OPTIMUM CHAOTIC QUANTIZED SEQUENCES FOR ASYNCHRONOUS DS-CDMA SYSTEM

Calin Vladeanu

Telecommunications Department, “Politehnica” University of Bucharest
1-3 Iuliu Maniu Bvd., Bucharest, Romania
phone: + (40) 21 402 4765, fax: + (40) 21 402 4765, email: calin@comm.pub.ro
web: www.elcom.pub.ro

ABSTRACT

Using chaotic sequences for spreading is a new approach for optimising the BER (Bit Error Rate) performances in the DS-CDMA (Direct Sequence – Code Division Multiple Access) systems. This paper presents the use of a very well known family of PWAM (Piece-Wise Affine Markov) maps, namely (n, t) – tailed shifts maps, and their optimum quantized versions. This optimisation involves the variance minimisation for the mean MAI (Multiple Access Interference) term, which minimises also the mean BER under the SGA (Standard Gaussian Approximation) condition.

1. INTRODUCTION

The BER (Bit Error Rate) performances of DS-CDMA (Direct Sequence – Code Division Multiple Access) systems depend mainly on the correlation properties of the spreading sequences set [1], [2], [3]. The use of low cross-correlation sets of sequences increases the BER performances and the system capacity as well. Hence, it is imperative to design optimum spreading sequences sets that minimise the BER.

Classical sets of spreading sequences used in actual standards of DS-CDMA mobile communications systems are binary sequences generated by LFSR (Linear-Feedback Shift Register) schemes. Even for minimum cross-correlation sequences, forming Gold and Kasami sets, the set dimension and the period of the sequences are limited by the LFSR polynomial degree. Another drawback of these sequences is induced by the generator linearity, which increases the interception probability.

A new direct-sequence spreading method assumes the use of discrete-time non-linear dynamical systems trajectories. These systems are used to generate truly random sequences when working in so called “chaotic” regimes. These chaotic sequences present noise-like features that make them good for spreading in DS-CDMA systems [4], [5]. A single system, described by its discrete chaotic map, can generate a very large number of distinct chaotic sequences, each sequence being uniquely specified by its initial value. This dependency on the initial state and the non-linear character of the discrete map make the DS-CDMA system using these sequences more secure.

It is known that binary quantized sequences generated by PWAM (Piece-Wise Affine Markov) (n, t) – tailed shifts maps minimize the BER under the SGA (Standard Gaussian Approximation) assumption in the asynchronous DS-CDMA system [5], [6]. However, this paper considers the more general case of multilevel quantized (n, t) – tailed shifts sequences [7].

This paper is organised as follows. The second paragraph is presenting the BER estimation for the asynchronous DS-CDMA system for optimal spreading sequences and perfectly random (white) spreading sequences, assuming a frequency non-selective fading channel with AWGN noise. The third part of the paper describes the design method for optimal sets of chaotic multilevel quantized (n, t) – tailed shifts sequences based on their auto-correlation shaping. The fourth part presents some simulation results compared to the theoretical average values for both optimal and white sequences. Finally, some conclusions are drawn.

2. AVERAGE BER ESTIMATION FOR ASYNCHRONOUS DS-CDMA SYSTEM

Let’s consider the block scheme of the asynchronous DS-CDMA system in figure 1 [1-3].

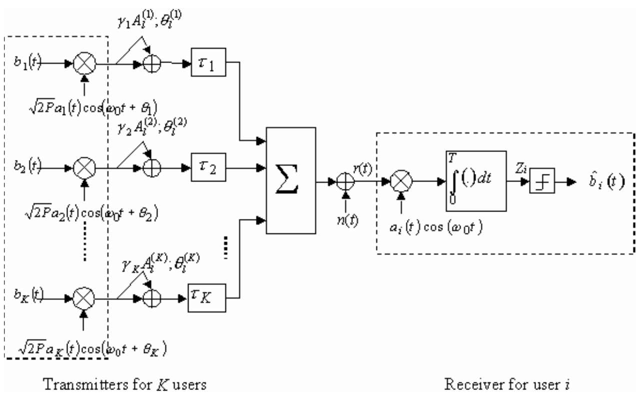

Fig.1. The asynchronous DS-CDMA system block scheme.

The notations in figure 1 are as follows: K is the number of users, P is the common received power, $\omega_0 = 2\pi f_0$ is the common carrier pulsation, $a_k(t)$ is the waveform for the spreading sequence $(a_j^{(k)})$, with the chip period $T_c=T/N$, such that $\sum_{j=0}^{N-1}[a_j^{(k)}]^2 = N$, N being the sequence period,

$b_k(t)$ is the binary data sequence for user k, $k=\overline{1,K}$. The asynchronous DS-CDMA system consists of random initial phases of the carrier $0 \le \theta_k < 2\pi$ and random propagation delays $0 \le \tau_k < T$ for all the users $k=\overline{1,K}$. The transmission channel is assumed to be a non-selective fading Rician channel with two-sided PSD (Power Spectral Density) $N_0/2$ additive Gaussian noise. The output signal of a Rician nonselective fading channel is the sum of a non-faded version of the input signal (specular component) and a non-delayed faded version of the input signal (scatter component). All communications links are assumed to fade independently. A_l^k is a positive random variable satisfying the normalisation constraint $E\{[A_l^{(k)}]^2\}=1$ and θ_l^k is the phase shift due to fading, both being uniformly distributed. We also assume that all users have the same faded power ratio $\gamma^2=\gamma_1^2=\gamma_2^2=...=\gamma_K^2$. There is no loss of generality to assume that $\theta_i=0$ and $\tau_i=0$ for the desired user i, and to consider only $0 \le \tau_k < T$ and $0 \le \theta_k < 2\pi$ for any $k \ne i$.

Under the assumptions considered above the mean and the variance of the correlator output Z_i are given by [1-3]:

$$E\{Z_i\}=\sqrt{\frac{P}{2}}T \quad , \quad \mathrm{var}\{Z_i\}=\frac{N_0T}{4}+\frac{PT^2\gamma^2}{4}+(1+\gamma^2)\sigma_A^2(i) \qquad (1)$$

where $\sigma_n^2=N_0T/4$ is the variance for the additive Gaussian noise [2], the second term represents the faded component power from the user i, and $\sigma_A^2(i)$ is the overall (non-faded) interference (MAI – Multiple Access Interference) power for the desired ith user.

The MAI variance for the desired ith user can be computed as [1-3], [7]:

$$\sigma_A^2(i)=\frac{PT^2}{12N^3}\left(\sum_{\substack{k=1\\k\ne i}}^{K} r_{k,i}\right), \qquad (2)$$

where $r_{k,i}$ represents the interference term corresponding to the interfering user k. The interference term $r_{k,i}$ from expression (2) can be written in terms of the cross-correlation function as [1]:

$$r_{k,i}=2\sum_{l=1-N}^{N-1}C_{k,i}^2(l)+\sum_{l=1-N}^{N-1}C_{k,i}(l)C_{k,i}(l+1), \qquad (3)$$

where $C_{k,i}(l)$ is the discrete aperiodic cross-correlation function for the sequences $(a_j^{(k)})$ and $(a_j^{(i)})$, defined as:

$$C_{k,i}(l)=\begin{cases}\sum_{j=0}^{N-1-l} a_j^{(k)}a_{j+l}^{(i)}, \text{for } 0\le l\le N-1\\ \sum_{j=0}^{N-1+l} a_{j-l}^{(k)}a_j^{(i)}, \text{for } 1-N\le l<0\end{cases}, \qquad (4)$$

and $C_{k,i}(l)=0, \text{for } |l|\ge N$.

The BER for the desired user i, with the spreading sequence $(a_i)\overset{\Delta}{=}(a_j^{(i)})_{j=\overline{1,N}}$, may be estimated under the SGA (Standard Gaussian Approximation) assumption as [1-3]:

$$BER(i)=Q\left(\frac{E\{Z_i\}}{\sqrt{\mathrm{var}\{Z_i\}}}\right)=Q\left(\frac{\sqrt{\frac{P}{2}}T}{\sqrt{\frac{N_0T}{4}+\frac{PT^2\gamma^2}{4}+(1+\gamma^2)\sigma_A^2(i)}}\right), \qquad (5)$$

where the Q function is given by $Q(z)=\int_z^\infty \frac{1}{\sqrt{2\pi}}e^{-y^2/2}dy$.

In order to derive a general result it is necessary to estimate an average BER over the whole set of spreading sequences:

$$BER=\mathop{\mathrm{E}}_{\{a_i\}}[BER(i)]=\mathop{\mathrm{E}}_{\{a_i\}}\left[Q\left(\frac{\sqrt{\frac{P}{2}}T}{\sqrt{\frac{N_0T}{4}+\frac{PT^2\gamma^2}{4}+(1+\gamma^2)\sigma_A^2(i)}}\right)\right] \qquad (6)$$

Expression (6) is difficult to work with because the Q function is non-linear. Hence, under the SGA condition a good approximation of (6) is given by:

$$BER\approx Q\left(\frac{\sqrt{\frac{P}{2}}T}{\sqrt{\frac{N_0T}{4}+\frac{PT^2\gamma^2}{4}+(1+\gamma^2)\mathop{\mathrm{E}}_{\{a_i\}}[\sigma_A^2(i)]}}\right). \qquad (7)$$

It is known that when perfectly random sequences (white noise-like sequences) are employed, the average MAI variance from (7) may be written as [1]:

$$\mathop{\mathrm{E}}_{\{a_i\}}[\sigma_{A,\,white}^2(i)]=\frac{PT^2(K-1)}{6N}. \qquad (8)$$

According to [6] the lower bound of average BER for all the users can be attained if using spreading sequences that have the following auto-correlation ensemble (2nd order moment) expression:

$$A_k(l)\overset{\Delta}{=}\mathop{\mathbf{E}}_{\{a_k\}}[a_j^{(k)}a_{j+l}^{(k)}]=(-1)^l\frac{r^{l-N}-r^{N-l}}{r^{-N}-r^N}, \quad l=0,1,2,...,N-1,\ \forall k \qquad (9)$$

where $r=2-\sqrt{3}$.

Note that when $l<<N$, $A_k(l)\approx(-r)^l$, which decays exponentially with alternate sign. By introducing (9) into (2) and (3), the minimum interference power is obtained for user i [6]:

$$\mathop{\mathrm{E}}_{\{a_i\}}[\sigma_{A,\,optimum}^2(i)]\cong\frac{PT^2(K-1)\sqrt{3}}{12N} \qquad (10)$$

Comparing the optimum case with the case when white sequences are employed for spreading, the first one offers an increase in the system BER performances. By writing the ratio of the minimum interference variance over the variance term for the white sequences case we have

$$\frac{\underset{\{a_i\}}{\mathrm{E}}[\sigma^2_{A,\, optimum}(i)]}{\underset{\{a_i\}}{\mathrm{E}}[\sigma^2_{A,\, white}(i)]} = \frac{\sqrt{3}}{2} \tag{11}$$

which increases the number of users accommodated for the same mean BER, from the white sequences case to the optimum case, by $K_{optimum} / K_{white} \to 2/\sqrt{3} \cong 1.1547$ for large numbers of users. It is obvious from (11) that the optimum case increases the number of users by more than 15% than the white spreading case, for the same mean BER.

3. OPTIMUM QUANTIZED SPREADING SEQUENCES GENERATED BY (N, T) – TAILED SHIFTS MAPS

One of the well known family of PWAM (Piece-Wise Affine Markov) maps that generate chaotic sequences is the (*n*, *t*) – *tailed shifts* map, defined as [4], [5], [6]:

$$M(x) = \begin{cases} ((n-t)\,x)_{(\mathrm{mod}(n-t)/n)} + \dfrac{t}{n}, & \text{for } 0 \le x < \dfrac{n-t}{n} \\ \left(t\left(x - \dfrac{n-t}{n}\right)\right)_{(\mathrm{mod}\, t/n)}, & \text{otherwise} \end{cases} \tag{12}$$

for $t < n/2$, which is represented in figure 2.

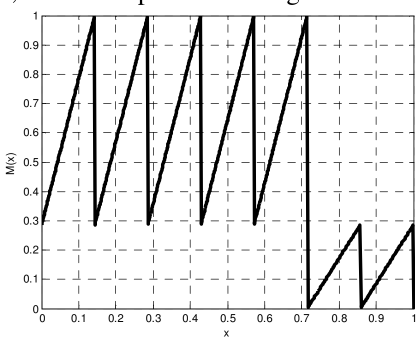

Fig. 2. The (7, 2) – *tailed shifts* map.

It is known that these maps are exact and have a uniform invariant probability density function [5].

Using the tensorial algebra as in [5] it can be demonstrated that for the (*n*, *t*) – *tailed shifts* map *M*-levels uniformly quantized sequences, the second order moment defined in (9) has the following expression [7]:

$$A_k(l) = \underset{\{a_k\}}{\mathbf{E}}[f(a_0^{(k)})f(a_l^{(k)})] = \cong \begin{cases} \dfrac{M+1}{3(M-1)}, & \text{for } l = 0 \\ \dfrac{M^2}{(M-1)^2}\left(\dfrac{n-t}{n}\right)^2 g^l, & \text{for } l \neq 0, \forall k. \end{cases} \tag{13}$$

for any *k*, where $g = -t/(n-t)$, and the *M* levels uniform quantization function $f_i(x)$ for any *i*, is given by:

$$f_i(x) = \frac{1}{M-1}\sum_{j=1}^{M-1} sign\left(x - \frac{j}{M}\right), \quad \forall i \in \{1, 2, ..., n\} \tag{14}$$

M taking values larger than 2. Following the same procedure for the binary quantized sequences, when $M = 2$ in (14), the second order moment has the following expression [5], [6]:

$$A_k(l) = \underset{\{a_k\}}{\mathbf{E}}[f(a_0^{(k)})f(a_l^{(k)})] = g^l, \quad \forall l, k \tag{15}$$

It is important to note that if the number of quantizing levels *M* is very large ($M \to \infty$ at the limit, but practical values larger than 20 are enough as approximation) the normalised second order moment in (13) may be written as follows:

$$A_k(l) = \underset{\{a_k\}}{\mathbf{E}}[f(a_0^{(k)})f(a_l^{(k)})] = \cong \begin{cases} 1, & \text{for } l = 0 \\ 3\left(\dfrac{n-t}{n}\right)^2 g^l, & \text{for } l \neq 0, \forall k. \end{cases} \tag{16}$$

Comparing expressions (15) and (16) with (9) it is easy to see that the optimum auto-correlation ensemble can be approximated by these sets of sequences if the following designing condition is met:

$$-g = \frac{t}{n-t} \cong r = 2 - \sqrt{3}, \tag{17}$$

by fixing the *n* and *t* parameters of the map, accordingly. This result is consistent with the results in [6].

Considering these two limits of quantized sequences and the approximation in (17), then the average MAI variance from (2) may be estimated as [7]:

$$\underset{\{a_i\}}{\mathrm{E}}[\sigma^2_{A,\,(n,t)-TS,2levels}(i)] \cong \frac{PT^2(K-1)}{12N^3}\left(\sqrt{3}N^2 - \frac{2\sqrt{3}}{3}N + \frac{4-\sqrt{3}}{6}\right) \tag{18}$$

for $M = 2$ levels, and

$$\underset{\{a_i\}}{\mathrm{E}}[\sigma^2_{A,\,(n,t)-TS,M\to\infty\ levels}(i)] \cong \frac{PT^2(K-1)}{12N^3}\left[\left(\frac{6+\sqrt{3}}{4}\right)N^2 - \left(\frac{6+2\sqrt{3}}{3}\right)N + \frac{16+9\sqrt{3}}{24}\right] \tag{19}$$

for a very large number of levels, $M \to \infty$.

The quantized sequences with average MAI variances in (18) and (19) increase the number of users accommodated for the same mean BER, from the white sequences case by the following factors:

$$\begin{aligned} K_{M\to\infty levels} / K_{white} &\to 1.0715 \\ K_{M=2levels} / K_{white} &\to 1.167 \end{aligned}, \tag{20}$$

for large numbers of users. From relations (20) it is obvious that the use of binary quantized sequences determines a system capacity increase by more than 16% for the same mean BER. This case is consistent with the optimum case derived in section 2. When a very large number of levels is used for quantizing the sequences a system capacity increase by more than 7% is noted, for the same mean BER.

4. SIMULATION RESULTS

The asynchronous DS-CDMA system presented in section 2 using quantized (50, 11) – *tailed shifts* sequences and Gold sequences generated by primitive polynomials of degree $n = 6$, having the period $N = 2^n - 1 = 63$, was considered for simulation. The maximum number of quantizing levels is M=128. The parameters values of the (n, t) – *tailed shifts* map were chosen to match the condition in (17):

$$-g = \frac{t}{n-t} = \frac{11}{39} \cong 0.282 \cong 0.268 \cong 2 - \sqrt{3} = r \, , \quad (21)$$

The estimated average BER was evaluated for $K = 10$ users and the energy-per-bit to noise DSP ratio E_b / N_0 taking values from 0 to 30 dB. The common faded power ratio is taking the value $\gamma^2 = 0.1$. The resulting BER as a function of the ratio E_b / N_0 is depicted in figure 3.

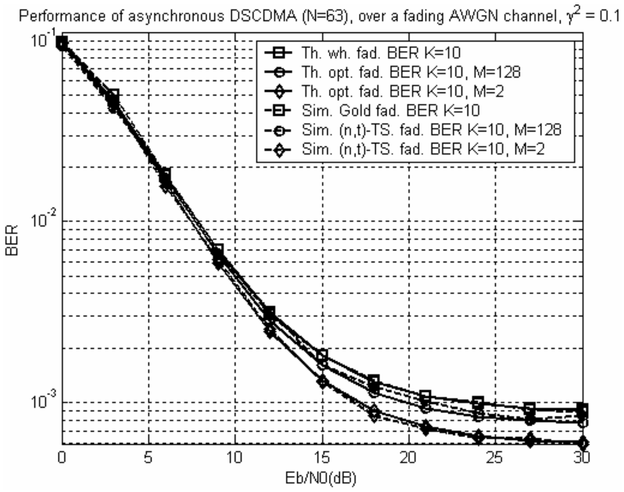

Fig. 3. Theoretical and simulated BER as function of E_b/N_0 ratio for asynchronous DS-CDMA system.

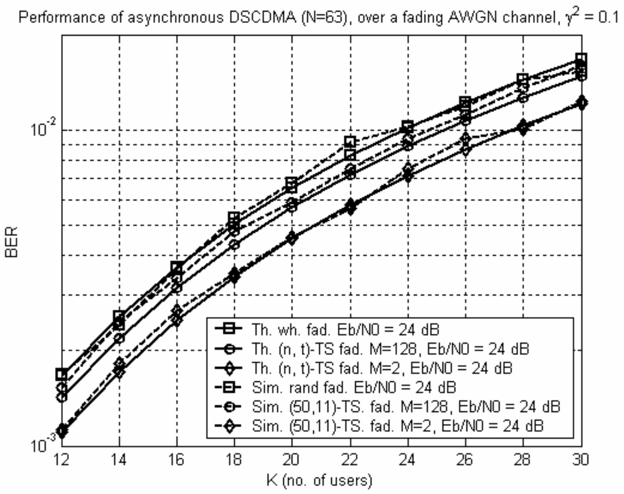

Fig. 4. Theoretical and simulated BER as function of the number of users K for asynchronous DS-CDMA system.

The asynchronous DS-CDMA system capacity is also an important parameter to measure. The average BER was estimated considering several values for the number of users $K \in \{12 \ldots 30\}$ having the same value of the energy-per-bit to noise DSP ratio E_b/N_0=24dB. The resulting BER values as a function of the number of users K are depicted in figure 4.

The simulation results show that binary optimal (M=2) and multilevel (M=128) quantized (50, 11) – *tailed shifts* sequences are better than Gold sequences in terms of allowable number of users (figures 3 and 4) by more than 15%, and 7%, respectively. This result is consistent with the analytical result presented in section 3. However, there are some differences between the simulation and analytical results given the fact that Gold sequences are not perfectly white, quantized (50, 11) – *tailed shifts* sequences are in fact pseudo-optimal, and the SGA approximation is not quite valid for a small number of users.

5. CONCLUSIONS

A family of optimal spreading sequences for the asynchronous DS-CDMA system for the SGA approximation hypothesis was considered for minimising the average BER. The sequence generation method for the quantized (n, t) – *tailed shifts* map and their correlation properties were also presented. The BER performance of the asynchronous DS-CDMA system was estimated assuming a frequency non-selective fading channel with AWGN noise. Hence, an asynchronous DS-CDMA system using optimal sequences offers a capacity increase of about 15% than when white sequences or Gold codes are used. The sensitive dependency of chaotic maps on the initial condition offers both a greater number of available sequences and security increase.

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