SUBBAND ADAPTIVE EQUALISER TRACKING FOR FRACTIONALLY-SAMPLED FADING BROADBAND MIMO CHANNELS

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ABSTRACT

This paper demonstrates the performance of a subband adaptive multiple-input multiple-output (MIMO) equaliser tracking the inverse of a Doppler-filtered Rayleigh-fading broadband channel. The subband adaptive equaliser is shown to outperform the fullband equivalent for channels with a high spectral dynamic range and long MMSE equaliser response in terms of tracking MSE and computational cost.

1. INTRODUCTION

Huge radio channel capacity increases can be achieved through the use of Multiple-Input Multiple-Output (MIMO) channels. The complexity of equalising such channels at symbol rates greater than the coherence bandwidth can rapidly spiral to unrealistic numbers of computations, especially for MIMO systems of large dimensions and fractional-spaced architectures. Further, when the receiving device is moving, the problem is exacerbated by fading. An adaptive equalisation algorithm will have to track the inverse of the channel. Commonly, the Least Mean Squares (LMS) or Recursive Least Squares (RLS) algorithms [1] are use to adaptively invert unknown systems. Although the RLS can converge to stationary systems very quickly, the cost of inverting and tracking these can make this approach unfeasible. Further, in a dynamic environment the RLS algorithm may perform worse than the LMS [2]. Even using the LMS algorithm can require great computational resources for such channels [3].

An approach used in [3] to combat this problem employs a subband adaptive system which has been shown to reduce computational cost while in some cases improving convergence performance for the stationary case. It has been shown that in terms of computational cost and speed a better approach to finding the MMSE equaliser is to adaptively identify the channel and use an efficient analytic inversion method to obtain the equaliser [4]. We may then use subband adaptive inversion in [3] to track dynamic channels. Therefore, in this paper we explore the performance of a subband based equaliser for symbol-spaced (SS) and fractionally-spaced (FS) broadband Doppler-filtered Rayleigh-fading MIMO channels.

2. SYSTEM DESCRIPTION

A fractionally-spaced (FS) fading broadband MIMO system can be described by

$$\mathbf{y}[n] = \mathbf{H}[n]\mathbf{x}[n] + \quad [n], \qquad (1)$$

where

$$\begin{aligned}
\mathbf{x}[n] &= [\,x_1[n] \cdots x_1[n-L_h+1]\; x_2[n] \cdots x_M[n-L_h+1]]^T, \\
[n] &= [\quad _{1,0}[n] \cdots \quad _{1,R-1}[n] \quad _{2,0}[n] \cdots \quad _{P,R-1}[n]]^T, \\
\mathbf{y}[n] &= [\,y_{1,0}[n] \cdots y_{1,R-1}[n]\; y_{2,0}[n] \cdots y_{P,R-1}[n]]^T,
\end{aligned}$$

$$\mathbf{H}[n] = \begin{bmatrix} \mathbf{H}_{11}[n] & \cdots & \mathbf{H}_{M1}[n] \\ \vdots & \ldots & \vdots \\ \mathbf{H}_{1P}[n] & \cdots & \mathbf{H}_{MP}[n] \end{bmatrix}, \qquad (2)$$

$$\mathbf{H}_{mp}[n] = \begin{bmatrix} h_{mp,0}[n] & \cdots & h_{mp,0}[n-L_h+1] \\ \vdots & \ldots & \vdots \\ h_{mp,R-1}[n] & \cdots & h_{mp,R-1}[n-L_h+1] \end{bmatrix}.$$

The vector $\mathbf{y}[n]$ contains the multiple received signals and $\mathbf{x}[n]$ the stacked multi-channel filter tapped delay line values, while $[n]$ is a similarly defined vector of additive white Gaussian noise. The FS MIMO channel is expressed in polyphase form [5], where the phases are effectively represented as separate channels, and the coefficient of phase r of the sub-channel between input m and output p at time n is denoted $h_{mp,r}[n]$. Finally, L_h is the length of each phase of each sub-channel comprising the MIMO channel, M is the number of input channel (e.g. transmitters), P is the number of output channels (e.g. receivers) and R is the number of phases, equal to the FS oversampling factor.

We now apply a MIMO equaliser, $\mathbf{G}[n]$ defined similarly to (2), to the output of the channel. The equaliser matrix is of dimensions $M \times PRL_g$, where L_g is the length of each phase of each sub-channel of the MIMO equaliser. If we assume that the input signal powers $^2_{x_m}$ are the same for all transmitters and the noise powers ^2_p are equal at all receivers, then the infinite-length MMSE solution to the problem is given in the frequency domain by the pseudo-inverse regularised by the noise to signal power ratio

$$\mathbf{G}_{\text{MMSE}}(f) = \left((\mathbf{H}(f))^H \mathbf{H}(f) + \frac{^2}{^2_x}\mathbf{I} \right)^{-1} (\mathbf{H}(f))^H, \qquad (3)$$

where the dependency on time has been dropped for simplicity of notation. Note the dimensions of $\mathbf{G}(f)$ are $M \times PR$. Assuming that the fading channel is known at some point, for example by adaptive identification, we may use (3) to calculate a near-MMSE equaliser. Using this as initialisation, an adaptive MIMO equaliser can thereafter track the inverse of the fading channel.

3. ADAPTIVE TRACKING

Figure 1 shows an adaptive system that can be used for tracking a $T/2$-spaced equaliser for a fading 2×2 MIMO channel. We use a multi-channel normalised LMS algorithm due to its low cost and stability. However, for highly time-dispersive MIMO broadband channels the cost of this algorithm may become problematic [3]. This problem is further exacerbated by the use of fractional-spacing which effectively increases the number of channels R, with a cost accruing to

$$C_{\text{M-NLMS}} = M(8RPL_g + 8RP), \qquad (4)$$

multiply-accumulate (MAC) operations per iteration.

The convergence and tracking characteristics of the NLMS algorithm are severely limited by the potential spectral colouring introduced by the frequency-selectivity of the channel [1]. In addition, the channel may fade at such a speed that the algorithm is unable to adequately track its inverse. Subband techniques have in the past provided a reduction of cost and an increase in convergence speed for adaptive inversion of stationary MIMO channels. Therefore, here we want to exploit this technique and investigate the tracking ability of subband equaliser structures.

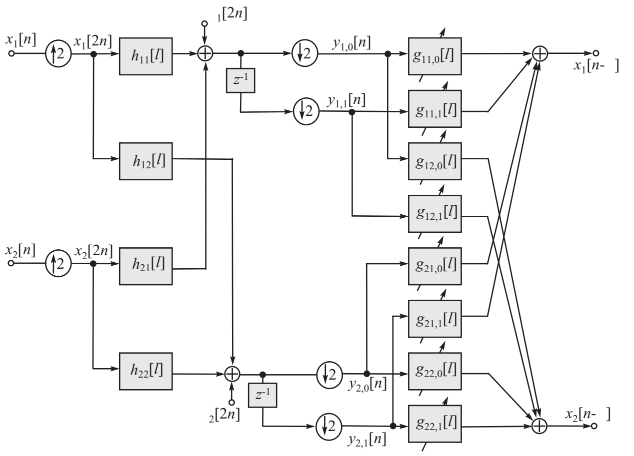

Figure 1: A polyphase representation of a 2×2 MIMO $T/2$ fractionally-spaced channel-equaliser system.

4. SUBBAND ADAPTATION

Subband adaptive filtering [6] involves passing the input fullband signal through a bank of parallel band-pass filters, a so-called analysis filter bank, to split the signal into K consecutive frequency bands. Any signal processing task such as equalisation can then be performed in subbands prior to reconstructing the fullband signal by means of a synthesis filter bank. The ability to process subbands independently relies vitally on oversampling, i.e. decimating the subband signals by only a factor $N < K$ in order to avoid aliasing [7].

A number of advantages are associated with subband processing. Firstly, the subbands can be processed in parallel. Secondly, the division into a number of frequency bands results in a "spectral whitening" which enhances the convergence speed of LMS-type algorithms for coloured signals [3, 7]. Thirdly, since the adaptation update rate is reduced by N, and the filters for subband processing can be shortened by up to a factor of N, a cost reduction of up to K/N^2 for LMS-type algorithms becomes possible [3].

For subband MIMO equalisation we require a multi-channel subband adaptive equaliser shown in Figure 2. Each phase of each received signal is separated by shifting and downsampling as shown in Figure 1 to create PR equaliser input signals. Each of these is passed through an analysis bank and sent to the appropriate subband block. Within each block we have PR filters to perform the equalisation. The output of each subband block is compared to the corresponding subband desired signal for the m^{th} transmitter, and thus an error signal is generated from which the filters are adapted. The subband outputs may be synthesised to created the equalised fullband output signal for the m^{th} transmitted signal. The MIMO system is implemented by using M of these MISO structures in parallel, each using a different desired (transmitted) signal. The cost of the subband multi-channel NLMS algorithm is

$$\begin{aligned} C_{\text{sb,NLMS}} &= \frac{KM}{N}(8RPL_{gs} + 8RP) + \\ &\quad (RP + 2M)(2L_p + 4K\log_2 K + 8K)/N, \qquad (5) \end{aligned}$$

where L_{gs} and L_p are the subband equaliser filter and analysis/synthesis filter lengths respectively . The relationship between L_{gf} and L_{gs} is not exact and depends on the channel characteristics, but values in the range $L_{gs} = [L_{gf}/N, (L_{gf} + 2L_p)/N]$ are often used.

A problem with the subband implementation is concerned with its application in a real system. Normally, an adaptive equaliser will track in a blind decision-directed mode. With the fullband system,

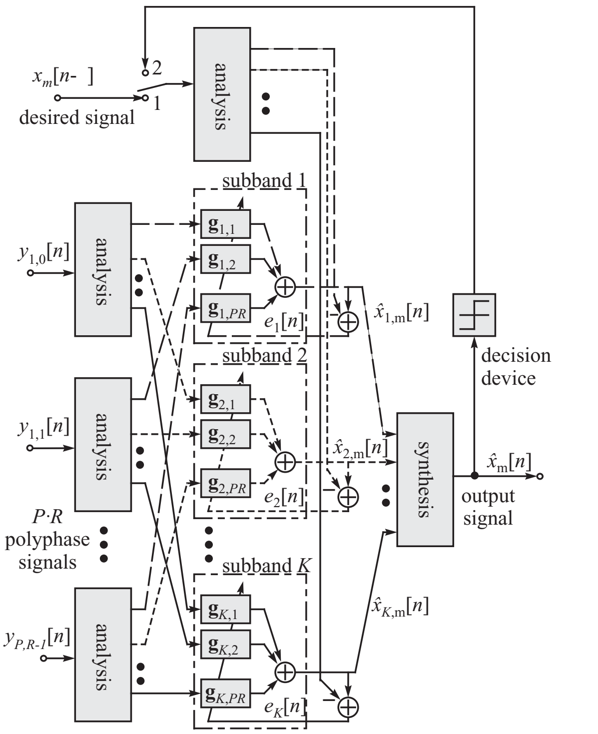

Figure 2: A system for performing multi-channel subband adaptive equaliser tracking for a fading broadband MIMO channel.

this presents no particular problem— as long as the decisions are error-free we may expect the tracking performance to be the same as if a known training signal was used for the entire period. The subband system in Figure 2 with the switch in position 1 requires the use of a training signal known to the receiver, and hence the adaptation is performed in a delayless subband mode. However, to function in decision-directed mode with the switch in position 2 the output subband signals must be reconstructed by the synthesis bank into the fullband before a decision can be made for each symbol. Then the signal must be transformed back into subbands through an analysis bank to form the subband "desired" signals. Together these filter banks introduce a delay on the order of the analysis or synthesis filter length into the error path. Therefore the input signals to the algorithm must also be delayed by the same amount [8]. This results in a special case of the filtered-x LMS algorithm, where the nominal filter simply consists of a delay, which can have a negative effect on the tracking ability of the algorithm to the extent that the delay may counteract any improvement in adaptation, and even potentially cause instability. If this is the case, a solution is to use a smaller adaptive step-size coefficient . The delayed subband simulations are equivalent to error-free decision-directed performance.

5. SIMULATIONS

We simulate the system using SS and FS $T/2$ spaced SPIB and two different statistical Saleh-Valenzuela (SV) channels. The SPIB channels are measured channels from the Signal Processing Information Base [9] and have been sampled at twice the symbol rate of 30 MHz. The SV channels are generated using the parameters suggested in [10], i.e. a mean cluster arrival time $1/\ = 300$ ns, a mean ray arrival time $1/\ = 5$ ns, a cluster power decay time constant $= 60$ ns and the ray power decay time constant $= 20$ ns. One set of SV channels are sampled at 1 or 2 GHz, depending on whether SS or FS channels are required, respectively, and then sufficiently band-limited. In this case the response is truncated after 300 or 600 sampling periods of 1 or 2 ns, which captures all of the significant

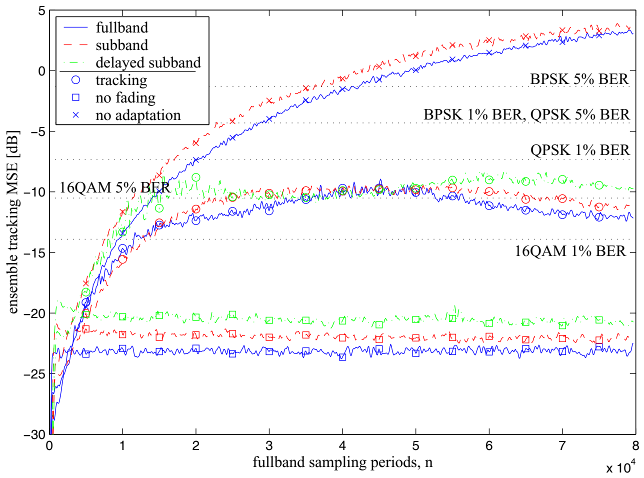

Figure 3: Channel-Equaliser Tracking MSE of a SS SPIB 2×2 MIMO channel.

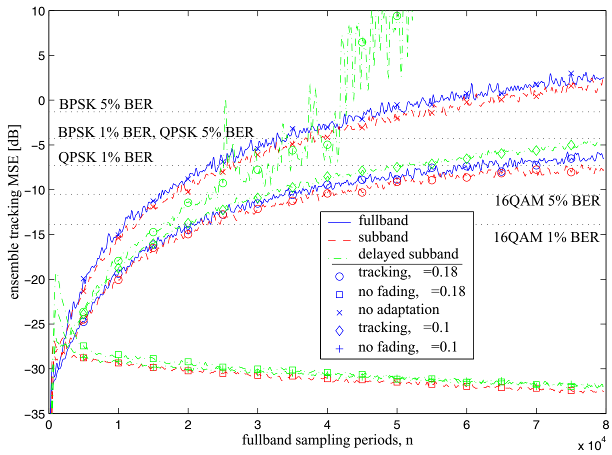

Figure 4: Channel-Equaliser Tracking MSE of a SS SV100M 2×2 MIMO channel.

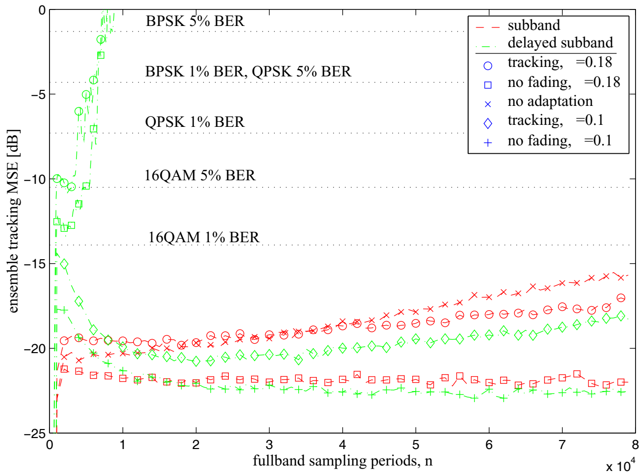

Figure 5: Channel-Equaliser Tracking MSE of a SS SV1G 2×2 MIMO channel.

part of the response. We refer to these channels as "SV1G". The second set is similarly sampled at 100 or 200 MHz, and truncated after 30 or 60 samples of period 10 or 20 ns. These channels are referred to as "SV100M". The $T/2$-spaced SV channels are band-limited using a raised-cosine filter with a roll-off factor of 0.1.

For each simulation four channels create the 2×2 MIMO system. All the channels are subject to Doppler-filtered Rayleigh fading corresponding to a mobile speed of 120 km/h. The equalisers are initialised to a near-MMSE solution, after which the channel begins to fade and the adaptive equaliser begin its tracking using the multi-channel NLMS algorithm with a step-size coefficient of 0.18, unless otherwise stated. The subband systems use $K = 16$, $N = 14$ and $L_p = 448$. Finally, we use $L_{gf} = 280$ and $L_{gs} = 20$ for the SPIB and SV100M simulations, and $L_{gs} = 70$ for the SV1G simulations.

Figure 3 shows the simulation for the SS SPIB channel. With no fading all three adaptation methods — fullband, subband and delayed subband — immediately jump to their steady-state MSE of between about -20 dB and -23 dB. The reason for the relatively high fullband MSE is that the MMSE inverse response is long relative to the equaliser lengths thus causing a large excess MSE. The subband MSE is also limited by factors such as analysis filter stop-band attenuation and analysis-synthesis distortion from the perfect reconstruction property [7]. The upper curves show the fading characteristic for the fullband and subband methods when there is no adaptation, in which case the delayed subband is identical to the subband adaptation. The difference between these arises because the calculated subband equaliser at the start is based on the subband channel, which was found by performing adaptive identification in subbands. Hence the subband equaliser at the start is an approximation to the near-MMSE inverse, whereas the fullband calculated equaliser is based on perfect channel knowledge. Since the SPIB channels are only mildly frequency-selective we find that there is no great difference between the fullband and subband tracking. The delayed subband method fairs slightly worse but still manages to achieve a satisfactory steady-state BER of around 5% using 16-QAM, or better than 1% using QPSK.

Figure 4 shows the simulation for the SS SV100M channel. Here we see that for the delayed subband simulations the step-size factor $= 0.18$ now causes instability. In fact, the MSE oscillates with a period of half the delay. By using $= 0.1$ however, the algorithm is made stable, although the performance is worsened over the delayless subband algorithm. The latter exhibits slightly better performance in terms of MSE than the fullband algorithm, due to its superior ability to converge to highly frequency-selective channels.

Figure 5 show the tracking performance for the SS SV1G channel. There is no fullband curve since, as we shall see later, the computational cost of performing this task is too great to be feasible. Even the subband algorithm has difficulty tracking this channel, managing only a slightly better MSE than no adaptation by the end of the simulation. This is due to the length of the equaliser, and the highly dynamic frequency-selective nature. It is known that for channels where the frequency attenuation varies highly dynamically across the band of interest the subband adaptive inversion will not be a great deal better than for the fullband case, although we still benefit from the associated computational savings. Due to the great cost of this simulation, the steady-state MSE however could not be found, but this is less of a problem for such a high bandwidth channel as there is far greater scope to insert additional training sequences, and hence re-identify and analytically re-invert the channel more often. The delayed subband algorithm is once again unstable with $= 0.18$ but by using $= 0.1$ the performance is even better than for the delayless subband performance with $= 0.18$. Clearly the step-size coefficient is especially important when tracking the inverse of fading channels using a delayed subband algorithm.

Figure 6 shows the performance for the FS SPIB channel. Here we see that the fullband FSE performs better than its SS counterpart, but for the subband simulation the performance is comparable. Evidently the relatively small range of the spectral dynamics of the SPIB channel means that the subband algorithm does not improve

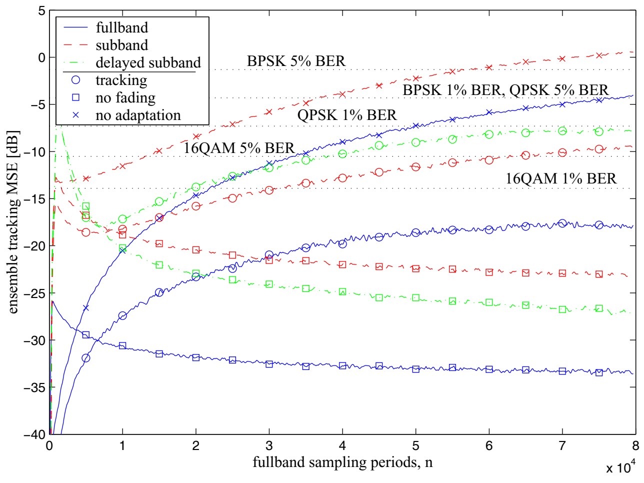

Figure 6: Channel-Equaliser Tracking MSE of a FS $T/2$ SPIB 2×2 MIMO channel.

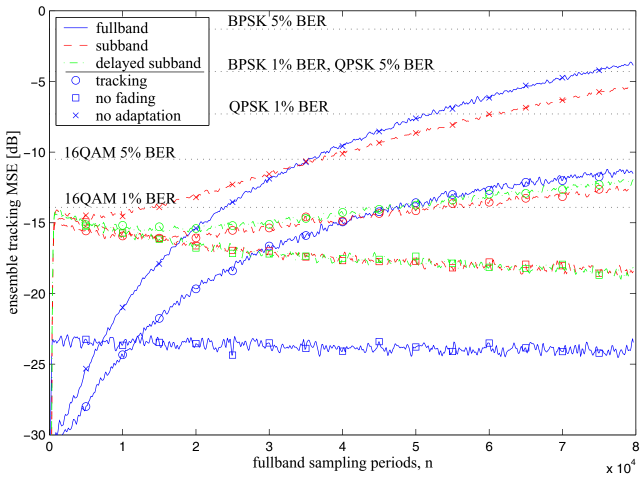

Figure 7: Channel-Equaliser Tracking MSE of a FS $T/2$ SV100M 2×2 MIMO channel.

convergence much. Further the fullband MMSE equaliser response is relatively long but the subband equaliser length $L_{gs} = L_{gf}/N$ is too short to model the system well, hence the performance is worsened still.

Figure 7 shows tracking performance for the FS SV100M channel. Both the subband and delayed subband algorithm appear to give a superior steady-state MSE to the fullband method for tracking, even though for the stationary channel the fullband achieve the far better minimum achievable MSE. This is again due to the subband algorithm's better performance for frequency-selective channels. Comparing these results to those of the SS SV100M channel we see that the fractional-spacing results in a much improved BER performance, where now we can achieve a BER of less than 5% using 16-QAM by the end of the simulation whereas for the SS the BER was worse than 1% for QPSK.

Finally, Figure 8 shows comparisons in the per-fullband-sampling-period cost of the fullband and subband algorithms for a range of total equaliser lengths, and using three different rules for the equivalent subband equaliser length. For all our simulations we have used $L_{gs} = L_{gf}/N$, however for short fullband equalisers this may be inadequate and a longer subband equaliser should be used. Assuming $L_{gs} = L_{gf}/N$ we see that the subband algorithm is computationally cheaper for $L_{gf} > 100$ and for very long equalisers the

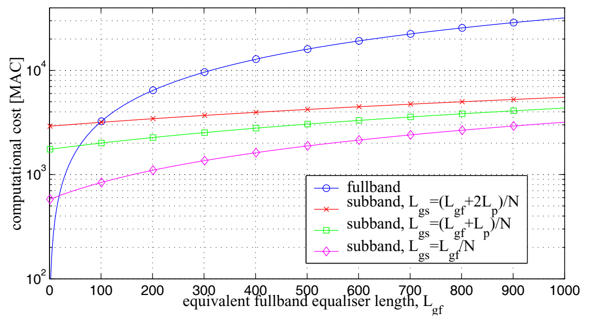

Figure 8: Computational cost per fullband sampling period of the fullband and subband adaptive NLMS algorithm, for varying equaliser filter lengths and update factors.

saving approaches factor N.

6. CONCLUSIONS

We have seen that for fading frequency-selective channels with a large spectral dynamic range and where a long equaliser length is required a subband adaptive equaliser can track the changes with superior performance to the fullband equivalent in both terms of steady-state MSE and computational cost. For fractionally-spaced equalisation the steady-state MSE is often better than for SS, and the computational savings offered by the subband approach are greater still.

REFERENCES

[1] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Englewood Cliffs, 3rd edition, 1996.

[2] N.J Bershad and O.M. Macchi, "Adaptive Recovery of a Chirped Sinusoid in Noise, Part 1: Performance of the RLS Algorithm," *IEEE Transactions on Signal Processing*, vol. 39, no. 3, pp. 583–594, March 1991.

[3] V. Bale and S. Weiss, "A Low-Complexity Subband Adaptive MIMO Equaliser for Highly Frequency-Selective Channels," in *Spectral Methods and Multi-rate Signal Processing*, Vienna, Austria, September 2004, pp. 61–68.

[4] V. Bale and S. Weiss, "Equalisation of Broadband MIMO Channels by Subband Adaptive Identification and Analytic Inversion," in *Proceedings of International ITG/IEEE Workshop on Smart Antennas*, Duisberg, Germany, April 2005, vol. CD.

[5] C.B. Papadias and D.T.M. Slock, "Fractionally Spaced Equalization of Linear Polyphase Channels and Related Blind Techniques based on Multichannel Linear Prediction," *IEEE Transactions on Signal Processing*, vol. 47, pp. 641–654, March 1999.

[6] S. Weiss and R. W. Stewart, *On Adaptive Filtering in Oversampled Subbands*, Shaker Verlag, Aachen, Germany, 1998.

[7] S. Weiss, A. Stenger, R.W. Stewart, and R. Rabenstein, "Steady-state performance limitations of Subband Adaptive Filters," *IEEE Transactions on Signal Processing*, vol. 49, no. 9, pp. 1982–1991, September 2001.

[8] M. Rupp and R. Frenzel, "Analysis of LMS and NLMS Algorithms with Delayed Co-efficient update under the Presence of Spherically Invariant Processes," *IEEE Transactions on Signal Processing*, vol. 42, pp. 668–672, March 1994.

[9] "Signal Processing Information Base: Micro-wave Channel Data," Rice University, http://spib.rice.edu.

[10] A. A. M. Saleh and R. A. Valenzuela, "A Statistical Model for Indoor Multipath Propagation," *IEEE Journal on Selected Areas of Communications*, vol. 5, no. 2, pp. 128–137, February 1987.