ROBUST SUBSPACE TECHNIQUE FOR JOINT ANGLE/DOPPLER ESTIMATION IN NON-GAUSSIAN CLUTTER

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ABSTRACT

We consider the problem of joint angle and doppler estimation for Space-Time Adaptive Processing (STAP) airborne radar in non-gaussian clutter which is modeled as a complex symmetric alpha stable SαS process. We introduce a sign covariance estimate which has almost robust performance in heavy tailed noise [1]. The subspace estimate is calculated via the propagator method [2] to reduce the computational load in the way that it does not require the eigendecomposition. Performance of the proposed technique is assessed through simulations and it is shown that the method reveals better performance than FLOM-MUSIC [3] and ROC-MUSIC[4].

1. INTRODUCTION

Airborne surveillance radars are faced with the difficult task of detection, identification and parameter estimation of weak moving targets in strong clutter and interferences environments. As a result the problem of clutter and jamming suppression has been the focus of considerable research in the radar engineering community. In the early 1970s Brennan and Reed [5] proposed what is known as Space-Time Adaptive Processing by exploiting the information in the spatial and temporel domain for interference suppression and target detection.

Most of the work in detection and estimation for radar application assumes that the clutter has a gaussian distribution. This is partly because of the desirable properties that the gaussian model processes, which leads to tractable solutions. Spikes due to clutter sources such as mountains, forest and sea waves (at low grazing angles), and glints due to reflections from large flat surfaces such as buildings and vehicles are usually present in radar returns. A statistical model of impulsive interference has been proposed recently as a good fit to radar returns [4, 6], which is based on the theory of symmetric alpha-stable (SαS) processes. In this paper we address the target parameter estimation problem through the use of STAP radar array sensor in the presence of impulsive interference of an uncorrelated nature. We introduce a new subspace algorithm for joint angle/doppler target estimation based on the sign covariance matrix (SCM) [1]. The proposed algorithm outperforms the classical Music algorithm and fractional lower order statistics Music algorithms in resolution capability [3, 4] with a low computational cost.

This paper is organized as follows. In section 2, we briefly review some preliminaries on α-stable distributions. In section 3, we formulate the STAP problem for airborne radar. In section 4, we present the FLOM-matrix and our proposed subspace algorithm. Finally, some simulation examples are presented in section 5, and concluding remarks are given in section 6.

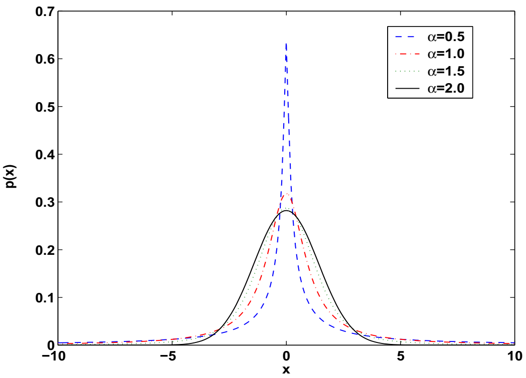

Fig. 1. Symmetric α-stable PDFs with $\mu = 0, \gamma = 1$, and different characteristic exponents

2. SαS DISTRIBUTIONS

Gaussian distribution have long been accepted as a useful tool for stochastic modelling. In this section, we introduce a statistical model based on the class of symmetric α stable (SαS)distributions which is well suited for describing signals that are impulsive in nature [7]. A complex random variable (r.v.) $X = X_1 + jX_2$ is isotropic SαS if X_1 and X_2 are jointly S αS and have a symmetric distribution. This class of distribution is best defined by its characteristic function

$$\varphi(\omega) = \exp\{j\mu\omega - \gamma|\omega|^{\alpha}\} \tag{1}$$

where α is the characteristic exponent restricted to the values $(0 < \alpha \leq 2)$, μ $(-\infty < \mu < \infty)$ is the location parameter and γ $(\gamma > 0)$ is the dispersion of the distribution. The dispersion parameter γ determines the spread of the distribution around its location parameter μ in the same way that the variance of a Gaussian distribution determines the spread around the mean [7]. The characteristic exponent determines the shape of the distribution see Figure.1. The smaller α, the heavier the tails of the alpha stable density. This implies that random variables following alpha-stable distributions with small characteristic exponents are highly impulsive. We should also note that for $\alpha = 2$ the distribution coincides with the Gaussian density. For α-stable processes only the moments of order $p < \alpha$ exist. So estimation methods based on second order statistics (SOS) of the data cannot be applied.

3. STAP PROBLEM FORMULATION

STAP is a two dimensional adaptive filtering algorithm that combines signals from multiple array elements and pulses to suppress interferences (clutter and jamming) and achieves both target detection and parameter estimation in airborne or space borne radar [8]. Consider a uniformly linear radar array consisting of N elements, which transmits a coherent burst of M pulses at a constant pulse repetition frequency (PRF) $f_r = 1/T_r$ over a set of range directions of interest. The array receives signals generated by q narrowband moving targets which are located at azimuth $\{\theta_k; k = 1,...,q\}$ with doppler frequencies $\{f_k; k = 1,...,q\}$. The array output can be expressed as [8]

$$\mathbf{x}(t) = \mathbf{V}(\Theta,\Omega)\mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

where

- $\mathbf{x}(t) = [x_1(t),...,x_{MN}]^T$ is the array output vector
- $\mathbf{s}(t) = [s_1(t),...,s_q]^T$ is the signal vector
- $\mathbf{V}(\Theta,\Omega) = [\mathbf{v}(\varpi_1,\nu_1),...,\mathbf{v}(\varpi_q,\nu_q)]$ is the space-time steering matrix.
  $\mathbf{v}(\varpi_k,\nu_k) = \mathbf{b}(\varpi) \otimes \mathbf{a}(\nu_k)$ is the space-time steering vector with
  - $\mathbf{a}(\nu_k) = [1\ \ e^{j2\pi\nu_k} \cdots\ \ e^{j2\pi(M-1)\nu_k}]^T$ is the temporel steering vector ($\nu_k = \frac{f_k}{f_r}$).
  - $\mathbf{b}(\varpi_k) = [1\ \ e^{j2\pi\varpi_k} \cdots\ \ e^{j2\pi(N-1)\varpi_k}]^T$ is the spatial steering vector ($\varpi_k = \frac{d}{\lambda} sin(\theta_k)$, d is the element separation distance and λ is the wavelength).
- $\mathbf{n}(t) = [n_1(t),...,n_{MN}(t)]^T$

The interference vector $\mathbf{n}$ is supposed to be due to clutter and thermal noise.

$$\mathbf{n} = \mathbf{n}_c + \mathbf{n}_w \tag{3}$$

The component $\mathbf{n}_w$ is due to the thermal noise and it is spatially and temporally white.

The Radar clutter returns for each range will be modelled as a superposition of a large N_c clutter sources that are evenly distributed in a circular ring about the radar platform. The location of the *ith* clutter patch is described by its azimuth θ_i and normalized doppler frequency ν_i, the clutter component of the space-time snapshot is given by

$$\mathbf{n_c} = \sum_{i=1}^{N_c} \gamma_i \mathbf{v}_i(\varpi_i,\nu_i) \tag{4}$$

where $\mathbf{v}_i(\varpi_i,\nu_i)$ and γ_i are the space-time steering vector and the random amplitude of the *ith* clutter patch respectively. Assuming the availability of K coherent processing intervals CPI's at $t_1,...,t_K$, the data can be expressed as

$$\mathbf{X} = \mathbf{V}(\Theta,\Omega)\mathbf{S} + \mathbf{N} \tag{5}$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $MN \times K$ matrices

$$\mathbf{X} = [\mathbf{x}(t_1),\cdots,\mathbf{x}(t_K)] \tag{6}$$

$$\mathbf{N} = [\mathbf{n}(t_1),\cdots,\mathbf{n}(t_K)] \tag{7}$$

and $\mathbf{S}$ is the $q \times K$ matrix

$$\mathbf{S} = [\mathbf{s}(t_1),\cdots,\mathbf{s}(t_K)] \tag{8}$$

Our aim is to jointly estimate the directions of arrivals $\{\theta_k;\ k = 1,...,q\}$ and the doppler frequencies $\{\nu_k; k = 1,...,q\}$ of the source targets using subspace techniques.

4. SUBSPACE TECHNIQUES IN SαS DISTRIBUTED CLUTTER

We assume that the signals are complex circular gaussian random variables that are statistically independent of each other. The noise vector $\mathbf{n}(t)$ is assumed Sα S with characteristic exponent α.

The subspace techniques exploit the geometric properties of the measurement signal and noise characteristics to estimate the targets parameters. One of the most popular among these is the MUSIC algorithm. The usual second order statistics Music algorithm utilises the sample covariance matrix $\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{x}(t_k)\mathbf{x}(t_k)^H$. When the noise is impulsive, SOS can not be applied. In this case preprocessing the data or introducing a new covariance estimate can alleviate the problem.

4.1. FLOM-MUSIC

As the performance of the standard Music algorithm degrades in the presence of impulsive noise due to the unboundness of the sample covariance matrix, new classes of matrices based on fractional lower order statistics have been introduced in literature [3, 4]. Among these classes are the Robust covariation [4] and Fractional lower order based (FLOM) matrices [3]. Tsung et *al.* [3] showed that the two classes of matrices yield almost the same performance when used with Music to estimate DOAs of circular signals. In the following we consider FLOM-based matrices [3]. The $(i,j)^{th}$ entry of the FLOM-based matrix $\hat{\mathbf{C}}^p$ [3] is defined by

$$\hat{C^p}_{ij} = \frac{1}{K}\sum_{k=1}^{K} x_i(t_k)|x_j(t_k)|^{p-2}x_j(t_k)^* \tag{9}$$

where the fractional moment p must satisfy the inequality $1 < p < \alpha \leq 2$ so that the $\hat{\mathbf{C}}^p$ is bounded. For Robust covariation matrices $\hat{\mathbf{\Gamma}}^p$ the (i,j)th entry [4] is defined by

$$\hat{\Gamma^p}_{ij} = \frac{\frac{1}{K}\sum_{k=1}^{K} x_i(t_k)|x_j(t_k)|^{p-2}x_j(t_k)^*}{\frac{1}{K}\sum_{k=1}^{K}|x_j(t_k)|^p}, \quad 1 < p \leq 2 \tag{10}$$

Denoting the eigenvectors of $\hat{\mathbf{C}}^p$ by $\{\mathbf{u}_i\}_{i=1}^{NM}$ then the 2D spectrum Music based on $\hat{\mathbf{C}}^p$ named 2D FLOM-Music can be expressed as

$$S_{FLOM-MUSIC}(\varpi,\nu) = \frac{1}{\mathbf{v}^H(\varpi,\nu)\mathbf{U_n}\mathbf{U_n}^H\mathbf{v}(\varpi,\nu)} \tag{11}$$

where $\mathbf{U}_n = [\mathbf{u}_{q+1},...,\mathbf{u}_{NM}]$ and $\mathbf{v}(\varpi,\nu)$ is the space-time search steering vector.

4.2. SCM-MUSIC

We here propose the use of the sample sign covariance matrix (SCM) shown to be a consistent estimate for ARMA process in [9] and used recently for DOAs estimate in [1] which is shown to be robust in heavy tailed noise and does not require any tuning parameter in contrast to FLOM-Music which needs the parameter (p) to be adjusted.

$$\hat{\mathbf{\Sigma}} = \frac{1}{K}\sum_{k=1}^{K} \rho(\mathbf{x}(t_k))\rho(\mathbf{x}(t_k))^H \tag{12}$$

where $\rho(.)$ is the sign function defined as

$$\rho(\mathbf{x}) = \begin{cases} \frac{\mathbf{x}}{\|\mathbf{x}\|} & \mathbf{x} \neq 0 \\ 0 & \mathbf{x} = 0 \end{cases} \tag{13}$$

and $\|\mathbf{x}\| = (\mathbf{x}^H\mathbf{x})^{1/2}$

The 2D SCM-Music spectrum can be obtained by applying the eigendecomposition to the calculated SCM ([1], theorem 5).

4.3. SCM-Propagator

Performing the eigendecomposition of the SCM is a computationally time consuming process $O(NM)^3$, for this reason we propose an extension of the propagator method [2, 10] proposed as a subspace approximate technique without eigendecomposition to angle doppler target estimation. This method is summarized in the following steps

- Calculate the propagator operator $\mathbf{P}$ by partitioning the SCM into two submatrices $\mathbf{G}$ ($NM \times L$) of full rank (a good choice is to take L=number of sources) and $\mathbf{H}$ ($NM \times (NM-L)$)

$$\mathbf{\Sigma} = [\ \mathbf{G} \quad \mathbf{H}\ ] \tag{14}$$

$$\mathbf{P} = \mathbf{G}^{\dagger}\mathbf{H}, \tag{15}$$

  where $(.)^{\dagger}$ denotes the Moore-Penrose pseudo inverse

- The noise subspace estimate is given by

$$\hat{\mathbf{U}}_{\mathbf{n}} = \begin{bmatrix} \mathbf{P} \\ \mathbf{I}_{NM-L} \end{bmatrix} \tag{16}$$

  where $\mathbf{I}_{NM-L}$ is the identity matrix of dimension $(NM-L, NM-L)$.

- Orthonormalisation of $\hat{\mathbf{U}}_{\mathbf{n}}$

The computational complexity for each step is described below:

Operartion	Computational cost
Calculation Of $\mathbf{P}$	$O(NML^2) + O((NM)^2L) + O(L^3)$
Orthonormalisation	$O(NM(NM-L)^2)$

5. SIMULATION RESULTS

In all simulations, the radar array is linear with five elements spaced by half wavelength N=5. The number of transmitted pulses is M=10. The amplitude of the targets is modelled by complex gaussian random variables. The FLOM parameter p=1.1[1]. We define the generalized signal to noise ratio (GSNR) [4]

$$GSNR = 10\log(E\{|s(t)|^2\}/\gamma \tag{17}$$

for FLOM-Music and ROC-Music Figure 2 shows the isosurface of the 2D spectra of ROC-Music, FLOM-Music and the proposed method in impulsive environment wit $(\alpha = 1.2)$[2]. We note that Music can not resolve the two closely moving targets.The SCM-Propagator reveals better resolution capability compared to FLOM-Music and ROC-Music [4].

In Figures 3 and 4, we evaluated the estimation accuracy of ROC-Music, FLOM-Music and the proposed algorithm in function of the number of snapshots and the GSNR(effect of the dispersion parameter γ). In every experiment we perform 100 Monte-Carlo runs to compute the mean square error (MSE) of the parameter estimates. As it is expected the performance of ROC-Music and FLOM-Music [3] are approximately the same. The complete failure of Music is apparent in both figures. On the other hand, we observe that the number of snapshots does not affect the overall performance of ROC-Music and FLOM-Music (the curves are flat). In both figures the SCM-Propagator shows better performance than FLOM-Music and ROC-Music.

[1]It is shown in [3, 4] that using p close to 1 gives better performance

[2]For purpose of comparaison, values of $\alpha < 1$ are not considered because of the unboundness of the FLOM-based covariance matrix and using higher values close to 2 (gaussian case) the proposed method as well as FLOM based covariance matrices estimators will converge to the conventional sample covariance matrix estimator

6. CONCLUSION

In this paper, we have considered the problem of joint target angle doppler estimation in SαS noise using subspaces techniques. We proposed the use of a sample sign covariance matrix along with the propagator method for joint angle/doppler estimation in symmetric alpha-stable (SαS) additive noise. The proposed method shows high resolution capability and lower estimation error compared to FLOM-Music and ROC-MUSIC with a low computational load. The proposed method can also be applied even for $\alpha < 1$ and does not require any parameter adjustment; in contrast to the methods mentioned earlier applicable only for $\alpha \geq 1$ and which require the fractional moment p to be adjusted. To reduce further the computational complexity, the SCM estimator can be combined with other subspace tracking algorithms.

7. REFERENCES

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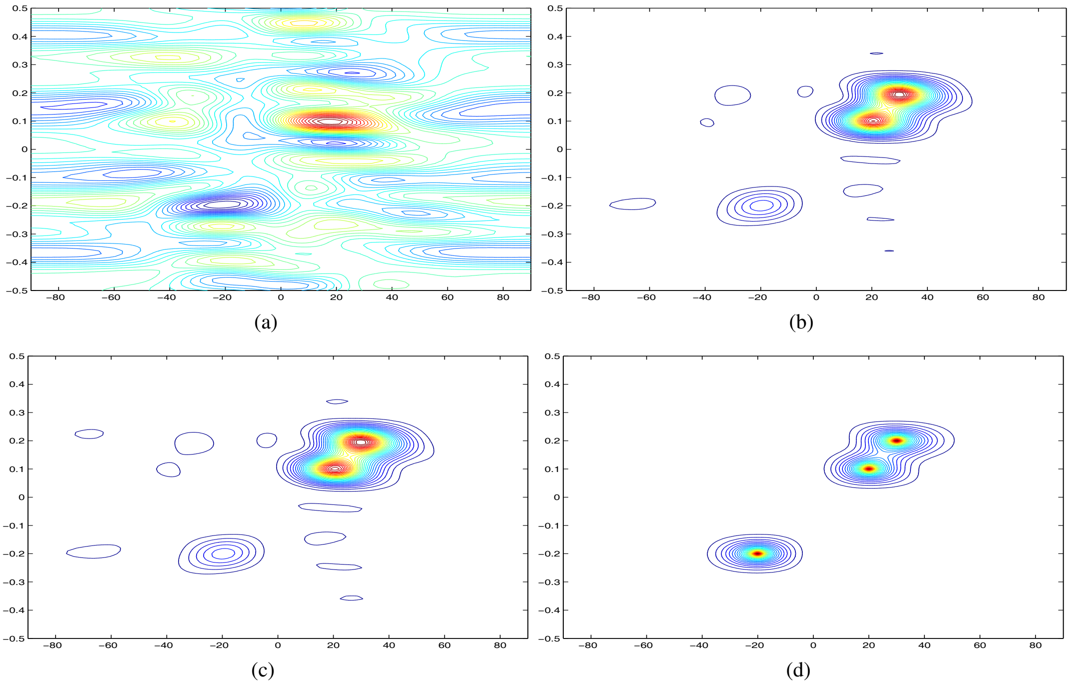

Fig. 2. 2D Angle, doppler spectra for(a)Music (b) FLOM-Music (c) ROC-Music (d) SCM-Propagator (N=5,M=10,2 moving target at azimuth angles $[20°, 30°, -20°]$ with normalized dopplers 0.1, 0.2 and -0.2 additive noise (α=1.2), GSNR=10dB, $p = 1.1$

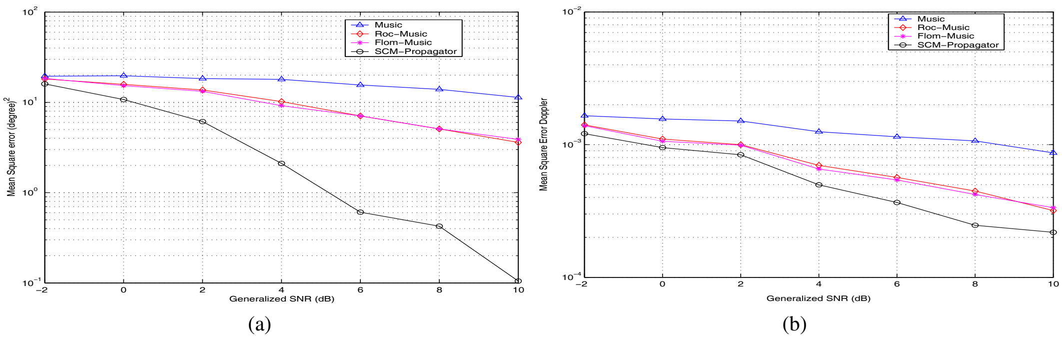

Fig. 3. MSE of estimated (a)Angle (b) Doppler as function of the GSNR N=5, M=10, 2 moving target at azimuth angles $[20°, 30°]$ with normalized dopplers 0.1, 0.2, additive noise (α=1.2), $K = 50$, $p = 1.1$

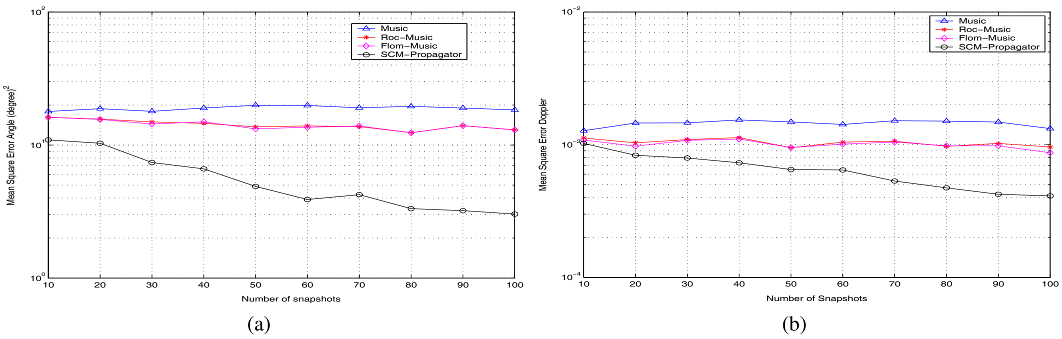

Fig. 4. MSE of estimated (a)Angle (b) Doppler as function of number of snapshots K N=5, M=10, 2 moving target at azimuth angles $[20°, 30°]$ with normalized dopplers 0.1, 0.2, additive noise (α=1.2), GSNR=2dB, $p = 1.1$