MMSE ESTIMATION OF BASIS EXPANSION MODELS FOR RAPIDLY TIME-VARYING CHANNELS*

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ABSTRACT

In this paper, we propose an estimation technique for rapidly time-varying channels. We approximate the time-varying channel using the basis expansion model (BEM). The BEM coefficients of the channel are needed to design channel equalizers. We rely on pilot symbol assisted modulation (PSAM) to estimate the channel (or the BEM coefficients of the channel). We first derive the optimal minimum mean-square error (MMSE) interpolation based channel estimation technique. We then derive the BEM channel estimation, where only the BEM coefficients are estimated. We consider a BEM with a critically sampled Doppler spectrum, as well as a BEM with an oversampled Doppler spectrum. It has been shown that, while the first suffers from an error floor due to a modeling error, the latter is sensitive to noise. A robust channel estimation can then be obtained by combining the MMSE interpolation based channel estimation and the BEM channel estimation technique. Through computer simulations, it is shown that the resulting algorithm provides a significant gain when an oversampled Doppler spectrum is used (an oversampling rate equal to 2 appears to be sufficient), while only a slight improvement is obtained when the critically sampled Doppler spectrum is used.

1. INTRODUCTION

Communication systems are currently designed to provide high data rates to high mobility terminals. High mobility and/or frequency offsets between the transmitter and the receiver result into rapidly time-varying channels. Such channels have a coherence time in the order of the symbol period, and thus cannot be considered time-invariant. Many equalization techniques have already been developed to combat the effect of such channels. In [1, 2, 3] linear and decision feedback equalizers have been developed for single carrier transmission. A per-tone frequency-domain equalization technique for multicarrier transmission over doubly selective channels has been proposed in [4]. In these works, the time-varying channel was modeled using the basis expansion model (BEM). The BEM coefficients are then used to design the equalizer (linear or decision feedback). The above equalizers thus assume perfect knowledge of the channel at the receiver. The BEM coefficients can be obtained by using least squares fitting. In practice, the BEM coefficients have to be estimated, e.g. using training. This is the focus of this paper.

In this paper, we will rely on pilot symbol assisted modulation (PSAM), which consists of inserting known pilot symbols at known positions. We first derive the optimal minimum mean-square error (MMSE) interpolation based channel estimation technique. Then we derive the conventional BEM channel estimation technique. It has been shown that the modeling error (between the true channel and the BEM channel model) is quite high for the case when the BEM period equals the time window [4]. This case corresponds to a critical sampling of the Doppler spectrum. Reducing this modeling error can be achieved by setting the BEM period equal to a multiple of the time window. In other words, we can reduce the modeling error by oversampling the Doppler spectrum. In [5] the authors treated the first case ignoring the modeling error. However, when an oversampling of the Doppler spectrum is used, the BEM based PSAM channel estimation is sensitive to noise. Here, we show that robust PSAM based channel estimation can be obtained by combining the optimal MMSE interpolation based channel estimation with the BEM considering an oversampling rate greater than one (an oversampling rate equal to 2 appears to be sufficient).

This paper is organized as follows. In Section 2, we present the system model. In Section 3, we derive the PSAM MMSE channel estimation. BEM channel estimation is introduced in Section 4. In Section 5, we show through computer simulations the performance of the proposed channel estimation techniques. Finally, our conclusions are drawn in Section 6.

Notations: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, ^{H}, and † represent conjugate, transpose, Hermitian, and pseudo-inverse, respectively. Continuous-time variables (discrete-time) are denoted as $x(\cdot)$ $(x[\cdot])$. $\mathcal{E}\{\cdot\}$ denotes expectation. Finally, we denote the $N \times N$ identity matrix as $\mathbf{I}_N$.

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2. SYSTEM MODEL

We assume a single-input single-output (SISO) system, but the results can be easily extended to a single-input multiple-output (SIMO) system or a multiple-input multiple-output (MIMO) system. Focusing on a baseband-equivalent description, when transmitting a symbol sequence $s[n]$ at rate $1/T$, the received signal $y(t)$ can be written as:

$$y(t) = \sum_{n=-\infty}^{\infty} g(t; t - nT)s[n] + v(t),$$

where $g(t;\tau)$ is the baseband-equivalent of the doubly selective channel (time- and frequency-selective) from the transmitter to the receiver, $v(t)$ is the baseband-equivalent filtered additive noise at the receiver. $g(t;\tau)$ includes the physical channel $g_{ch}(t;\tau)$ as well as the transmit filter $g_{tr}(t)$ and receive filter $g_{rec}(t)$:

$$g(t;\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_{rec}(s)g_{tr}(\tau - s - \theta)g_{ch}(t - s;\theta)\mathrm{d}s\mathrm{d}\theta.$$

Sampling the received signal at the symbol rate $1/T$, the received sample sequence $y[n] = y(nT)$, can be written as:

$$y[n] = \sum_{\nu=-\infty}^{\infty} g[n;\nu]s[n - \nu] + v[n], \tag{1}$$

where $v[n] = v(nT)$ and $g[n;\nu] = g(nT;\nu T)$.

Multipath is a common phenomenon in wireless links due to scattering and reflection of the transmitted signal. Each resolvable path corresponds to a superposition of a large number of scattered rays, called a cluster, that arrive at the receiver almost simultaneously with a common propagation delay τ_c. Each of these rays within the cluster is characterized by its own complex gain and frequency offset. Hence, the physical channel $g_{ch}(t;\tau)$ can be written as [6, 7]:

$$g_{ch}(t;\tau) = \sum_c \delta(\tau - \tau_c)\sum_\mu G_{c,\mu}e^{j2\pi f_{c,\mu}t} \tag{2}$$

where $G_{c,\mu}$ and $f_{c,\mu}$ are the complex gain and frequency offset of the μth ray of the cth cluster.

Assuming the time variation of the physical channel $g_{ch}(t;\tau)$ is negligible over the span of the receive filter $g_{rec}(t)$ and the transmit filter $g_{tr}(t)$, we obtain:

$$\begin{aligned} g(t;\tau) &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} g_{rec}(s)g_{tr}(\tau - s - \theta)\mathrm{d}s\right)g_{ch}(t;\theta)\mathrm{d}\theta \\ &= \int_{-\infty}^{\infty}\psi(\tau - \theta)g_{ch}(t;\theta)\mathrm{d}\theta \\ &= \sum_c \psi(\tau - \tau_c)\sum_\mu G_{c,\mu}e^{j2\pi f_{c,\mu}t}. \end{aligned} \tag{3}$$

Hence, we can express $g[n;\nu]$ as:

$$g[n;\nu] = \sum_c \psi(\nu T - \tau_c)\sum_\mu G_{c,\mu}e^{j2\pi f_{c,\mu}nT}. \tag{4}$$

For simplicity (but without loss of generality) we will focus on a flat fading rapidly time-varying channel, where, only one cluster of rays is considered, i.e. $c = 0$ that arrives at τ_0. Then $g_{ch}(t;\tau)$ can be written as:

$$g_{ch}(t;\tau) = \sum_\mu G_\mu e^{j2\pi f_\mu t}\delta(\tau - \tau_0).$$

For simplicity, we consider $\tau_0 = 0$. Hence, the received sequence can be written as:

$$y[n] = g[n]s[n] + v[n].$$

See also a comment on the generality of our approach at the end of Section 4.

3. MMSE CHANNEL ESTIMATION

In this section we derive the minimum mean-square error (MMSE) channel estimator. We rely on pilot-symbol assisted modulation (PSAM) [8], which consists of inserting a few known pilot symbols at know positions. Defining $\mathbf{s}_t = [s[n_0], s[n_1], \ldots, s[n_{P-1}]]^T$ as the vector of the transmitted known symbols, where n_p is the position of the pth pilot symbol, and P is the total number of pilot symbols inserted in a block of N symbols. A noisy estimate of the channel is simply obtained by:

$$\hat{g}_t[p] = \frac{y[n_p]}{s[n_p]} = g[n_p] + \tilde{v}[n_p], \text{ for } p = 0, \ldots, P-1. \tag{5}$$

where $\tilde{v}[n_p] = v[n_p]/s[n_p]$. Define $\hat{\mathbf{g}}_t = [\hat{g}_t[0], \ldots, \hat{g}_t[P-1]]^T$, which is a vector containing the noisy estimates of the channel on the pilot positions. From these noisy estimates, we have to reconstruct the channel response for all $n \in \{0, \ldots, N-1\}$. In other words, we need to design a $P \times N$ interpolation matrix $\mathbf{W}$ such that:

$$\hat{\mathbf{g}} = \mathbf{W}^H\hat{\mathbf{g}}_t, \tag{6}$$

where $\hat{\mathbf{g}} = [\hat{g}[0], \ldots, \hat{g}[N-1]]^T$ is the channel estimate. The mean square-error (MSE) can be written as:

$$\begin{aligned} \epsilon &= \frac{1}{N}\sum_{n=0}^{N-1}\mathcal{E}\{|\hat{g}[n] - g[n]|^2\} \\ &= \frac{1}{N}\mathcal{E}\{\|\mathbf{W}^H\hat{\mathbf{g}}_t - \mathbf{g}\|^2\} \end{aligned} \tag{7}$$

The minimum mean-square error (MMSE) interpolation matrix $\mathbf{W}$ is obtained by solving:

$$\min_{\mathbf{W}} \epsilon$$

The solution of this problem is obtained as follows:

$$\mathbf{W} = \left(\mathbf{R}_p + \mathbf{R}_{\tilde{v}}\right)^{-1}\mathbf{R}_h \tag{8}$$

where $\mathbf{R}_p$ is the channel correlation matrix on the pilots given by:

$$\mathbf{R}_p = \begin{bmatrix} r_h[0] & \cdots & r_h[n_{P-1} - n_0] \\ r_h[n_1] & \cdots & r_h[n_{P-1} - n_1] \\ \vdots & \ddots & \vdots \\ r_h[n_{P-1}] & \cdots & r_h[0] \end{bmatrix},$$

and $\mathbf{R}_h$ is given by:

$$\mathbf{R}_h = \begin{bmatrix} r_h[n_0] & \cdots & r_h[N - n_0 - 1] \\ r_h[n_1] & \cdots & r_h[N - n_1 - 1] \\ \vdots & \ddots & \vdots \\ r_h[n_{P-1}] & \cdots & r_h[N - n_{P-1} - 1] \end{bmatrix},$$

with $r_h[k] = \mathcal{E}\{g[n]g^*[n - |k|]\}$. $\mathbf{R}_{\tilde{v}}$ is the noise correlation matrix. Both $\mathbf{R}_p$ and $\mathbf{R}_h$ are assumed to be known. Note that we used the assumption that the channel is wide sense stationary (WSS). Assuming independent identically distributed (i.i.d) input symbols $s[n]$ with variance σ_s^2, and white noise with variance σ_v^2, then $\mathbf{R}_{\tilde{v}} = \beta\mathbf{I}_P$, where $\beta = \sigma_v^2/\sigma_s^2$.

4. BEM CHANNEL ESTIMATION

The channel model in (4) has a rather complex structure due to the large (possibly infinite) number of parameters to be identified, which complicates, if not prevents, the development of low complexity equalizers. This motivates the use of alternative models, which have fewer number of parameters. This is the motivation behind the Basis Expansion Model (BEM) [9, 10, 11]. In this BEM, the time-varying channel $g[n]$ over a window of N samples, is expressed as a superposition of complex exponential basis functions with frequencies on a discrete grid. In other words, the time-varying channel $g[n]$ is modeled for $n \in \{0, \cdots, N-1\}$ by a BEM:

$$h[n] = \sum_{q=-Q/2}^{Q/2} h_q e^{j2\pi qn/K}, \tag{9}$$

where Q is the number of basis functions, and K is the BEM period. Q and K should be chosen such that $Q/(KT)$ is larger than the maximum Doppler frequency, i.e. $Q/(KT) \geq f_{\max}$. h_q is the coefficient of the qth basis function of the channel, which is kept invariant over a period of NT, but may change from block to block. In earlier work on equalization of doubly selective channels [1, 2, 3], the time-invariant coefficients of the BEM channel model are required to design the equalizer. Define $\mathbf{h}_b = [h_{-Q/2}, \ldots, h_{Q/2}]^T$ as a vector containing the channel BEM coefficients. In the ideal case, where the time-varying channel $\mathbf{g} = [g[0], \ldots, g[N-1]]^T$ is perfectly known at the receiver $\forall n \in \{0, \ldots, N-1\}$, the channel BEM coefficients can be obtained by solving the following least squares (LS) problem:

$$\min_{\mathbf{h}_b} \|\mathbf{g} - \boldsymbol{\mathcal{L}}\mathbf{h}_b\|^2, \tag{10}$$

where

$$\boldsymbol{\mathcal{L}} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{-j2\pi Q/2/K} & \cdots & e^{j2\pi Q/2/K} \\ \vdots & & \vdots \\ e^{-j2\pi Q/2(N-1)/K} & \cdots & e^{j2\pi Q/2(N-1)/K} \end{bmatrix}.$$

The solution of (10) is given by:

$$\mathbf{h}_b = \boldsymbol{\mathcal{L}}^{\dagger}\mathbf{g}.$$

In practice, only a few pilots are available for channel estimation. Assuming the noisy estimates are obtained as in (5), then the channel BEM coefficients can be obtained by solving the following LS problem:

$$\min_{\mathbf{h}_b} \|\hat{\mathbf{g}}_t - \tilde{\boldsymbol{\mathcal{L}}}\mathbf{h}_b\|^2, \tag{11}$$

where

$$\tilde{\boldsymbol{\mathcal{L}}} = \begin{bmatrix} e^{-j2\pi Q/2n_0/K} & \cdots & e^{j2\pi Q/2n_0/K} \\ \vdots & & \vdots \\ e^{-j2\pi Q/2n_{P-1}/K} & \cdots & e^{j2\pi Q/2n_{P-1}/K} \end{bmatrix}.$$

The solution of (11) is obtained by:

$$\mathbf{h}_b = \tilde{\boldsymbol{\mathcal{L}}}^{\dagger}\hat{\mathbf{g}}_t \tag{12}$$

It has been shown in [5], that for the case of a critical sampling of the Doppler spectrum ($K = N$), the optimal training strategy consists of inserting equipowered, equispaced pilot symbols. However, critical sampling of the Doppler spectrum results into an error floor due to the high modeling error. On the other hand, oversampling the Doppler spectrum ($K = rN$, with r integer $r > 1$) reduces the modeling error when the ideal case is considered [4], i.e. when (10) is applied. However, this channel estimate is sensitive to noise when PSAM channel estimation is used.

A robust channel estimate can then be obtained by combining the optimal MMSE interpolation based channel estimate obtained in (6) with the BEM channel estimate obtained in (10) as follows:

- First, obtain the channel estimate $\hat{\mathbf{g}}$ as in (6).
- Second, obtain the LS solution of the following problem:

$$\min_{\mathbf{h}_b} \|\hat{\mathbf{g}} - \boldsymbol{\mathcal{L}}\mathbf{h}_b\|^2, \tag{13}$$

The solution of (13) can be obtained as:

$$\mathbf{h}_b = \boldsymbol{\mathcal{L}}^{\dagger}\hat{\mathbf{g}}, \tag{14}$$

or equivalently in one step as:

$$\mathbf{h}_b = \boldsymbol{\mathcal{L}}^{\dagger}\mathbf{W}^H\hat{\mathbf{g}}_p. \tag{15}$$

Even though this applies to both critical sampling and the oversampling case, little gain is obtained when combining the MMSE interpolation based channel estimate with the critically sampled BEM ($K = N$), as will be clear in Section 5.

In our discussion so far, we considered a time-selective channel. Extending this to a doubly selective channel of order L is rather straightforward when applying the optimal training strategy that consists of equipowered equispaced pilot symbols surrounded by L zeros on each side [5]. Doing this, enables us to treat each tap separately.

5. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed channel estimation techniques. We consider a rapidly time-varying channel simulated according to Jakes' model with $f_{\max} = 1/(400T) = 250\ Hz$, where the sampling time $T = 10\mu sec$. The channel autocorrelation function is given by $r_h[k] = \sigma_h^2 J_0(2\pi f_{\max} kT)$, where J_0 is the zero-th order Bessel function and σ_h^2 denotes the variance of the channel. We consider a window length of $N = 800$ symbols. For the BEM, we consider the critically sampled Doppler spectrum $K = N$, as well as the oversampled Doppler spectrum with oversampling rate 2 ($K = 2N$). The number of basis functions is therefore chosen to be $Q = 4$ for the critical sampling case, and $Q = 8$ for the oversampling case. We use PSAM to estimate the channel. We consider equipowered and equispaced pilot symbols with M the spacing between the pilots. The number of pilots is then computed as $P = \lfloor N/M \rfloor + 1$.

First, we consider the normalized channel MSE versus SNR. We evaluate the performance of the different estimation techniques, in particular, BEM with $K = N$, combined BEM and MMSE with $K = N$, BEM with $K = 2N$, combined BEM and MMSE with $K = 2N$, and the MMSE channel estimate. We consider the case when the spacing between pilot symbols is 160 which corresponds to $P = 5$ pilot symbols dedicated for channel estimation. This choice is well suited for the case of $K = N$, where the number of BEM coefficients to be estimated is $Q + 1$. As shown in Figure 1, all the MSE channel estimates suffer from an early error floor. However, combining the critically sampled BEM with the MMSE results into slight better performance. We further

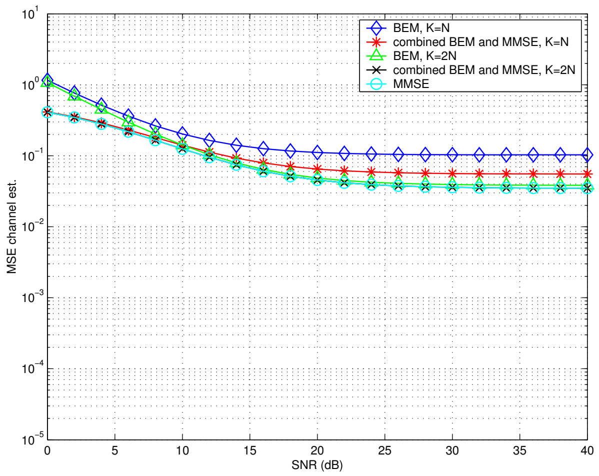

Figure 1: MSE vs. SNR for $P = 5$, $M = 160$

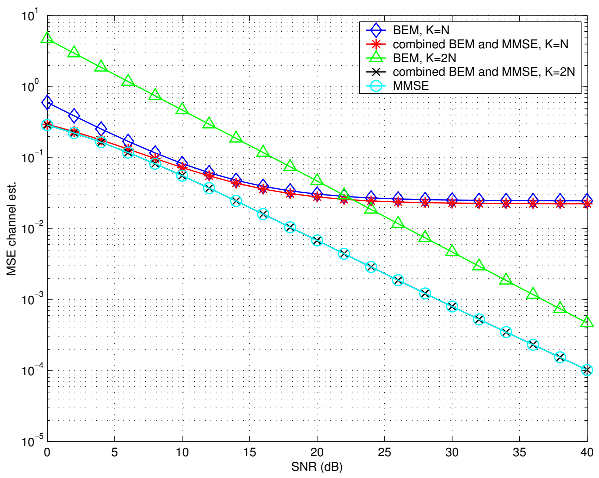

Figure 2: MSE vs. SNR for $P = 9$, $M = 95$

consider the case when the spacing between pilot symbols is $M = 95$ which corresponds to $P = 9$ pilot symbols dedicated for channel estimation. This case is well suited for the case when $K = 2N$. As shown in Figure 2, the performance of BEM with $K = N$ suffers from an early error floor, which means that increasing the number of pilot symbols does not enhance the channel estimation technique. Whilst for the case when $K = 2N$, the MSE curves do not suffer from an early error floor. However, the oversampled BEM channel estimate is sensitive to noise. A significant improvement is obtained when the combined BEM and MMSE method is used, where a gain of 9 dB at $MSE = 10^{-2}$ is obtained over the conventional BEM method, when the oversampling rate is 2. Note also that, the performance of the combined BEM and MMSE method when $K = 2N$ coincides with the performance of the MMSE only.

Second, the estimated channel BEM coefficients are used to design a time-varying (TV) FIR equalizer. We consider here a single-input multiple-output (SIMO) system with $N_r = 2$ receive antennas. The channel is considered to be doubly selective with order $L = 3$. The TV FIR equalizer is designed to have order $L' = 12$ and number of TV basis functions $Q' = 12$. We use the minimum mean-square error (MMSE) criterion to design the TV FIR filter (see [2]):

$$\mathbf{w}^T = \mathbf{e}_d^T(\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}} + SNR^{-1}\mathbf{I}_{(Q+Q'+1)(L+L'+1)})^{-1}\boldsymbol{\mathcal{H}}^H \quad (16)$$

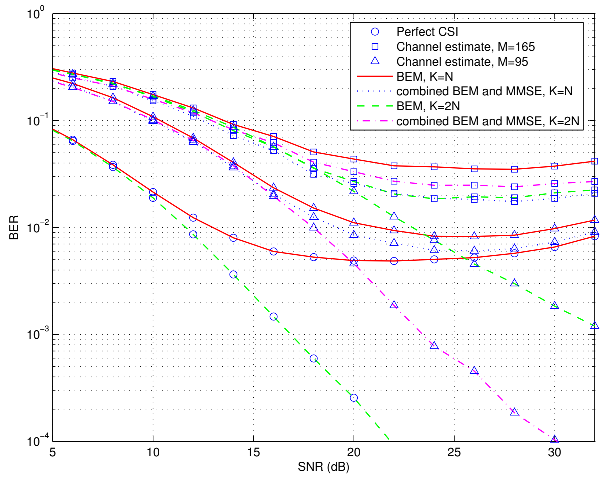

Figure 3: BER vs. SNR using MMSE TV FIR linear equalizer.

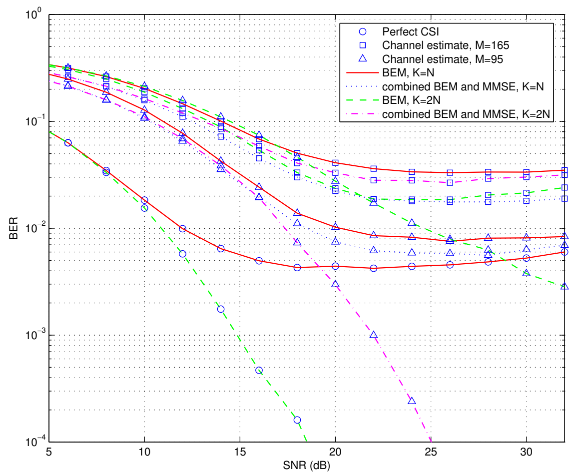

Figure 4: BER vs. SNR using the MMSE TV FIR DFE.

where $\mathbf{w}$ is a vector contains the TV FIR equalizer BEM coefficients, $\mathbf{e}_d$ is a $(Q+Q'+1)(L+L'+1)\times 1$ unit vector with the 1 in the $(d(Q+Q'+1)+(Q+Q')/2+1)$st position, d is the decision delay chosen as: $d = \lfloor\frac{L+L'}{2}\rfloor + 1$, and $\boldsymbol{\mathcal{H}}$ is a $N_r(Q'+1)(L'+1)\times(Q+Q'+1)(L+L'+1)$ matrix containing the doubly selective channel BEM coefficients (as obtained by the different scenarios). The BEM resolution of the TV FIR equalizer matches that of the channel. QPSK signaling is assumed. We define the SNR as $SNR = \sigma_h^2(L+1)E_s/\sigma_v^2$, where E_s is the QPSK symbol power. As shown in Figure 3, the BER curve experiences an error floor when $M = 165$ for the different scenarios. For the case of $M = 95$, we experience an SNR loss of 11.5 dB for the case of $K = 2N$ compared to the case when perfect channel state information (CSI) is known at $BER = 10^{-2}$, while the SNR loss is reduced to 6 dB for the case of combined BEM and MMSE when $K = 2N$. For $K = N$, both cases (BEM and combined BEM and MMSE) suffer from an error floor. We also consider MMSE decision feedback equalization (DFE) as explained in [3]. For the case of MMSE TV FIR DFE, the TV FIR feedforward filter is designed to have order $L' = 12$

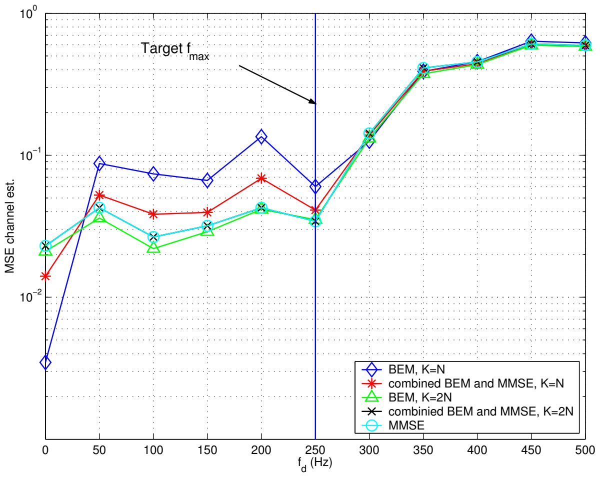

Figure 5: MSE vs. $f_{\max}$ for $P = 5$, $M = 160$, and $SNR = 25\ dB$

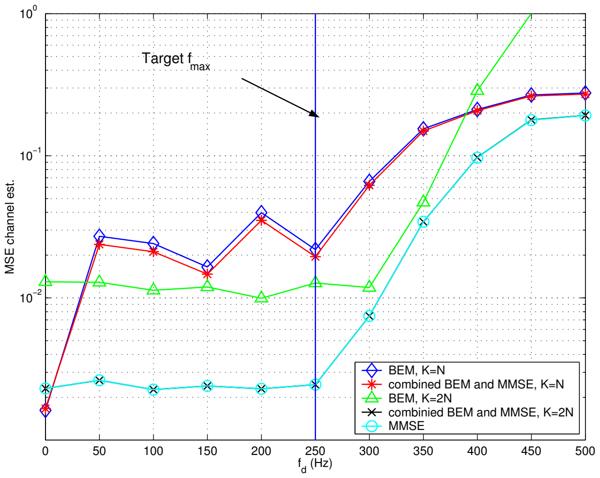

Figure 6: MSE vs. $f_{\max}$ for $P = 9$, $M = 95$, and $SNR = 25\ dB$

and number of TV basis functions $Q' = 12$, and the TV FIR feedback filter to have order $L'' = L$ and $Q'' = Q$. The results are shown in Figure 4. Similar observations to the MMSE TV FIR linear equalizer can be generally observed for the MMSE TV FIR DFE.

Finally, we measure the MSE of the channel estimation techniques as a function of the maximum Doppler frequency. We design the system to have a maximum target Doppler frequency of $f_{\max} = 1/(400T) = 250\ Hz$. We then examine the performance of the channel estimation techniques for different maximum Doppler frequencies at a fixed signal to noise ratio $SNR = 25\ dB$. The results are shown in Figure 5 for the case when $P = 5$ pilot symbols are used for channel estimation, and Figure 6 when $P = 9$ pilot symbols are used. For either case, the channel estimation techniques maintain a low MSE channel estimate as long as the channel maximum Doppler frequency is less than the target maximum Doppler frequency.

6. CONCLUSIONS

In this paper, we have proposed a channel parameters estimation technique for rapidly time-varying channels derived from an interpolation based MMSE estimation scheme. We use the BEM to approximate the time-varying channel. Using the BEM, we only require to estimate the time-invariant coefficients. We rely on PSAM to estimate the channel. We consider the case when the Doppler spectrum is critically sampled ($K = N$) and when it is oversampled (K is multiple of N). While in the first case, the estimation scheme suffers from an early error floor due to the large modeling error, the estimation is sensitive to noise in the oversampled case. It has been shown through computer simulations that combining the MMSE interpolation based channel estimate with the oversampled BEM significantly improves the channel estimation.

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