MULTIPATH CHANNEL ESTIMATION VIA THE MPM ALGORITHM

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ABSTRACT

In this paper, we consider the problem of multipath channel estimation from data observed at the receiver matched filter output. An approach based on the MPM algorithm is proposed in the literature assuming white noise at the output of the matched filter. We present an improvement to this method that takes into account the noise correlation involved by the matched filtering. Results obtained under both the white-noise and the correlated noise hypotheses are compared, showing that the latter approach performs significantly better in terms of false-alarm and paths detection probabilities, as well as in terms of mean quadratic error on the estimation of amplitudes and delays. Simulations examples are proposed.

1. INTRODUCTION

In many applications involving transmission and reception of a signal, the propagation medium between the transmitter and the receiver corresponds to a multipath channel whose impulse response $h(t)$ is

$$h(t) = \sum_{n=1}^{P} \ _n \exp(j \ _n) \ (t- \ _n), \tag{1}$$

where P is the number of paths and $\ _n$, $\ _n$ and $\ _n$ are respectively the attenuation, the phase and the time delay of path n. Estimation of the attenuations and time delays is often required, either for the first strongest paths as in communications where the transmitted digital information is searched for, or for all the paths, even the weakest ones, as in oceanic acoustic tomography (OAT) where estimation of all the paths is important to optimally recover physical parameters of the ocean.

We assume in this paper that the transmitted signal is known by the receiver, which happens with learning sequences in communications or with the whole signal for medium identification oriented applications such as active OAT. Maximum likelihood approaches [4] or MUSIC-like subspace techniques [9, 5] have been proposed for estimating the channel parameters. Unfortunately, they require prior estimation of the number of paths P that can be provided by methods such as AIC [1] or MDL criteria [11]. On another hand, iteratively substracting channel paths [3] may lead to errors in the presence of very close channel paths. To overcome such problems, penalized methods have been proposed [13]. In fact, it can be checked that the penalty term can be interpreted as some bayesian prior upon paths amplitudes.

An alternative bayesian channel estimation technique is considered in this paper, based on a Monte Carlo Markov Chain method, namely the MPM (Maximization of the *a Posteriori* Marginals) algorithm. This algorithm, first introduced in image processing to solve segmentation problems [2], has also been used in the seismic domain [8], and more recently in tomography [10] under a white noise assumption at the receiver matched filter output. In order to achieve improved channel estimation, we derive here a version of the MPM algorithm where the noise correlation is taken into account. For the sake of simplicity and without loss of generality, we only consider here noise correlation induced by matched filtering.

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This paper is organised as follows: Section 2 describes the transmitted and received signals. The MPM algorithm in the presence of correlated noise is derived in Section 3, and simulations results are given in Section 4 with comparison with the Cramer-Rao Lower Bounds (CRLB) for time-delay estimation and amplitude estimation. A conclusion is given in Section 5.

2. TRANSMITTED AND RECEIVED SIGNALS

Throughout this article, we consider transmitted signals that can be written in the form

$$e(t) = s(t)\cos(2 \ f_c t),$$

where $s(t)$ is the signal of interest and f_c is the frequency of the carrier.

From (1), the received signal $r(t)$ can be expressed as

$$r(t) = \sum_{n=1}^{P} \ _n s(t- \ _n)\cos(2 \ f_c(t- \ _n)+ \ _n)+n_0(t),$$

where $n_0(t)$ denotes an additive white gaussian noise with variance ^{2}. $r(t)$ is demodulated by means of two orthogonal carriers, and a matched filter with impulse response $s(-t)$ is then applied. The resulting signal can finally be written as

$$x(t) = \ (t)\star h(t)+n(t), \tag{2}$$

where (t) represents the autocorrelation of $s(t)$, and $n(t)$ is a complex circular gaussian noise. Due to the matched filtering, the autocorrelation function of $n(t)$ is $_n(t) = \ ^2 \ (t)$. After sampling, we obtain the equivalent matrix equation:

$$\mathbf{x} = \mathbf{S} \ \mathbf{h}+\mathbf{n}, \tag{3}$$

where $\mathbf{S}$ is the convolution matrix associated with (t).

3. DECONVOLUTION VIA THE MPM ALGORITHM

The deconvolution problem consists in recovering the impulse response $\mathbf{h}$ from the received signal $\mathbf{x}$. Solving this problem via the standard least square methods wouldn't take into account the sparseness of $\mathbf{h}$, that is, that there are few non-zero entries in $\mathbf{h}$, but would rather provide a time continuous impulse response.

3.1 A priori model

To account for the channel sparseness, we introduce an *a priori* model for $\mathbf{h}$. An underlying state vector $\mathbf{q} = (q_k)_{k=1,L}$ (L is the number of samples) of independent Bernoulli random variables q_k taking value 0 or 1 corresponding respectively to the absence or the presence of a path is introduced. The entries h_k of $\mathbf{h}$ are then modeled by a mixture of two gaussians:

$$\begin{aligned} h_k \sim & \ \ \left[\mathcal{N}(0, \ _1^2)+j\mathcal{N}(0, \ _1^2)\right] \\ & +(1- \ )\left[\mathcal{N}(0, \ _0^2)+j\mathcal{N}(0, \ _0^2)\right], \end{aligned}$$

where $_1 >> \ _0$. In this equation, $\ = P(q_k = 1)$ represents the probability that a path is present at sample k. Therefore the probability density function of h_k conditionally to q_k is

$$p(h_k|q_k = i) \sim \mathcal{N}(0, \ ^2_i) + j\mathcal{N}(0, \ ^2_i), \qquad i = 0, 1.$$

We denote by $\mathbf{z} = (\mathbf{q}, \mathbf{h})$ the so-completed unknown data.

Using Bayes formula and the fact that samples q_k are independent and samples h_k are independent conditionally to q_k [12], it can be shown that the posterior log-likelihood of $\mathbf{z}$ is given by

$$\begin{aligned} L(\mathbf{z}|\mathbf{x}) = \ & -(\mathbf{x} - S\ \mathbf{h})^H \mathbf{A}_n (\mathbf{x} - S\ \mathbf{h}) - \frac{\mathbf{h}^H \mathbf{D_q} \mathbf{h}}{2\ ^2_1} \\ & - \frac{\mathbf{h}^H (1 - \mathbf{D_q}) \mathbf{h}}{2\ ^2_0} + \mathbf{q}^H \mathbf{q} \ln \left(\frac{}{1 - } \frac{_0}{_1} \right), \end{aligned} \tag{4}$$

where the constant terms have been omitted, $\mathbf{D_q}$ represents the diagonal matrix whose k-th entry is q_k and $\mathbf{A}_n = \mathbf{\Gamma}_n^{-1}$ is the inverse of the autocorrelation matrix of $\mathbf{n}$.

3.2 The MPM algorithm

Estimating directly $\mathbf{q}$ and $\mathbf{h}$ from the log-likelihood (4) is a difficult problem since its complexity increases exponentially with L. It is therefore necessary to implement simulation methods, such as the MPM algorithm. This algorithm aims at simulating the pdf of $\mathbf{z}$ conditional to $\mathbf{x}$. Since this probability is not computable directly, the algorithm uses a Gibbs sampler to simulate realisations of samples z_k following the *a posteriori* marginals $p(z_k|\mathbf{x}, \mathbf{z}_{-k})$, where $\mathbf{z}_{-k} = (z_0, \ldots, z_{k-1}, z_{k+1}, \ldots, z_L)$.

The Gibbs sampler is implemented in the following way:

1. Initialization: $\mathbf{q} = \mathbf{q}^{(0)}$ and $\mathbf{h} = \mathbf{h}^{(0)}$.
2. For $i \leq I$ and for k randomly covering $\{1, \ldots, L\}$,
   - drawing of a uniform random variable $v \sim \mathcal{U}_{[0,1]}$
   - Detection step: $q_k^{(i)} = \mathbb{I}_{[v,1]}(d_k)$
   - Estimation step:
     $h_k^{(i)} \sim \mathcal{N}(m_{q_k^{(i)}}, V_{q_k^{(i)}}) + j\mathcal{N}(m_{q_k^{(i)}}, V_{q_k^{(i)}})$,

where $\mathbb{I}_A(t)$ is the index function of A ($\mathbb{I}_A(t) = 1$ if $t \in A$ and 0 otherwise) and $d_k = p(q_k = 1|\mathbf{x}, \mathbf{z}_{-k}^{(i)})$ is the *a posteriori* detection probability (see Appendix A):

$$d_k = \left[1 + \frac{1 - }{} \frac{V_0\ ^2_1}{V_1\ ^2_0} \exp\left(\frac{|m_0|^2}{2V_0} - \frac{|m_1|^2}{2V_1} \right) \right]^{-1}.$$

We can remark that the expression of the MPM algorithm for correlated noise little differs from the white noise case [12]: the noise correlation only appears in the computation of conditional means and variances m_0, m_1, V_0 and V_1. The simulated samples $(\mathbf{z}^{(i)})_{i=I_0,I}$ (for $i < I_0$, the samples are removed to account for the learning period) are used to compute Monte Carlo estimators $\mathbf{q}^e$ and $\mathbf{h}^e$ of $\mathbf{q}$ and $\mathbf{h}$ as:

$$\begin{aligned} q_k^e &= 1 \quad \text{if} \quad \mathbb{E}[q_k] = \tfrac{1}{I - I_0} \textstyle\sum_{i=I_0+1}^{I} q_k^{(i)} > s, \\ h_k^e &= \begin{cases} \dfrac{1}{\mathbb{E}[q_k]} \displaystyle\sum_{i=I_0+1}^{I} q_k^{(i)} h_k^{(i)} & \text{if} \quad q_k^e = 1 \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

Remark: In the simulations, s has been chosen equal to 0.5. Besides performance little depend on the choice of $(\ , \ ^2_1, \ ^2_0)$ that can anyway be estimated precisely via an SEM algorithm [12]).

4. RESULTS

For the simulations, $s(t)$ was chosen to be a Pseudo Noise (PN) sequence of length $N = 63$ binary symbols with the following circular autocorrelation shape: it takes value $1 - (1 + N)|t|/(NT_b)$ when $t \in [-T_b, T_b]$ and value $-1/N$ otherwise [6], where T_b is the binary symbol duration such that $T_b = 8T_s$ and T_s is the sampling rate. In order to benefit from this circular autocorrelation property, the sequence is transmitted several times (10 times here) and then averaged after matched filtering. Figure 1 presents the results obtained for a multipath channel with 20 paths and for an average SNR by paths, defined as $\overline{\text{SNR}} = (P\ ^2_n)^{-1} \ _{k=1,P}\ ^2_k$, equal to -15 dB.

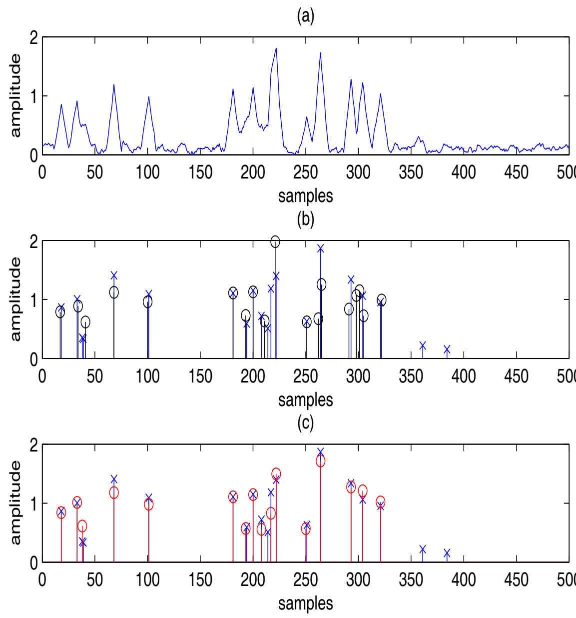

Figure 1: Channel estimation via MPM algorithm: (a) output of the matched filter, (b) real channel ($\times$) and estimated channel with white noise assumption ($\circ$), (c) real channel ($\times$) and estimated channel with correlated noise assumption ($\circ$).

Let us remark that for a given path, under the white noise assumption several neighbouring estimated peaks may be found [12]. This phenomenon disappears under the correlated noise assumption.

For performance computation, we consider three different channels: a single path channel, a multipath channel and a channel with two close paths. For each case, J simulations are run. Besides the average SNR by path is considered for keeping coherence among the results obtained for channels with different numbers of paths.

4.1 Single path channel

For a single path channel, at simulation j ($1 \leq j \leq J$), the estimated channel may contain more than one path. Therefore we must define a criterion for detection of the true path. Let's call K the number of estimated paths, $\ _k$ the k-th estimated path time delay and $\ _r$ the true path time delay. Then we consider that the true path is detected when the estimated path whose time delay verifies

$$_e = \arg\min_k (|\ _k - \ _r|)$$

is closer to the real path than the duration T_b of one symbol: $|\ _e - \ _r| \leq T_b$. Otherwise it is considered as a false alarm, as well as any additional estimated path. From this definition, we can compute the detection probability P_D and the false alarm probability P_{FA}

$$P_D = \frac{1}{J} \sum_{j=1}^{J} N_d^j, P_{FA} = \frac{1}{J} \sum_{j=1}^{J} N_f^j,$$

where $N_d^j = 1$ if the single path has been detected at simulation j and 0 otherwise, and $N_f^j = 1$ if there was at least one false alarm at simulation j and 0 otherwise. We can also compute the time delay mean square error $e^2 = \mathbb{E}[| \ _e - \ _r|^2]$ and the corresponding amplitude mean square error $e^2 = \mathbb{E}[| \ _e - \ _r|^2]$. Results obtained under the white and correlated noise assumptions are presented in Table 1. They show that the algorithm performs significantly better for any of the four criteria considered when the noise correlation is taken into account.

4.2 Multipath channel

A channel with 10 paths is simulated. In order to compare the true channel with the estimated one, the matched filter output is reconstructed for a noisefree signal and using both the true impulse response $\mathbf{h}$ and the estimated impulse response $\hat{\mathbf{h}}$. Then we compute the mean square error between both filter outputs: $\mathbb{E}[\|\mathbf{S} \ \mathbf{h} - \mathbf{S} \ \hat{\mathbf{h}}\|^2]$. Results presented in Figure 2 show that the case when the noise correlation is taken into account outperforms the white noise case.

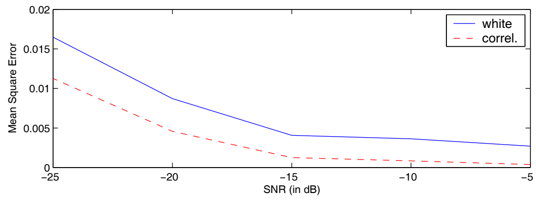

Figure 2: Matched filter output MSE for a 10 paths channel under white noise and correlated noise assumptions.

4.3 Channel with two close paths

We now consider a channel with two paths delayed by $T_b/2$. These paths are generally not directly distinguishable at the output of the matched filter. Let's call again K the number of estimated paths, $\ _k$ the k-th estimated path time delay and $\ _{r,j}$ the true j-th path time delay (j =1, 2). Each true path j is now considered to be detected when the estimated path whose time delay $\ _{e,j}$ verifies

$$\ _{e,j} = \arg\min_k \left(| \ _k - \ _{r,j}|\right)$$

is closer to the corresponding real path than *a quarter* of T_b: $| \ _{e,j} - \ _{r,j}| \leq T_b/4$. We define also a criterion for *exact* detection: when $| \ _{e,j} - \ _{r,j}| < T_s$, the j-th path is said to be perfectly detected. Results are presented in Table 1. Again the algorithm is shown to perform better when the noise correlation is taken into account, and especially to be more precise for the time delay estimation.

4.4 Cramer-Rao Lower Bounds

The Cramer-Rao Lower Bound (CRLB) for the variance of the estimate $\hat{\ }$ of a vector parameter is given by the inverse of the Fisher information matrix $\mathbf{J}$ with entry (i, j) expressed as

$$\mathbf{J}_{ij} = -\mathbb{E}\left[\frac{\ ^2 \log p(\mathbf{x}| \ )}{\ _i \ _j}\right], \tag{5}$$

where $\mathbf{x}$ is the data vector [7].

For amplitude estimation of a single path channel, the received data can be written $\mathbf{x} = \ \ + \mathbf{n}$ where is the amplitude and is the autocorrelation of signal $s(t)$. From equation (5), it comes that

$$\mathrm{CRLB}(\ ) = \frac{1}{2 \ ^H \mathbf{A_n} \ }.$$

For time delay estimation of a single path channel, since has a triangular shape spreading over $M = \lceil 2T_b/T_s \rceil$ samples (we assume it takes value zero outside that interval), it comes from equation (11) (see Appendix B) that the CRLB for the time delay estimate is

$$\mathrm{CRLB}(\ ) = \frac{N^2 T_b^2}{8(N+1)^2 \ \sum_{p=1}^{L} \frac{1}{L} d_{pp} \frac{\sin^4\left(2 \ \frac{p}{2L} \frac{M}{2}\right)}{\sin^2\left(2 \ \frac{p}{2L}\right)}},$$

where the coefficients d_{pp} are the eigenvalues of $\mathbf{A_n}$ when it is diagonalized in the Fourier basis (see Appendix B).

Figure 3 shows that variances of the estimate of and are significantly closer to CRLB( ) when the noise correlation is taken into account.

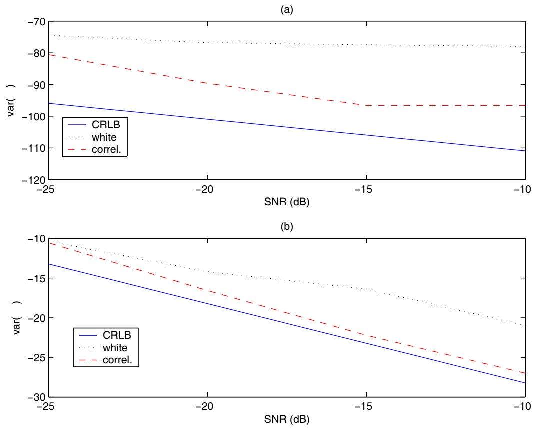

Figure 3: CRLB and estimate variances for: (a) time delay estimation (b) amplitude estimation

5. CONCLUSION

In this paper we have proposed a multipath channel identification technique based on the MPM algorithm. Previously used under the assumption of white noise, we have extended it to take into account the noise correlation at the matched filter output. Results obtained show that performance are significantly improved when the noise correlation is taken into account, especially for the time-delay estimates, which explains the better results obtained for detection and false alarm probabilities, and for the overall channel estimate.

A. APPENDIX A

We derive here the *a posteriori* detection probability $d_k = p(q_k = 1|\mathbf{x}, \mathbf{z}_{-k})$.

On one hand, the *a posteriori* pdf $p(z_k|\mathbf{x}, \mathbf{z}_{-k}, \ )$ can be written as a gaussian mixture:

$$p(z_k|\mathbf{x}, \mathbf{z}_{-k}) = \begin{array}{l}\left[\frac{d_k}{2 \ V_1}\exp\left(-\frac{|h_k - m_1|^2}{2V_1}\right)\right]^{q_k} \times \\ \left[\frac{1-d_k}{2 \ V_0}\exp\left(-\frac{|h_k - m_0|^2}{2V_0}\right)\right]^{1-q_k}.\end{array} \tag{6}$$

Therefore:

$$\frac{p(z_k = (0, h_k)|\mathbf{x}, \mathbf{z}_{-k})}{p(z_k = (1, h_k)|\mathbf{x}, \mathbf{z}_{-k})} = \frac{1-d_k}{d_k}\frac{V_1}{V_0}\exp\left(\frac{|h_k - m_1|^2}{2V_1} - \frac{|h_k - m_0|^2}{2V_0}\right). \tag{7}$$

On the other hand, applying the Bayes rule $p(z_k|\mathbf{x}, \mathbf{z}_{-k}) \sim p(\mathbf{x}|\mathbf{z})p(h_k|q_k)p(q_k)$ and according to the Bernoulli-gaussian

		$\overline{\text{SNR}} = -10$ dB		$\overline{\text{SNR}} = -15$ dB		$\overline{\text{SNR}} = -20$ dB		$\overline{\text{SNR}} = -25$ dB	
		white	correl.	white	correl.	white	correl.	white	correl.
single path	P_D	1.000	1.000	0.995	1.000	0.980	0.995	0.937	0.961
	P_{FA}	0.038	0.000	0.115	0.000	0.078	0.005	0.405	0.434
	e^2	$1.6\cdot 10^{-8}$	0.000	$1.8\cdot 10^{-8}$	$2.2\cdot 10^{-10}$	$2.1\cdot 10^{-8}$	$1.1\cdot 10^{-9}$	$3.6\cdot 10^{-8}$	$8.8\cdot 10^{-9}$
	e^2	0.008	0.002	0.023	0.006	0.038	0.022	0.092	0.088
Close paths	P_D	0.911	1.000	0.890	1.000	0.890	0.995	0.720	0.835
	$P_{D,e}$	0.246	1.000	0.233	0.995	0.245	0.873	0.165	0.618

Table 1: Results

model presented in section 3.1, $p(z_k|\mathbf{x},\mathbf{z}_{-k})$ can also be written:

$$p(z_k|\mathbf{x},\mathbf{z}_{-k},\ )\sim \left[\frac{}{2\ \ _1^2}\exp\left(-\frac{|h_k|^2}{2\ _1^2}-\mathbf{X}\right)\right]^{q_k}\times \left[\frac{1-}{2\ \ _0^2}\exp\left(-\frac{|h_k|^2}{2\ _0^2}-\mathbf{X}\right)\right]^{1-q_k}, \qquad (8)$$

where $\mathbf{X}=(\mathbf{x}-S\ \ \mathbf{h})^H\mathbf{A}_n(\mathbf{x}-S\ \ \mathbf{h})$. Then:

$$\frac{p(z_k=(0,h_k)|\mathbf{x},\mathbf{z}_{-k})}{p(z_k=(1,h_k)|\mathbf{x},\mathbf{z}_{-k})}=\frac{1-}{}\frac{\ _1^2}{\ _0^2}\exp\left(\frac{|h_k|^2}{2\ _1^2}-\frac{|h_k|^2}{2\ _0^2}\right). \qquad (9)$$

Besides $\mathbf{X}$ can be written

$$\mathbf{X}=\ \sum_{n=0}^{M}\sum_{p=0}^{M}\left(U_{k,n}-s_nh_k\right)^*a_{n+k,p+k}\left(U_{k,p}-s_ph_k\right)+f(\mathbf{z}_{-k}),$$

where $\mathbf{A}_n=(a_{ij})$, s_p representing the p-th entry among the M coefficients of the autocorrelation function of $s(t)$ and

$$U_{k,n}=x_{n+k}-\sum_{\substack{i=0\\ i\neq n}}^{M}s_ih_{n+k-i}.$$

Then, comparing equations (6) and (8) provides

$$m_i=2V_i\sum_{n=0}^{M}\sum_{p=0}^{M}s_p^*a_{n+k,p+k}U_{k,n},$$
$$V_i=\left(\frac{1}{\ _i^2}+2\sum_{n=0}^{M}\sum_{p=0}^{M}s_n^*a_{n+k,p+k}s_p\right)^{-1},$$

while comparing the constant terms in (7) and (9) gives:

$$d_k=\left[1+\frac{1-}{}\frac{V_0}{V_1}\frac{\ _1^2}{\ _0^2}\exp\left(\frac{|m_0|^2}{2V_0}-\frac{|m_1|^2}{2V_1}\right)\right]^{-1}.$$

B. APPENDIX B

We derive the general CRLB for time delay estimation of a single path channel. For a scalar parameter , when the elements of $\mathbf{J}$ are complex circular gaussian with mean $(\ )$ and variance $\mathbf{\Gamma}(\ )$, the Fisher information can be written [7]:

$$\mathbf{J}=2\left[\frac{(\ )}{}\right]^H\mathbf{\Gamma}^{-1}(\ )\left[\frac{(\ )}{}\right]+\mathrm{tr}\left[\left(\mathbf{\Gamma}^{-1}(\ )\frac{\mathbf{\Gamma}(\ )}{}\right)^2\right]. \qquad (10)$$

Here $\mathbf{\Gamma}(\ )/\ \ =0$, and thus the second term in (10) vanishes. Furthermore we assume that the path contribution to $\mathbf{x}$ spreads over an interval denoted by $[n_0,n_0+M-1]$. Therefore (10) yields

$$\begin{aligned}\mathbf{J}\ \ &=2\sum_{k=n_0}^{n_0+M-1}\sum_{l=n_0}^{n_0+M-1}[\mathbf{\Gamma}^{-1}]_{kl}\frac{(kT_e-\ _0)}{_0}\frac{(lT_e-\ _0)}{_0}\\ &=2\sum_{k=0}^{M-1}\sum_{l=0}^{M-1}[\mathbf{\Gamma}^{-1}]_{k+n_0,l+n_0}\left(\frac{d\ (t)}{dt}\right)_{t=kT_e}\left(\frac{d\ (t)}{dt}\right)_{t=lT_e}.\end{aligned}$$

$\mathbf{\Gamma}$ is a toeplitz matrix and even a circulant matrix if we use a PN sequence with periodic circular autocorrelation. Therefore, using Whittle's approximation for the Toeplitz case [14] and without any approximation in the latter case, it comes that the EVD of $\mathbf{\Gamma}^{-1}$ is $\mathbf{\Gamma}^{-1}=\mathbf{W}\mathbf{D}\mathbf{W}^H$, where $\mathbf{W}$ is the Fourier transform matrix, that is $\mathbf{W}=(w_{ik})=(e^{2j\ \frac{ik}{L}}/\sqrt{L})$ (L is the matrix size), and $\mathbf{D}=(d_{ik})$ is the eigenvalues diagonal matrix.

This leads to

$$\mathbf{J}=\sum_{p=1}^{L}d_{pp}\left|\sum_{k=0}^{M-1}\frac{1}{L}\left(\frac{d\ (t)}{dt}\right)_{t=kT_e}e^{2j\ \frac{kp}{L}}\right|^2. \qquad (11)$$

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