

ADAPTIVE LAYERED SPACE-FREQUENCY EQUALIZATION FOR SINGLE-CARRIER MIMO SYSTEMS

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ABSTRACT

We propose an adaptive layered space-frequency equalization (ALSFE) structure to deal with the multiple-input multiple-output (MIMO) time-varying frequency selective channels, where at each stage of detection, a group of selected data streams are detected and are then cancelled from the received signals. Two types of adaptive channel estimation approaches are employed for ALSFE, assuming respectively uncorrelated and correlated frequency bins. Noise power estimation is also exploited, which is based on the maximum likelihood (ML) criterion. It is shown that our proposed multistage ALSFE significantly outperforms the previous RLS based single-stage adaptive FDE without channel estimation, at comparable complexity. In particular, ALSFE based on the least-mean-square structured channel estimation (LMS-SCE) approaches the performance of LSFE with perfect channel state information (CSI) and has a fast convergence speed.

1. INTRODUCTION

Frequency-domain equalization (FDE) [1-4] has been shown to be an effective solution for frequency selective channels in a single-carrier (SC) system. In a highly dispersive channel, FDE provides enhanced performance over time domain decision feedback equalization (DFE), and requires less complexity than maximum likelihood sequence estimation (MLSE). FDE also has superiority over orthogonal frequency division multiplexing (OFDM), with lower peak-to-average ratio (PAR) and less sensitivity to carrier synchronization. In [5], FDE was used in multiple-input multiple-output (MIMO) systems, where all the signals are detected simultaneously. In [6], a layered space-frequency equalization (LSFE) structure was proposed, which combines FDE and successive interference cancellation to improve the performance of the single-stage MIMO FDE [5]. However, [5] and [6] only investigated quasi-static channels.

Adaptive FDE structures have been investigated in [2] and [7], where the equalizer coefficients are directly calculated based on the least-mean-square (LMS) or recursive-least-square (RLS) criterion, without channel estimation required. Another type of adaptive FDE structures are based on adaptive channel estimation [8] where the equalizer coefficients are computed based on the channel estimates. However, the work in [8] only assumed single-input single-output (SISO) and single-input multiple-output (SIMO) systems.

In this paper we propose an adaptive LSFE (ALSFE) structure for MIMO systems in time-varying frequency selective channels. Our work is different in that we incorporate LSFE with both adaptive channel estimation and noise power estimation for MIMO systems. Two types of adaptive channel estimation methods are proposed. The first one operates independently on each frequency bin and is referred to as *unstructured channel estimation* (UCE). The second one is called *structured channel estimation* (SCE) which utilizes the fading correlation between adjacent frequency bins. The channel estimates are updated by the LMS and RLS algorithms and are used for computing the FDE coefficients. Noise power estimation is also employed, which is based on the maximum likelihood (ML) criterion. The channel estimates and noise power estimates are then used to compute the FDE coefficients in LSFE, where at each stage a group of the best data streams in the minimum mean square error (MMSE) sense are detected and are then canceled from the received signals. The ALSFE structure provides performance enhancement especially at high SNR compared to RLS based single-stage FDE without explicit channel estimation [7]. In particular, the LMS-SCE based ALSFE performs significantly better than RLS FDE with negligible increase in computational complexity. Also the LMS-SCE ALSFE performs much better than RLS-UCE ALSFE with much less computation and this performance tends to reach that of LSFE with perfect channel state information (CSI).

Section 2 presents the system model. The proposed ALSFE structure is described in Section 3. The computational complexity is analyzed in Section 4. Section 5 shows the simulation results and the conclusion is drawn in Section 6.

2. SYSTEM MODEL

We investigate an uncoded MIMO system with K transmit antennas and L receive antennas. Let $d_k^{(p)}[i]$ denote the i th ($i = 0, \dots, M-1$) data symbol in the p th block of M symbols transmitted by the k th ($k = 1, \dots, K$) antenna, with unit average symbol energy. The noise is AWGN with single-sided power spectral density $N_0^{(p)}$ over the p th block. The channel memory is assumed to be N , and $h_{kl}^{(p)}[i]$ ($i = 0, \dots, N$) denotes the channel impulse response (CIR) between the k th transmit antenna and the l th receive antenna over the p th block. Each block is appended with a length- N cyclic prefix (CP) which is discarded at the receiver to prevent the interblock interfer-

ence (IBI) and to make the received block appear to be periodic with period M . The received signals transferred into the frequency domain by the FFT operation is given by

$$X_l^{(p)}[m] = \sum_{k=1}^K D_k^{(p)}[m] H_{kl}^{(p)}[m] + N_l^{(p)}[m] \quad (1)$$

where $X_l^{(p)}[m] = \sum_{i=0}^{M-1} x_l^{(p)}[i] e^{j2\pi im/M}$, $H_{kl}^{(p)}[m] = \sum_{i=0}^N h_{kl}^{(p)}[i] e^{j2\pi im/M}$, $D_k^{(p)}[m] = \sum_{i=0}^{M-1} d_k^{(p)}[i] e^{j2\pi im/M}$, $N_l^{(p)}[m] = \sum_{i=0}^{M-1} n_l^{(p)}[i] e^{j2\pi im/M}$. Defining $\mathbf{D}^{(p)}[m] = [D_1^{(p)}[m] \dots D_K^{(p)}[m]]$ and $\mathbf{H}_l^{(p)}[m] = [H_{1l}^{(p)}[m] \dots H_{Kl}^{(p)}[m]]^T$, (1) becomes

$$X_l^{(p)}[m] = \mathbf{D}^{(p)}[m] \mathbf{H}_l^{(p)}[m] + N_l^{(p)}[m] \quad (2)$$

3. ADAPTIVE LAYERED SPACE-FREQUENCY EQUALIZATION

3.1 Algorithm Description

The proposed ALSFE structure is the same as LSFE [6] except that adaptive channel estimation and noise power estimation are employed. ALSFE consists of 1 to K detection stages (K stages in fig.1), determined by the number of output data streams at each stage. At a particular stage, it is assumed that m_o data streams are selected for detection, the index set of which is denoted by Det . Letting $\tilde{d}_k^{(p)}[i]$ and $\hat{d}_k^{(p)}[i]$ ($k \in Det$) denote the soft estimate and hard estimate of $d_k^{(p)}[i]$ respectively, the mean square error (MSE) is defined as

$$MSE_k^{(p)} = E \left[\tilde{d}_k^{(p)}[i] - d_k^{(p)}[i] \right]^2 \quad (3)$$

The selection process is based on the MMSE criterion, i.e., the m_o data streams with the smallest MSEs are selected.

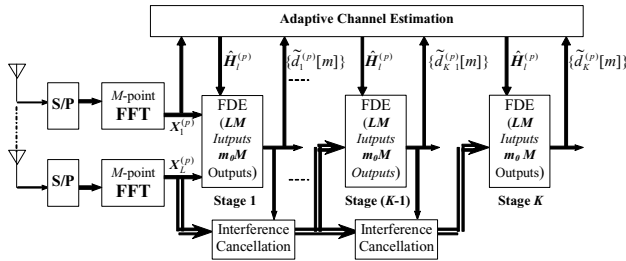


Figure 1. Block diagram of ALSFE with total K stages ($m_o=1$)

3.2 FDE Coefficients

As shown in fig.1, the modified incoming signal vector $\mathbf{X}^{(p)}[m]$ at the m th frequency tone at a particular stage is composed of L entries $X_l^{(p)}[m]$ ($l=1, \dots, L$) and can be written as

$$\mathbf{X}^{(p)}[m] = \sum_n D_n^{(p)}[m] \hat{\mathbf{H}}_n^{(p)}[m] + N^{(p)}[m] \quad (4)$$

where n denotes the summation over the undetected data streams, and $\hat{\mathbf{H}}_n^{(p)}[m]$ is the estimate of $\mathbf{H}_n^{(p)}[m]$ composed of L entries $H_{nl}^{(p)}[m]$ ($l=1, \dots, L$). Let $\mathbf{W}^{(p)}[m]$ ($m=0, \dots, M-1$) denote the FDE weight matrix with respect to the m th frequency tone, which is of size $L \times m_o$. The soft estimate is expressed as

$$\tilde{\mathbf{d}}^{(p)}[i] = \frac{1}{M} \sum_{m=0}^{M-1} \mathbf{W}^{(p)H}[i] \mathbf{X}^{(p)}[m] e^{j2\pi im/M} \quad (5)$$

where $(\cdot)^H$ denotes the complex-conjugate transpose. For simplicity of presentation and without loss of generality,

$Det = \{1, \dots, m_o\}$. Defining the error vector $\mathbf{e}^{(p)}[i] = \tilde{\mathbf{d}}^{(p)}[i] - \mathbf{d}^{(p)}[i]$, the optimum weights are determined to minimize

$$\sum_{k \in Det} MSE_k^{(p)} = Tr \left(E \left[\mathbf{e}^{(p)}[i] (\mathbf{e}^{(p)}[i])^H \right] \right) = Tr \left(\mathbf{R}_{ee}^{(p)} \right) \quad (6)$$

where $\mathbf{R}_{ee}^{(p)}$ denotes the error autocorrelation matrix, and $Tr(\cdot)$ represents the trace of a matrix. It can be shown that the optimum weight matrix $\mathbf{W}^{(p)}[m]$ is given by

$$\mathbf{W}^{(p)}[m] = \mathbf{R}^{(p)-1}[m] \mathbf{F}^{(p)}[m] \quad (7)$$

where

$$\mathbf{R}^{(p)}[m] = \sum_n \hat{\mathbf{H}}_n^{(p)}[m] \hat{\mathbf{H}}_n^{(p)H}[m] + \hat{N}_0^{(p)} \mathbf{I} \quad (8)$$

$$\mathbf{F}^{(p)}[m] = [\hat{\mathbf{H}}_1^{(p)}[m] \dots \hat{\mathbf{H}}_{m_o}^{(p)}[m]] \quad (9)$$

3.3 Adaptive Channel Estimation

We propose two types of adaptive channel estimation schemes by extending the work in [8] to MIMO systems. The first one is based on the assumption of independent frequency bins, referred to as *unstructured channel estimation* (UCE). The second one utilizes the correlation between adjacent frequency bins, and is referred to as *structured channel estimation* (SCE). As illustrated in fig.1, we define a vector $\mathbf{x}_l^{(p)}$ as

$$\mathbf{x}_l^{(p)} = \tilde{\mathbf{D}}^{(p)} \mathbf{H}_l^{(p)} + N_l^{(p)} \quad (10)$$

where $\mathbf{x}_l^{(p)} = [X_l^{(p)}[0] \dots X_l^{(p)}[M-1]]^T$, $N_l^{(p)} = [N_l^{(p)}[0] \dots N_l^{(p)}[M-1]]^T$,

and $\tilde{\mathbf{D}}^{(p)} = \begin{bmatrix} \mathbf{D}^{(p)}[0] & & \\ & \ddots & \\ & & \mathbf{D}^{(p)}[M-1] \end{bmatrix}$ which is of size $M \times KM$.

$\mathbf{H}_l^{(p)} = [\mathbf{H}_l^{(p)T}[0] \dots \mathbf{H}_l^{(p)T}[M-1]]^T$ which can also be written as

$$\mathbf{H}_l^{(p)} = \tilde{\mathbf{F}} \mathbf{h}_l^{(p)} \quad (11)$$

where $\mathbf{h}_l^{(p)} = [h_{1l}^{(p)}[0] \dots h_{1l}^{(p)}[N] \dots h_{kl}^{(p)}[0] \dots h_{kl}^{(p)}[N]]^T$ is the CIR vector of length $K(N+1)$ at the l th receive antenna. $\tilde{\mathbf{F}} = (\mathbf{F}_0^T \dots \mathbf{F}_M^T)^T$ where \mathbf{F}_m ($0 \leq m \leq M-1$) is a $K \times K(N+1)$ block Toeplitz matrix

$$\mathbf{F}_m = \begin{bmatrix} f_m & 0 & \dots & 0 \\ 0 & f_m & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & f_m \end{bmatrix} \quad (12)$$

with $f_m = [e^{-j2\pi 0m/M} \dots e^{-j2\pi Nm/M}]$

3.3.1 LMS Unstructured Channel Estimation (LMS-UCE)

The LMS-UCE minimizes the cost function

$$J_{LMS\ UCE}(\hat{\mathbf{H}}_l^{(p)}) = E \left\{ \left\| \mathbf{X}_l^{(p)} - \tilde{\mathbf{D}}^{(p)} \hat{\mathbf{H}}_l^{(p)} \right\|^2 \right\} \quad (l=1, \dots, L) \quad (13)$$

with respect to $\hat{\mathbf{H}}_l^{(p)}$ which is the estimate of $\mathbf{H}_l^{(p)}$. This produces

$$\hat{\mathbf{H}}_l^{(p+1)} = \hat{\mathbf{H}}_l^{(p)} + \mathbf{E}_l^{(p)} \quad (14)$$

where μ is the step size and $\mathbf{E}_l^{(p)}$ is given by

$$\mathbf{E}_l^{(p)} = \tilde{\mathbf{D}}^{(p)H} \left[\mathbf{X}_l^{(p)} - \tilde{\mathbf{D}}^{(p)} \hat{\mathbf{H}}_l^{(p)} \right] \quad (15)$$

3.3.2 RLS Unstructured Channel Estimation (RLS-UCE)

The RLS-UCE aims at minimizing the cost function

$$J_{RLS\ UCE}(\hat{\mathbf{H}}_l^{(p)}) = \sum_{i=0}^p \mu^i \left\| \mathbf{X}_l^{(i)} - \tilde{\mathbf{D}}^{(i)} \hat{\mathbf{H}}_l^{(p)} \right\|^2 \quad (l=1, \dots, L) \quad (16)$$

where $(0 < \alpha < 1)$ is the forgetting factor. $\hat{\mathbf{H}}_l^{(p)}$ satisfies the recursive equation

$$\hat{\mathbf{H}}_l^{(p+1)} = \alpha \hat{\mathbf{H}}_l^{(p)} + \mathbf{G}^{(p)} \mathbf{E}_l^{(p)} \quad (17)$$

where $\mathbf{E}_l^{(p)}$ is defined in (15) and

$$\mathbf{G}^{(p)} = \text{diag}(\mathbf{G}^{(p)}[0] \dots \mathbf{G}^{(p)}[M-1]) \quad (18)$$

is a block diagonal matrix, with $\mathbf{G}^{(p)}[m]$ expressed as

$$\mathbf{G}^{(p)}[m] = \frac{\mathbf{S}^{(p)}[m]}{\mathbf{D}^{(p)}[m] \mathbf{S}^{(p)}[m] \mathbf{D}^{(p)H}[m]} \quad (19)$$

where $\mathbf{S}^{(p)}[m]$ satisfies the recursion

$$\mathbf{S}^{(p+1)}[m] = \alpha \left[\mathbf{I} + \mathbf{G}^{(p)}[m] \mathbf{D}^{(p)H}[m] \mathbf{D}^{(p)}[m] \right] \mathbf{S}^{(p)}[m] \quad (20)$$

Note that $\mathbf{G}^{(p)}[m]$ and $\mathbf{S}^{(p)}[m]$ are independent of the index l , implying that these are the same at each receive antenna.

3.3.3 LMS Structured Channel Estimation (LMS-SCE)

The cost function of LMS-UCFE is given by

$$J_{LMS-SCE}(\hat{\mathbf{h}}_l^{(p)}) = E \left\{ \left\| \mathbf{X}_l^{(p)} \tilde{\mathbf{D}}^{(p)} \tilde{\mathbf{F}} \hat{\mathbf{h}}_l^{(p)} \right\|^2 \right\} \quad (l=1, \dots, L) \quad (21)$$

with respect to $\hat{\mathbf{h}}_l^{(p)}$ which is the estimate of $\mathbf{h}_l^{(p)}$. This produces

$$\hat{\mathbf{h}}_l^{(p+1)} = \hat{\mathbf{h}}_l^{(p)} + \tilde{\mathbf{F}}^H \mathbf{E}_l^{(p)} \quad (22)$$

$$\hat{\mathbf{H}}_l^{(p+1)} = \hat{\mathbf{H}}_l^{(p)} + \tilde{\mathbf{F}} \tilde{\mathbf{F}}^H \mathbf{E}_l^{(p)} \quad (23)$$

3.3.4 RLS Structured Channel Estimation (RLS-SCE)

The objective of RLS-SCE is to minimize the cost function

$$J_{RLS-SCE}(\hat{\mathbf{h}}_l^{(p)}) = \sum_{i=0}^p \lambda^i \left\| \mathbf{X}_l^{(i)} \tilde{\mathbf{D}}^{(i)} \tilde{\mathbf{F}} \hat{\mathbf{h}}_l^{(p)} \right\|^2 \quad (l=1, \dots, L) \quad (24)$$

This however requires prohibitive complexity as no recursion can be used to compute the inverse of a matrix. Therefore, we do not consider this method in the following.

3.4 Noise Power Estimation

We assume that the noise power is the same at each receive antenna and constant over a frame. Therefore, the noise variance $\sigma^2 = N_0^{(p)}$ can be estimated frame by frame exploiting the training blocks. Collecting (10) and (11) yields

$$\mathbf{X}_l^{(p)} = \tilde{\mathbf{D}}^{(p)} \tilde{\mathbf{F}} \mathbf{h}_l^{(p)} + \mathbf{N}_l^{(p)} \quad (25)$$

where $\tilde{\mathbf{D}}^{(p)}$ is known (training block), the joint ML estimates of $\mathbf{h}_l^{(p)}$ and σ^2 ($l=1, \dots, L$) based on the observation of $\mathbf{X}_l^{(p)}$ are found by maximizing the log-likelihood function

$$J(\hat{\sigma}^2, \hat{\mathbf{h}}_l^{(p)}) = -M \ln(\hat{\sigma}^2) - \frac{1}{\hat{\sigma}^2} \left\| \mathbf{X}_l^{(p)} - \tilde{\mathbf{D}}^{(p)} \tilde{\mathbf{F}} \hat{\mathbf{h}}_l^{(p)} \right\|^2 \quad (26)$$

with respect to $\hat{\sigma}^2$ and $\hat{\mathbf{h}}_l^{(p)}$. Keeping $\hat{\sigma}^2$ fixed and maximizing (26) with respect to $\hat{\mathbf{h}}_l^{(p)}$ produces

$$\hat{\mathbf{h}}_l^{(p)} = (\mathbf{Z}^{(p)H} \mathbf{Z}^{(p)})^{-1} \mathbf{Z}^{(p)H} \mathbf{X}_l^{(p)} \quad (27)$$

where $\mathbf{Z}^{(p)} = \tilde{\mathbf{D}}^{(p)} \tilde{\mathbf{F}}$. Substituting (27) into (26) and maximizing with respect to $\hat{\sigma}^2$ gives the ML estimate of $\hat{\sigma}^2$

$$\hat{\sigma}^2 = \frac{1}{M} \left\| \left(\mathbf{I}_{2N} - \mathbf{Z}^{(p)} (\mathbf{Z}^{(p)H} \mathbf{Z}^{(p)})^{-1} \mathbf{Z}^{(p)H} \right) \mathbf{X}_l^{(p)} \right\|^2 \quad (28)$$

Averaging (28) over the received antennas and available training blocks produces the following unbiased estimate

$$\hat{N}_0^{(p)} = \hat{\sigma}_{ML}^2 = \frac{M}{N_T L (M-N)} \sum_{m=1}^M \sum_{l=1}^L \hat{\sigma}^2 \quad (29)$$

with N_T being the number of training blocks.

TABLE I. Computational Complexity Per Detected block

	Channel Estimation	FDE Coefficients	FFT+FDE +IFFT	
LMS-UCFE ALSFE	$(0.5 \log_2 M + 2L)KM$	$\frac{KM^3}{3}$ (1 stage)	$0.5LM (\log_2 M)$ + KLM + $0.5KM (\log_2 M)$	
RLS-UCFE ALSFE	$(0.5 \log_2 M + 2L)KM$ + $(K+L+3)K^2M$ + KM			$K(K+1)L^2M/6$ (K stages)
LMS-SCE ALSFE	$(0.5 \log_2 M + 2L)KM$ + $KLM \log_2 M$			
RLS FDE [7]	0	$(0.5 \log_2 M + 2L)KM$ + $(L+K+3)L^2M$ + LM		

TABLE II. Normalized Computational Complexity Per Detected Block With $K=4, L=4$ and $M=64$

LMS-UCFE ALSFE	RLS-UCFE ALSFE	LMS-SCE ALSFE	RLS FDE [7]
64% (1 stage)	132% (1 stage)	101% (1 stage)	100%
113% (4 stages)	181% (4 stages)	149% (4 stages)	(1 stage)

4. COMPLEXITY ANALYSIS

The computational complexity is approximately evaluated by counting the number of complex multiplications per detected block of signals. The complexities of multi-stage LMS-UCFE ALSFE, RLS-UCFE ALSFE and LMS-SCE ALSFE are shown in Table I. With $K=4$ transmit antennas, $L=4$ receive antennas, and a data block size $M=64$, their normalized complexity is demonstrated in Table II. We focus on 1-stage and K -stage ALSFE structures.

It can be derived that 1-stage LMS-UCFE ALSFE requires the least complexity and K -stage RLS-UCFE ALSFE requires the most complexity.

5. SIMULATION RESULTS

We use simulation results to show the performance of ALSFE, using the three adaptive channel estimation methods shown in Tables I-II, with $K=4$ transmit and $L=4$ receive antennas. Each frame consists of 10 training blocks and 100 data blocks, each of which consists of $M=64$ QPSK symbols, with a data rate of 2 Mbps. The transmit and receive filters use a raised-cosine pulse with a roll-off factor of 0.35. We consider a typical urban environment where the channel is modeled by the following power delay profile [9] with a normalized RMS delay spread $\sigma = 0.625 \tau_s$. The overall channel memory is $N=6$. Noise power estimation is employed and the Doppler frequency f_d is 50Hz or 100Hz. The SNR is defined as the spatial average ratio of the received signal power to noise ratio. The step sizes for LMS-UCFE ALSFE, LMS-SCE ALSFE are $\mu = 2 \cdot 10^{-3}$ and $\mu = 1.4 \cdot 10^{-4}$, respectively. The forgetting factor for both RLS-UCFE ALSFE and RLS FDE is set to $\alpha = 0.8$.

Fig. 2 and fig.3 show the BER performance of the ALSFE structures in Tables II with 10 training blocks for $f_d=50$ Hz and $f_d=100$ Hz respectively. In fig.2, all the ALSFE structures outperform RLS FDE without channel estimation [7], espe-

cially at high SNR. We can observe that RLS-UCE ALSFE and LMS-SCE ALSFE significantly outperform RLS FDE without channel estimation. In particular, LMS-SCE ALSFE provides the best performance, approaching the performance of FDE with perfect CSI. In fig.3, we can observe that LMS-SCE ALSFE structure and RLS-UCE ALSFE structure still outperform RLS FDE without channel estimation, especially for LMS-SCE ALSFE at high SNR. In particular, LMS-SCE ALSFE provides the best performance approaching the performance of FDE with perfect CSI.

Fig. 4 illustrates the learning curves for the 4-stage LASFE structures, in terms of MSE versus the number of training blocks with $f_d=50\text{Hz}$ and an SNR of 16 dB. It can be seen that 4-stage LMS-SCE ALSFE has the fastest convergence speed with only 4 training blocks required, at the cost of a modest increase of complexity compared to RLS FDE without channel estimation. Meanwhile, 4-stage LMS-SCE ALSFE has the lowest MSE in the steady state close to that of 4-stage ALSFE with perfect CSI.

6. CONCLUSION

We have proposed an ALSFE structure which incorporates LSFE with adaptive channel estimation and noise power estimation to combat MIMO time-varying frequency selective channels. Two types of adaptive channel estimation methods based on SCE and UCE are proposed. The ALSFE structure provides performance enhancement especially at high SNR compared to RLS based single-stage FDE without explicit channel estimation [7]. Particularly, the LMS-SCE based ALSFE performs significantly better than RLS FDE with negligible increase in computational complexity. Also the LMS-SCE ALSFE performs much better than RLS-UCE ALSFE with much less computation and this performance tends to reach that of LSFE with perfect channel state information (CSI).

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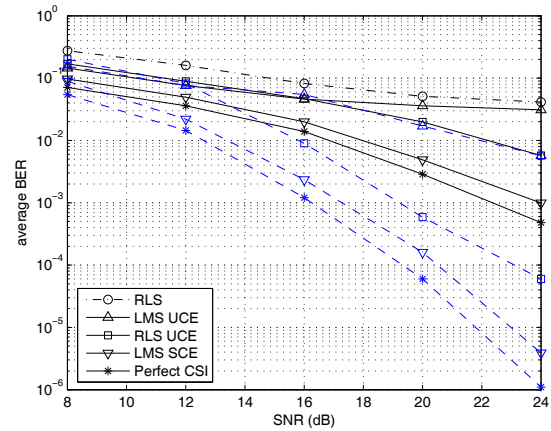


Figure 2. ALSFE with $K=4, L=4$ and $N_T=10$ ($f_d=50\text{Hz}$) (1-stage with solid lines and 4-stage with dashed lines)

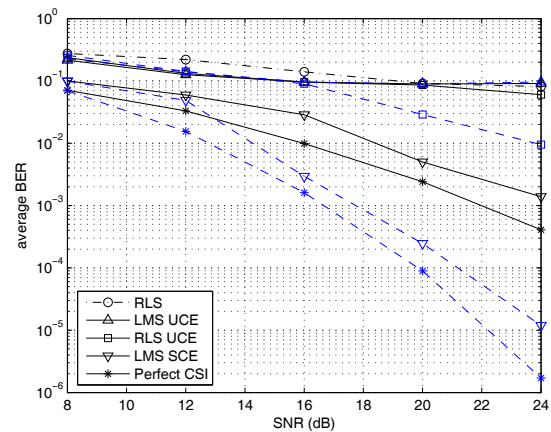


Figure 3. ALSFE with $K=4, L=4$ and $N_T=10$ ($f_d=100\text{Hz}$) (1-stage with solid lines and 4-stage with dashed lines)

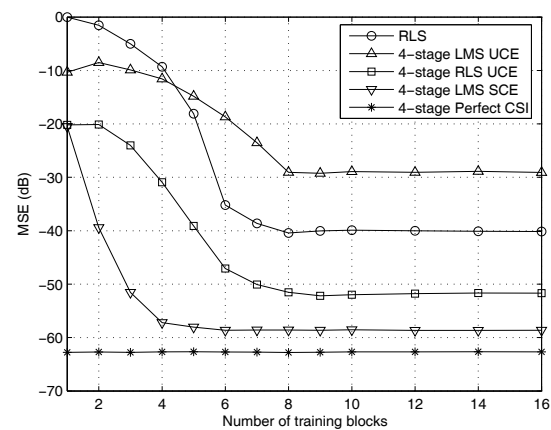


Figure 4. Learning curves for ALSFE with $K=4, L=4$ and SNR=16dB ($f_d=50\text{Hz}$)