication system that consists of an M_T-antenna transmitter, an M_R-antenna receiver and N subcarriers. On the k^{th} subcarrier, the transmitter optimally maps the $M_s[k]$-dimensional spatial data vector $\mathbf{s}[k] = [s_1[k] \cdots s_{M_s[k]}[k]]^T$, where $M_s[k] \leq \min(M_T, M_R)$, onto the M_T transmit antennas using a linear precoder $\mathbf{F}[k]$. If the cyclic prefix is larger than the channel length, the linear convolution with the frequency-selective MIMO channel is observed as cyclic. Thus, on the k^{th} subcarrier, it becomes equivalent to multiplication with the Discrete Fourier Transform (DFT) of the MIMO channel, given by the $M_R \times M_T$ channel matrix $\mathbf{H}[k]$, whose entries represent the channel gains experienced by subcarrier k [4]. Consequently, the M_R-dimensional received signal vector, on the k^{th} subcarrier, $\mathbf{y}[k]$ is given by $\mathbf{y}[k] = \mathbf{H}[k]\mathbf{F}[k]\mathbf{s}[k] + \mathbf{n}[k]$, where $\mathbf{n}[k]$ is the M_R-dimensional zero-mean spatially-white complex Gaussian receiver noise vector with covariance matrix $N_0\mathbf{I}_{M_R}$ and $E\{\mathbf{s}[k]\mathbf{s}[k]^H\} = \frac{\mathscr{E}_s}{M_s[k]}\mathbf{I}_{M_s[k]}$. Clearly, OFDM modulation decouples the convolutional MIMO channel into a set of N orthogonal flat-fading channels, on the N subcarriers. This property is exploited to carry out data detec-

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tion on each subcarrier independently. Accordingly, on subcarrier k, $\hat{\mathbf{s}}[k]$ is detected using the MMSE receiver $\mathbf{G}[k] = \left(\frac{M_s[k]N_0}{\mathscr{E}_s}\mathbf{I}_{M_s[k]} + \mathbf{F}[k]^H\mathbf{H}[k]^H\mathbf{H}[k]\mathbf{F}[k]\right)^{-1}\mathbf{F}[k]^H\mathbf{H}[k]^H$.

Let $\mathbf{H}[k] = \mathbf{U}[k] \quad [k]\mathbf{V}[k]^H$ be the Singular Value Decomposition (SVD) of the MIMO channel on the k^{th} subcarrier, where $\mathbf{U}[k]$ and $\mathbf{V}[k]$ are respectively $M_R \times M_R$ and $M_T \times M_T$-dimensional unitary matrices containing the left and right singular vectors associated with the singular values or modes of $\mathbf{H}[k]$. These modes are stacked in decreasing order in the $M_R \times M_T$ diagonal matrix . When perfect CSI is available at the transmitter, and the receiver is linear, it is well-known [1, 2] that the precoder $\mathbf{F}[k]$ consists of the $M_s[k]$ first columns of $\mathbf{V}[k]$, i.e. $\mathbf{F}[k] = [\mathbf{V}[k]]_{:,1:M_s[k]}$, where $M_s[k]$ is the number of spatial multiplexing streams to be transmitted on subcarrier k. In this contribution, however, the transmitter is assumed to perfectly know the unitary precoders only on a few pilot subcarriers. It is also assumed to know $\{p_{opt}[k]\}_{1\leq k\leq N}$ the optimal number of spatial multiplexing streams to be used on all subcarriers. Based on this information, we seek to deploy multi-mode precoding on all subcarriers.

3. INTERPOLATION-BASED MULTI-MODE PRECODING FOR MIMO-OFDM

Our strategy is to exploit the available knowledge of the optimal unitary precoders on U pilot subcarriers $\{\mathbf{V}[k_i]\}_{1\leq i\leq U}$, in order to try to recover the unitary precoders on the remaining subcarriers $\{\mathbf{V}[k]\}_{1\leq k\leq N}$. We then select the $p_{opt}[k]$ first columns of the resulting interpolated $\mathbf{V}[k]$, to form the precoder $\mathbf{F}[k]$ that enforces the optimal spatial multiplexing mode on each subcarrier k. More specifically, we propose 3 interpolation approaches that interpolate the $M_T \times M_T$ unitary matrices $\{\mathbf{V}[k_i]\}_{1\leq i\leq U}$ under a unitary constraint in subsection 3.1-3.3. Mode selection is applied in subsection 3.4. The simulation results in Section 4 assess the performance of our interpolation-based multi-mode precoding solutions.

3.1 Geodesic interpolation

This approach considers each 2 unitary precoders on successive pilot subcarriers, for instance $\mathbf{V}[k_i]$ and $\mathbf{V}[k_{i+1}]$ with $1 \leq i \leq U-1$, and interpolates to recover the unitary precoders on all subcarriers in between. More specifically, it considers these 2 pilot precoders as 2 frames on the *special unitary group* $\mathscr{SU}(M_T, M_T)$ [5] and tries to identify the smoothest trajectory on $\mathscr{SU}(M_T, M_T)$ between these 2 frames. The rotations constructing this trajectory, referred to as a *geodesic*, are the desired interpolated unitary precoders on the subcarriers between the 2 successive pilots. This so-called *geodesic* interpolation is widely known in the computer vision literature [6, 7], where it is the optimal way to perform grand tours of 3-D objects. In the following, we summarize the geodesic-interpolation solution.

Since the geodesic is known at the identity element of the $\mathscr{SU}(M_T, M_T)$, we make the following transformation on the pilot frames:

$$\begin{cases} \mathbf{V}[k_i] & \rightarrow \mathbf{I}_{M_T} \\ \mathbf{V}[k_{i+1}] & \rightarrow \mathbf{M} = \mathbf{V}^{-1}[k_i]\mathbf{V}[k_{i+1}]. \end{cases} \tag{1}$$

It was shown that the geodesic (tangent at the identity element) is defined as:

$$\text{I}(t) = \exp(t\mathbf{S}) \quad t \in [0,1], \tag{2}$$

where $\mathbf{S}$ is skew-Hermitian (i.e $\mathbf{S}^H = -\mathbf{S}$) and $\mathbf{M} = \exp(\mathbf{S}) = \text{I}(1)$. This form is known as the exponential map of the unitary matrix $\mathbf{M}$. In fact, every rotation can be written in that form where the exponent matrix is skew-Hermitian [6, 7]. In order to determine $\mathbf{S}$ starting from $\mathbf{M}$, we use the eigenvalue decomposition $\mathbf{M} = \mathbf{A} \quad \mathbf{A}^{-1}$. Since is a diagonal matrix, we can easily define its exponential map $= \exp(\mathbf{S} \quad)$. Consequently, $\mathbf{M}$ can be re-written as:

$$\mathbf{M} = \mathbf{A}\exp(\mathbf{S} \quad)\mathbf{A}^{-1} = \exp(\mathbf{S} = \mathbf{A}\mathbf{S} \quad \mathbf{A}^{-1}). \tag{3}$$

Finally, we can determine the skew-Hermitian matrix of the exponential map of $\mathbf{M}$ in (2) as $\mathbf{S} = \mathbf{A}\mathbf{S} \quad \mathbf{A}^{-1}$. After having determined the exponential map of $\mathbf{M}$, we can reverse the initial transformation of the pilot frames in (1) and consequently identify the geodesic or set of rotations between $\mathbf{V}[k_i]$ and $\mathbf{V}[k_{i+1}]$ as:

$$\text{V}(t) = \mathbf{V}[k_i] \quad \text{I}(t) = \mathbf{V}[k_i]\exp(t\mathbf{S}) \quad t \in [0,1], \tag{4}$$

where $\mathbf{S}$ is given by (3) and the step in the definition of t is determined by the number of subcarriers between the 2 successive pilot subcarriers.

3.2 Projection-based interpolation

While the previous method performs the interpolation directly on the group of rotations $\mathscr{SU}(M_T, M_T)$, this second method simply interpolates in the Grassmann manifold [5] using simple linear or polynomial interpolation and then projects the resulting matrices into $\mathscr{SU}(M_T, M_T)$. Indeed, the projection-based approach first interpolates in the Grassmann manifold between the precoders on 2 successive pilot subcarriers, $\mathbf{V}[k_i]$ and $\mathbf{V}[k_{i+1}]$. For illustration, we here consider a linear interpolation given by:

$$\mathbf{N} = \mathbf{V}[k_i] + (\mathbf{V}[k_{i+1}] - \mathbf{V}[k_i])t \quad t \in [0,1]. \tag{5}$$

Similarly to the previous approach, the step in the definition of t is determined by the number of subcarriers between the 2 pilots. After having determined the interpolated precoders in the Grassmann manifold, we need to identify the closest unitary matrices to these interpolated precoders. These unitary matrices represent the desired precoders on the subcarriers. It was shown [7] that for each linearly-interpolated matrix $\mathbf{N}$, whose Singular Value Decomposition (SVD) is given by $\mathbf{N} = \mathbf{A} \quad \mathbf{B}^H$, the closest unitary matrix is given by $\mathbf{R} = \mathbf{A}\mathbf{B}^H$, with respect to the metric defined on the Grassmann manifold as $\langle\mathbf{X},\mathbf{Y}\rangle = trace(\mathbf{Y}\mathbf{X}^H)$. It is worthwhile mentioning that the projection-based as well as the geodesic approach involve an optional step of optimizing the orientation of the singular vectors in $\mathbf{V}[k_i]$ and $\mathbf{V}[k_{i+1}]$. This step was skipped because of space limitation.

3.3 Conditional interpolation

This approach first considers channel interpolation, based on the MIMO channel matrices acquired on the U pilot subcarriers. Based on the results of this channel interpolation and the knowledge of the structure of the optimal precoders on the remaining subcarriers, this approach tries to identify an inherited precoder interpolation. Based on the MIMO channels on the U pilots, it is easy to reconstruct the MIMO channel on the k^{th} subcarrier as [8]:

$$\mathbf{H}[k] = \sum_{i=1}^{U}\left[\mathbf{F}\mathbf{F}_U^{\dagger}\right]_{k,k_i}\mathbf{H}[k_i], \tag{6}$$

where $\mathbf{F}$ represents the $N \times N$ DFT matrix and $\mathbf{F}_U$ is the $U \times N$ partial DFT matrix which corresponds to the U pilot positions. For notational brevity, we subsequently write $_{k,i} = [\mathbf{F}\mathbf{F}_U^{\dagger}]_{k,k_i}$. Since U as well as the position of the pilots are known both at the transmitter and the receiver, these parameters $(\ _{k,i})_{1 \leq k \leq N_c; 1 \leq i \leq U}$ are also known at the transmitter. Based on the previous interpolation expression, we try to extract the optimal precoder on the k^{th} subcarrier based on the knowledge of the precoders on the U pilots. As aforementioned, the optimal precoder on the i^{th} pilot, where $1 \leq i \leq U$, is given by $\mathbf{V}[k_i]$, where $\mathbf{H}[k_i] = \mathbf{U}[k_i]\ \ [k_i]\mathbf{V}^H[k_i]$. In order to identify the precoders on the remaining subcarriers, we recall that the precoder optimization criteria [1, 2] indicate that the optimal precoder on the k^{th} subcarrier is given by $\mathbf{V}[k]$, where $\mathbf{V}[k]$ contains the eigenvectors of $\mathbf{H}^H[k]\mathbf{H}[k]$. We now detail the expression of $\mathbf{H}^H[k]\mathbf{H}[k]$ using (6):

$$\mathbf{H}^H[k]\mathbf{H}[k] = \ _{i=1}^{U} |\ _{k,i}|^2 \mathbf{V}[k_i]\ \ ^2[k_i]\mathbf{V}^H[k_i] + \ _{i \neq j}\ ^*_{k,i}\ \ _{k,j}\mathbf{H}^H[k_i]\mathbf{H}[k_j]. \quad (7)$$

Clearly, the calculation of the optimal precoder on the k^{th} subcarrier would require not only the knowledge of the precoders on the pilots $(\mathbf{V}[k_i])_{1 \leq i \leq U}$ but also the knowledge of the eigenvalues $(\ ^2[k_i])_{1 \leq i \leq U}$ and that of the complete SVD of $(\mathbf{H}^H[k_i]\mathbf{H}[k_j])_{i<j}$. Since the former information is the only one available at the transmitter, we propose to consider the optimal precoder conditioned on the knowledge of the precoders on the U pilots. Rather than considering (7), the right expression to be evaluated is:

$$E_{cond}\{\mathbf{H}^H[k]\mathbf{H}[k]\} = \ _{i=1}^{U} |\ _{k,i}|^2 \mathbf{V}[k_i] E_{cond}\{\ ^2[k_i]\}\mathbf{V}^H[k_i] + \ _{i \neq j}\ ^*_{k,i}\ \ _{k,j} E_{cond}\{\mathbf{H}^H[k_i]\mathbf{H}[k_j]\}, \quad (8)$$

where $E_{cond}\{.\}$ denotes $E_{\mathbf{H}|(\mathbf{V}[k_i])_{1 \leq i \leq U}}\{.\}$. We now explicitly calculate, the conditional part of the second term in the sum of (8). For $i \neq j$, we can write:

$$E_{cond}\{\mathbf{H}^H[k_i]\mathbf{H}[k_j]\} = \mathbf{V}[k_i] E_{cond}\{\ \ [k_i]\mathbf{U}^H[k_i]\mathbf{U}[k_j]\ \ [k_j]\}\mathbf{V}^H[k_j]. \quad (9)$$

Exploiting the statistical independence of the singular values and the singular vectors of Rayleigh-fading MIMO channels and assuming that the i^{th} and j^{th} pilot subcarriers are far apart such that their fadings are independent, we can further develop (9) into:

$$E_{cond}\{\mathbf{H}^H[k_i]\mathbf{H}[k_j]\} = \mathbf{V}[k_i] E_{cond}\{\ \ [k_i]\} E_{cond}\{\mathbf{U}^H[k_i]\} \cdot E_{cond}\{\mathbf{U}[k_j]\} E_{cond}\{\ \ [k_j]\}\mathbf{V}^H[k_j]. \quad (10)$$

Recalling that, for the Rayleigh-fading MIMO channel $\mathbf{H} = \mathbf{U}\ \ \mathbf{V}^H$, $\mathbf{U}$ and $\mathbf{V}$ are isotropically-distributed in the unitary matrices groups $\mathscr{U}(M_R, M_R)$ and $\mathscr{U}(M_T, M_T)$, respectively, we can state that $E_{\mathbf{H}|\mathbf{V}[k_i]}\{\mathbf{U}^H[k_i]\} = \mathbf{0}_{M_R}$. Consequently, (10) reduces to $E_{cond}\{\mathbf{H}^H[k_i]\mathbf{H}[k_j]\} = \mathbf{0}_{M_T}$. Hence, (8) becomes:

$$E_{cond}\{\mathbf{H}^H[k]\mathbf{H}[k]\} = \ _{i=1}^{U} |\ _{k,i}|^2 \mathbf{V}[k_i] E_{\mathbf{H}}\{\ ^2[k_i]\}\mathbf{V}^H[k_i].$$

Since the calculation of $E_{cond}\{\ ^2[k_i]\}$ only requires the knowledge of the channel statistics, it can easily be acquired or made available beforehand at the transmitter. Finally, the optimal precoder, given only the knowledge of the precoders on the pilots, is given by:

$$\mathbf{T}_{opt} = \text{eigenvectors of}(\ _{i=1}^{U} |\ _{k,i}|^2 \mathbf{V}[k_i] E_{\mathbf{H}}\{\ ^2[k_i]\}\mathbf{V}^H[k_i]).$$

As a final step, we calculate $E_{\mathbf{H}}\{\ ^2[k_i]\}$ or equivalently $E_{\mathbf{H}[k_i]}\{\ ^2[k_i]\}$ through using the joint probability density function of the ordered eigenvalues of $\mathbf{H}^H[k_i]\mathbf{H}[k_i]$ [9]. For completeness, we highlight that this conditional interpolation is invariant with respect to the orientation of the singular vectors. As such, it avoids the additional optimization of these orientations needed by the 2 first interpolators for optimal performance.

3.4 Mode Selection

So far, we have interpolated the $M_T \times M_T$ unitary pilot precoding matrices $\{\mathbf{V}[k_i]\}_{1 \leq i \leq U}$, under a unitary constraint, to recover the $M_T \times M_T$ unitary precoding matrices on all subcarriers $\{\mathbf{V}[k]\}_{1 \leq k \leq N}$. Our strategy now is to select the $p_{opt}[k]$-first columns of each interpolated $\mathbf{F}[k]$ matrix to instantiate the optimal spatial multiplexing mode, where $p_{opt}[k]$ is the optimal number of spatial streams to be used on the k^{th} subcarrier. The optimality pertains to the minimization of an upper-bound on the symbol-vector error rate [10, 11], which can be shown to be achieved through the maximization of the Signal-to-Noise Ratio (SNR) on the weakest spatial stream. More specifically, the mode-selection optimization criterion, for the k^{th} subcarrier, reads:

$$\begin{cases} p_{opt}[k] = \max_{M_s[k]}\{\ \ _{\min}(\mathbf{H}[k][\mathbf{V}[k]]_{:,1:M_s[k]})\frac{\mathscr{E}_s}{M_s[k]N_0}\} \\ p_{opt}[k] \cdot log_2(M_{opt}[k]) = \frac{R}{N} \end{cases}, \quad (11)$$

where $\ _{\min}(\mathbf{B})$ denotes the smallest singular value of matrix $\mathbf{B}$, $M_{opt}[k]$ is the symbol constellation used to modulate the $p_{opt}[k]$ spatial-multiplexing data streams, such that the rate constraint of R/N per-subcarrier is fulfilled. On each subcarrier k, the transmitter finally enforces the optimal spatial multiplexing mode, $\{p_{opt}[k], M_{opt}[k]\}$.

4. PERFORMANCE RESULTS

In this section, we assess the performance of the 3 interpolation techniques proposed to reconstruct the optimal precoders for MIMO-OFDM systems where the optimal precoders are only known on a few pilot subcarriers. We also assess the performance of our previously proposed mode selection in such interpolation-based unitary precoding MIMO-OFDM systems. We consider a set-up consisting of a 3-antenna transmitter and a 2-antenna receiver at a rate of $R = 64$ Mbps. This rate corresponds to 2 QPSK-modulated streams per subcarrier over $N = 64$ subcarriers. Furthermore, we used the MIMO channel model provided by the IEEE 802.11 TGn [12] assuming the following assumptions: channel model B and F with dowlink and non line-of-sight, antenna spacings at the transmitter and the receiver are equal to , where is the carrier wavelength at 5.2 GHz and a sampling rate of 20 MHz. At this sampling rate, channel model B (rms delay spread 15 ns) exhibits $L = 10$ samples,

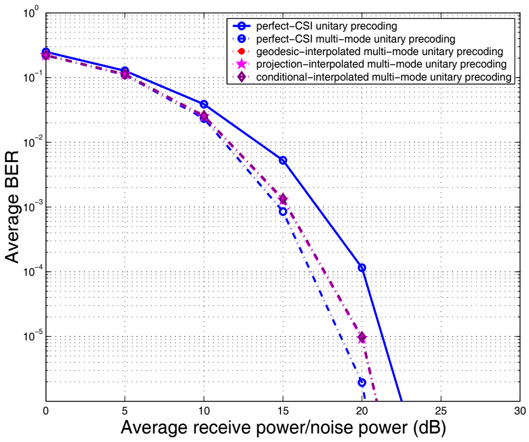

Figure 1: (3×2) *MIMO-OFDM set-up at rate* 64 *Mbps with channel B and* 4 *pilot precoders*

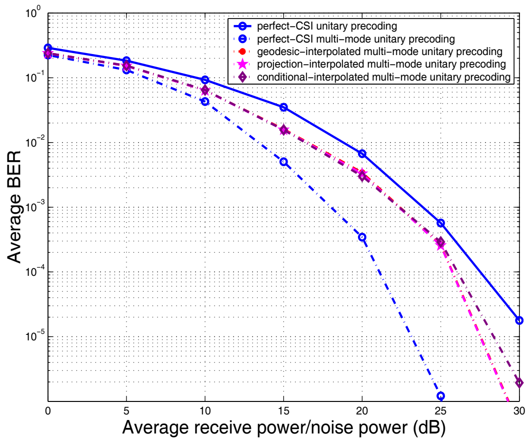

Figure 2: (3×2) *MIMO-OFDM set-up at rate* 64 *Mbps with channel F and* 10 *pilot precoders*

whereas channel model F (rms delay spread 150 ns) has a length $L = 29$. Finally, every point of the simulation results was obtained by averaging over more than 100 channel realizations. The performance of the proposed interpolation depends on the number of used pilot precoders U and how this number compares to the length of the delay-spread channel L. For $U \geq L$, the interpolation will succeed in reconstructing the precoders based on the pilots. Whereas, when $U < L$, the interpolation will make errors on the precoders and will consequently lead to a degradation of the BER performance. The latter case is clearly the most relevant, as it allows a higher reduction in the transmit CSI requirements. Consequently, the illustrated performance results are dedicated to scenarios where $U < L$. Figure 1 depicts the average BER performance of our multi-mode selection when the 3 proposed interpolation solutions are used to recover the unitary precoders, based on only $U = 4$ pilot precoders for IEEE 802.11 TGn channel 'B'. As it turns out, the 3 interpolation solutions lead to a very similar BER performance, which falls within 0.6 and 0.9 dB SNR loss at BER=10^{-3} with respect to the complete-CSI multi-mode unitary solution. Figure 1 also shows that our multi-mode optimization still significantly outperforms the unoptimized spatial multiplexing solutions, even with reduced transmit CSI. Figure 2 still confirms the latter remark for channels with larger delay spread, such as the IEEE 802.11 TGn channel 'F'. Clearly, the reduction in transmit CSI to only 10 pilots, while the channel length is $L = 29$, leads to a larger degradation of our proposed interpolation-based multi-mode solutions. Nevertheless, our solution still remains attractive even compared to complete-CSI unoptimized solutions.

5. CONCLUSIONS

In this contribution, we proposed a solution to limit the transmit CSI required for multi-mode precoding in MIMO-OFDM. Our approach was to use a few known unitary pilot precoders followed by interpolation to find the remaining precoders. We introduced three interpolation algorithms which preserve the unitary structure of the precoders, and showed that they provide reasonable BER performance even for small numbers of pilots.

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