ORTHOGONALIZATION OF QUASI-ORTHOGONAL SPACE-TIME BLOCK CODES IN MULTIPATH FADING ENVIRONMENTS BY USING FEEDBACK

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ABSTRACT

To combat the detrimental effect of the frequency-selective fading channel and the intersymbol interference in quasi-orthogonal space-time block codes (QO-STBCs), two feedback algorithms combined with multicarrier modulation scheme are proposed. One of the algorithms rotates the phase of the transmitted signal from certain antennas in a prescribed way. The other algorithm is based on selecting certain antennas and switching-off the others according a channel quality measure. A highly practical setup with only two bits of feedback can achieve considerable performance gain over the open-loop QO-STBC.

1. INTRODUCTION

Space-Time Block Codes (STBCs) are simple, yet effective tools to obtain transmit diversity in a MIMO communication channel. However, as shown by Tarokh, et al [1], STBCs which attain full code rate and full diversity do not exist for more than two transmit antennas for complex valued constellations. Orthogonal-STBCs (O-STBCs), [1], [2] and [3], designed for more than two antennas can achieve full diversity order but they have a code rate less than unity. Similarly, quasi-orthogonal STBCs (QO-STBCs), [4] and [5], for four transmit antennas were proposed to achieve full code rate, but they suffer from a loss in the diversity order due to coupling between the symbols in a codeword.

In the literature, several closed-loop methods are proposed to eliminate this coupling. In [6], [7], [8] and [9], the phase of the signals transmitted from certain transmit antennas are rotated in a prescribed way so that the coupling is made zero. Another method to avoid the coupling is to switch-off certain antennas and load the others, [6] and [7].

These algorithms are designed for frequency-flat fading channels. However, many practical communication channels experience frequency-selective fading. In [6], a code division multiple access (CDMA) based architecture is proposed to challenge this type of fading. Multicarrier modulation (MCM) is another method to mitigate multipath channels. By the aid of the FFT-IFFT operators, and after adding a cyclic prefix to the data packet, the channel can be approximated with narrower orthogonal frequency bins, each fading flat.

In this paper, an MCM based closed-loop method is proposed to achieve full transmit diversity with QO-STBCs using the above mentioned algorithms. It will be demonstrated that only two bits for feedback per frequency bin is adequate for a substantial gain in performance, making the algorithms highly suitable for practical applications.

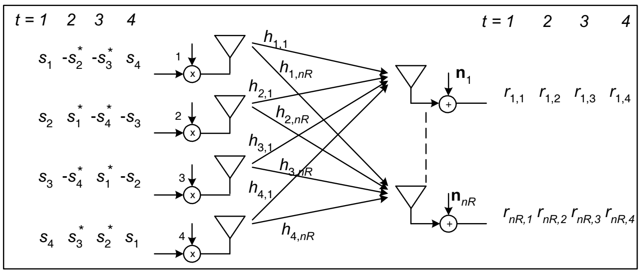

Figure 1: The structure of the transmitter preprocessing method for the orthogonalization of QO-STBC in a frequency-flat fading channel scenario with four transmit and n_R receive antennas.

The paper is organized as follows. In Section 2, an introduction of the QO-STBCs is provided together with the investigation of the intersymbol interference in QO-STBC. Also the transmit preprocessing algorithms for orthogonalization are derived. Section 3 extends these methods to multipath fading channels with the aid of MCM. Simulations and their results are depicted in Section 4, followed by conclusions in Section 5.

Some of the notation used throughout the paper is as follows. A scalar, a vector and a matrix will respectively be denoted by a lower-case italic, a lower-case boldface and an upper-case boldface letter. The entry on the i-th row and j-th column of a matrix is represented by $[\mathbf{X}]_{i,j}$,. $\text{Re}\{\cdot\}$ represents the real operator, $(\cdot)^*$ denotes the complex conjugate of the operand, $(\cdot)^T$ and $(\cdot)^H$ are respectively used for matrix transpose and Hermitian transpose.

2. ORTHOGONALIZATION OF QO-STBC

In this paper we will focus on QO-STBCs for four transmit antennas. A QO-STBC for three transmit antennas can be designed by simply deleting a column of the codematrix. Throughout this section the channel is assumed to be flat-fading, the frequency selective fading channel will be investigated in the next section.

There exist several QO-STBC codematrices, [4] and [5], but all of them can be shown to have identical properties. We will examine the following codematrix due to its structural similarity to the well known Alamouti's code [3]

$$\mathbf{C}_{QO} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix}. \quad (1)$$

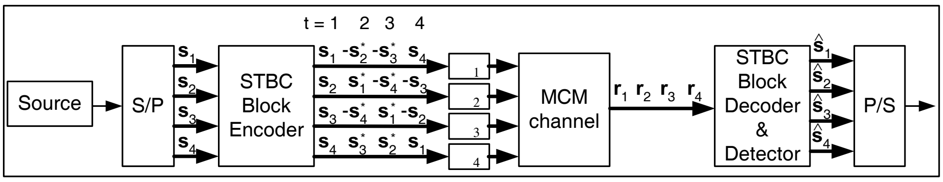

Figure 2: The structure of the transmitter processing scheme in an MCM environment. The diagonal matrices $\mathbf{\Theta_i}$ are used to orthogonalize the QO-STBC.

where s_i, $i = 1,...,4$, are the data symbols to be transmitted over a code period, i.e. four time slots. The complex valued element $[C_{QO}]_{l,i}$ corresponds to the symbol transmitted from the i-th transmit antennas at the l-th time slot.

Consider a MIMO communication channel scenario with four transmit antennas and n_R receive antennas as given in Figure 1.

All antennas are supposed to be positioned far apart so that they experience independent fading. The subchannel from the i-th transmit antenna to the j-th receive antenna is modelled with a zero-mean circularly symmetric complex valued Gaussian random variable, $h_{i,j}$. The channel coefficients are also assumed to be static over a code period. Prior to transmission, the signals from each antenna are multiplied by a certain variable, $\ _i$, $i = 1,...,4$, which will be clear shortly. The received signal at the j-th receive antenna can be expressed as

$$r'_{j,l} = \ _{i=1}^{4} [C_{QO}]_{l,i} \ _i h_{i,j} + n'_{j,l}$$

where the noise samples $n_{j,l}$ are mutually independent zero-mean circularly symmetric white Gaussian random variables with variance $\ _n^2$. Stacking the signal received from the j-th receive antenna over a code period into a vector and conjugating the second and third rows results in the following matrix expression

$$\begin{bmatrix} r'_{j,1} \\ r'^{*}_{j,2} \\ r'^{*}_{j,3} \\ r'_{j,4} \end{bmatrix} = \begin{bmatrix} \ _1 h_{1,j} & \ _2 h_{2,j} & \ _3 h_{3,j} & \ _4 h_{4,j} \\ \ _2^* h_{2,j}^* & - \ _1^* h_{1,j}^* & \ _4^* h_{4,j}^* & - \ _3^* h_{3,j}^* \\ \ _3^* h_{3,j}^* & \ _4^* h_{4,j}^* & - \ _1^* h_{1,j}^* & - \ _2^* h_{2,j}^* \\ \ _4 h_{4,j} & - \ _3 h_{3,j} & - \ _2 h_{2,j} & \ _1 h_{1,j} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n'_{j,1} \\ n'^{*}_{j,2} \\ n'^{*}_{j,3} \\ n'_{j,4} \end{bmatrix}$$

$$\mathbf{r}_j = \mathbf{H}_j \mathbf{s} + \mathbf{n}_j. \quad (2)$$

Then the vector for each receive antenna is stacked into a single vector yielding

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

where $\mathbf{r} = \begin{bmatrix} \mathbf{r}_1^T & \mathbf{r}_2^T & \cdots & \mathbf{r}_{n_R}^T \end{bmatrix}^T$, $\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \cdots & \mathbf{H}_{n_R}^T \end{bmatrix}^T$ and $\mathbf{n} = \begin{bmatrix} \mathbf{n}_1^T & \mathbf{n}_2^T & \cdots & \mathbf{n}_{n_R}^T \end{bmatrix}^T$. The matched filtering operation is performed by multiplying the received signal vector $\mathbf{r}$ by the matrix $\mathbf{H}^H$ [5], hence

$$\mathbf{r}_{mf} = \mathbf{H}^H \mathbf{r} = \mathbf{H}^H \mathbf{H}\mathbf{s} + \mathbf{H}^H \mathbf{n}.$$

If we investigate the matrix $\mathbf{H}^H\mathbf{H}$ in detail, we observe that

$$\mathbf{H}^H\mathbf{H} = \begin{bmatrix} & 0 & 0 & \\ 0 & & - & 0 \\ 0 & - & & 0 \\ & 0 & 0 & \end{bmatrix}$$

where $\ = \ _{j=1}^{n_R} \ _{i=1}^{4} |h_{i,j}|^2 | \ _i|^2$ and $\ = 2 \ _{j=1}^{n_R} \mathrm{Re}\{h_{1,j} h_{4,j}^* \ _1 \ _4^* - h_{2,j} h_{3,j}^* \ _2 \ _3^*\}$. Ideally $\ $ should be equal to 0 as in an O-STBC, so that there is no coupling between the data symbols estimated directly from the output of the matched filter. However, for QO-STBC $\ \neq 0$, yielding coupling in estimation and also a colored noise with autocorrelation matrix $\ _n^2 \mathbf{H}^H\mathbf{H}$. This causes loss in diversity order and the complexity of the maximum likelihood (ML) receiver is no longer linear in the number of transmit antennas in contrast to O-STBCs.

As given in [4], the optimum receiver for open-loop QO-STBC in the ML sense is

$$M_{1,4} = \ _{j=1}^{n_R} \Big(\ _{i=1}^{4} |h_{i,j}|^2 (|\hat{s}_1|^2 + |\hat{s}_4|^2)$$

$$-2\,\mathrm{Re}\{\mathbf{h}_{1,j}^H \mathbf{r}_j \hat{s}_1^* + \mathbf{h}_{4,j}^H \mathbf{r}_j \hat{s}_4^* - \mathbf{h}_{1,j}^H \mathbf{h}_{4,j} \hat{s}_1^* \hat{s}_4\}\Big), \quad (3)$$

$$M_{2,3} = \ _{j=1}^{n_R} \Big(\ _{i=1}^{4} |h_{i,j}|^2 (|\hat{s}_2|^2 + |\hat{s}_3|^2)$$

$$-2\,\mathrm{Re}\{\mathbf{h}_{2,j}^H \mathbf{r}_j \hat{s}_2^* + \mathbf{h}_{3,j}^H \mathbf{r}_j \hat{s}_3^* - \mathbf{h}_{2,j}^H \mathbf{h}_{3,j} \hat{s}_2^* \hat{s}_3\}\Big) \quad (4)$$

where $\hat{s}_i$ are the estimates chosen from the modulation constellation so as to minimize the metrics M_{14} and M_{23}. The vector $\mathbf{h}_{i,j}$ is the i-th column of the matrix $\mathbf{H}_j$. Note the coupling between the elements of the pairs (s_1, s_4) and (s_2, s_3). If the code was O-STBC, this coupling would not exist, and the metrics would only depend on the corresponding s_i resulting in a simpler receiver structure.

2.1 Signal Phase Rotation

One of the methods to orthogonalize the QO-STBC is to rotate the phase of the signals transmitted from certain antennas. Let $\ _1 = \ _2 = 1$, and $\ _3 = \ _4 = e^{j \ }$, i.e. the phase of the signals transmitted from the third and fourth antennas are rotated with the same phasor while the other two are kept unchanged. The coupling term $\ $ can be written as

$$\ = 2\,\mathrm{Re}\{ \ _{j=1}^{n_R} [h_{1,j} h_{4,j}^* - h_{2,j} h_{3,j}^*] e^{-j \ } \ \}.$$

Realizing $\mathrm{Re}\{\cdot\} = 0$ for a purely imaginary operand, in order to orthogonalize the QO-STBC it suffices to set the phase value $\ $ to

$$\ = m \ + (\measuredangle \ - \frac{\ }{2}), \ \forall m$$

where $\ = \ _{j=1}^{n_R} [h_{1,j} h_{4,j}^* - h_{2,j} h_{3,j}^*]$ and $\measuredangle\cdot$ is the angle operator. Note that the effective range of $\ $ can be limited to the range $[-pi/2, pi/2]$, without loss of generality.

Clearly, this method requires the feedback of the actual phase value $\ \in [-pi/2, pi/2]$ which needs a high precision if, for example, a floating or fixed point feedback is employed. In a practical application this may not be possible due to the very limited feedback bandwidth. Instead, the phase information can be quantized into 2^K levels i.e. $\hat{\ } \in \ = \{\pm((2k-1) \ )/2^{K+1}, k = 1,...,2^{K-1}\}$ according to

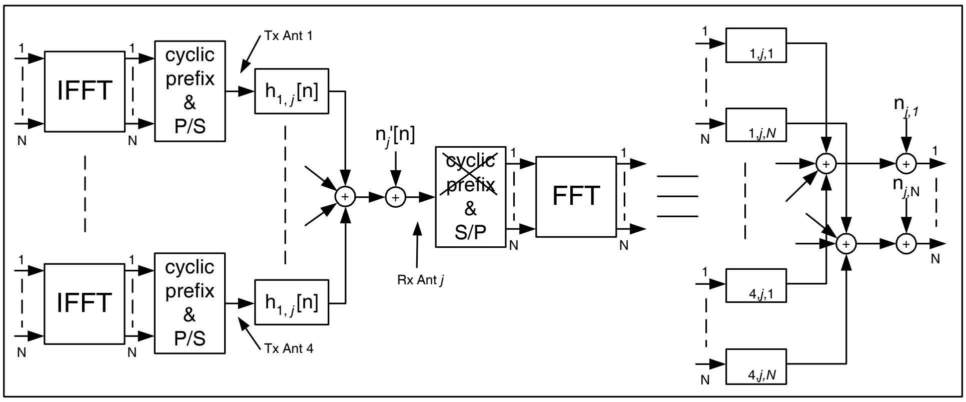

Figure 3: (left) Detail of the 'MCM Channel' block shown in Figure 2 for the j-th receive antenna. (right) The equivalent diagonalized MCM channel. Each subchannel experiences independent frequency-flat fading.

$$\hat{\ } = \arg\min_{\hat{\ } \in} \mathrm{Re}\{\ e^{-j\ \hat{\ }}\},$$

then these levels are fed back to the transmitter over K-bits.

2.2 Antenna Selection

Another method for orthogonalization is to select two out of four antennas according to the following rule related to the quality of the subchannels. If $\sum_{j=1}^{n_R} |h_{1,j}|^2 > \sum_{j=1}^{n_R} |h_{4,j}|^2$ then set $_1 = \sqrt{2}$ and $_4 = 0$, otherwise set $_1 = 0$ and $_4 = \sqrt{2}$. Similarly, if $\sum_{j=1}^{n_R} |h_{2,j}|^2 > \sum_{j=1}^{n_R} |h_{3,j}|^2$ then set $_2 = \sqrt{2}$ and $_3 = 0$, otherwise set $_2 = 0$ and $_3 = \sqrt{2}$. If these values for $_i$ are substituted into it can be seen that the coupling term is equal to zero. Hence the interference is eliminated with a two bit feedback.

3. ORTHOGONALIZATION IN MULTIPATH CHANNELS

The orthogonalization methods given in the previous section work in frequency-flat fading channels. However, most of the practical communication channels experience frequency-selective fading. One of the efficient ways to handle frequency-selective fading channels is to use a MCM scheme. By the aid of the FFT-IFFT operator pair, the transmission frequency range is partitioned into smaller frequency bins. If certain conditions are satisfied, these bins can be considered as experiencing frequency-flat fading. Hence, the algorithms provided in the previous section can be applied to each frequency bin separately.

Consider the system model depicted in Figure 2. The data symbols are first divided into four streams. Then data packets $\mathbf{s}_i$, $i = 1, \ldots, 4$, of length N are composed from these streams, i.e., $\mathbf{s}_i = [s_{i,1} \quad s_{i,2} \quad \cdots \quad s_{i,N}]^T$. The data packets are space-time coded in a similar way as (1)

$$\mathbf{C}_{QO} = \begin{bmatrix} \mathbf{s}_1 & \mathbf{s}_2 & \mathbf{s}_3 & \mathbf{s}_4 \\ -\mathbf{s}_2^* & \mathbf{s}_1^* & -\mathbf{s}_4^* & \mathbf{s}_3^* \\ -\mathbf{s}_3^* & -\mathbf{s}_4^* & \mathbf{s}_1^* & \mathbf{s}_2^* \\ \mathbf{s}_4 & -\mathbf{s}_3 & -\mathbf{s}_2 & \mathbf{s}_1 \end{bmatrix}$$

where now $\mathbf{C}_{QO}$ is a $4N \times 4$ matrix. Following this, all N frequency bins corresponding to the i-th transmit antenna are preprocessed with the diagonal matrix $\mathbf{\Theta}_i = \mathrm{diag}\{\ _{i,1},\ _{i,2}, \cdots,\ _{i,N}\}$, $i = 1, \ldots 4$.

The 'MCM Channel' block in Figure 2, is depicted with more detail in Figure 3. For simplicity only the j-th receive antenna is illustrated. The subchannel from the i-th transmit antenna to the j-th receive antenna is modeled with a linear tapped-delay line, i.e. $h_{i,j}[n] = \sum_{k=0}^{n_H-1} h_{i,j}^k\ [n-k]$ where $[n]$ is the Kronecker delta function and $h_{i,j}^k$ is the channel coefficient of the k-th path of the corresponding subchannel which can be represented with a zero-mean circularly symmetric Gaussian random variable. Each preprocessed data packet is processed by an IFFT operator of dimensions $N \times N$ and a cyclic prefix of length L is added to the packet, that is, the last L samples of the IFFT output is copied to the beginning of the packet. With the assumption $L \geq n_H - 1$ the channel become cyclic. It is well known that the unitary matrices of the eigendecomposition of a cyclic matrix are the FFT and IFFT operators, hence the channel is 'diagonalized' with the FFT and IFFT operators. After removal of the cyclic prefix at the receiver, the equivalent MCM channel is illustrated on the right hand side of Figure 2. The equivalent subchannel coefficients $_{i,j,n}$ $i = 1, \ldots, 4$, $j = 1, \ldots, n_R$ and $n = 1, \ldots, N$, are mutually independent zero-mean circularly symmetric complex Gaussian random variables, representing the frequency-flat fading subchannels.

Assuming that the channel is static over $4N$ symbol periods, the $4N \times 1$ received signal vector for the j-th receive antenna for this scenario can be written similar to (2) as

$$\begin{bmatrix} \mathbf{r}'_{j,1} \\ \mathbf{r}'^*_{j,2} \\ \mathbf{r}'^*_{j,3} \\ \mathbf{r}'_{j,4} \end{bmatrix} = \begin{bmatrix} \mathbf{\Theta}_1\mathbf{\Lambda}_{1,j} & \mathbf{\Theta}_2\mathbf{\Lambda}_{2,j} & \mathbf{\Theta}_3\mathbf{\Lambda}_{3,j} & \mathbf{\Theta}_4\mathbf{\Lambda}_{4,j} \\ \mathbf{\Theta}_2^*\mathbf{\Lambda}_{2,j}^* & -\mathbf{\Theta}_1^*\mathbf{\Lambda}_{1,j}^* & \mathbf{\Theta}_4^*\mathbf{\Lambda}_{4,j}^* & -\mathbf{\Theta}_3^*\mathbf{\Lambda}_{3,j}^* \\ \mathbf{\Theta}_3^*\mathbf{\Lambda}_{3,j}^* & \mathbf{\Theta}_4^*\mathbf{\Lambda}_{4,j}^* & -\mathbf{\Theta}_1^*\mathbf{\Lambda}_{1,j}^* & -\mathbf{\Theta}_2^*\mathbf{\Lambda}_{2,j}^* \\ \mathbf{\Theta}_4\mathbf{\Lambda}_{4,j} & -\mathbf{\Theta}_3\mathbf{\Lambda}_{3,j} & -\mathbf{\Theta}_2\mathbf{\Lambda}_{2,j} & \mathbf{\Theta}_1\mathbf{\Lambda}_{1,j} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \\ \mathbf{s}_4 \end{bmatrix} + \begin{bmatrix} \mathbf{n}'_{j,1} \\ \mathbf{n}'^*_{j,2} \\ \mathbf{n}'^*_{j,3} \\ \mathbf{n}'_{j,4} \end{bmatrix}$$

$$\mathbf{r}_j = \mathbf{\Lambda_j}\mathbf{s} + \mathbf{n}_j$$

where the matrix $_{i,j} = \mathrm{diag}\{\ _{i,j,1},\ _{i,j,2}, \cdots,\ _{i,j,N}\}$. After stacking $\mathbf{r}_j$ into $\mathbf{r} = [\ \mathbf{r}_1^T \quad \mathbf{r}_2^T \quad \cdots \quad \mathbf{r}_{n_R}^T\ ]^T$ and matched filtering we obtain

$$\mathbf{\Lambda}^H\mathbf{\Lambda} = \begin{bmatrix} \mathbf{\Gamma} & \mathbf{0} & \mathbf{0} & \mathbf{A} \\ \mathbf{0} & \mathbf{\Gamma} & -\mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{\Gamma} & \mathbf{0} \\ \mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{\Gamma} \end{bmatrix}$$

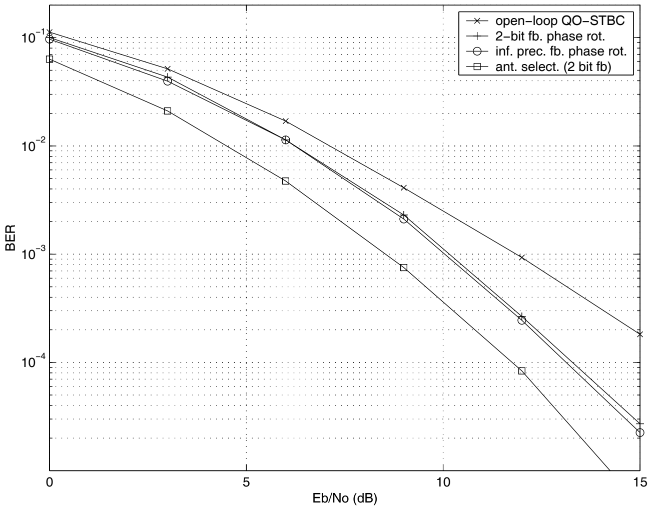

Figure 4: The BER performance comparison of the proposed transmit preprocessing schemes to the open-loop QO-STBC. A two bit feedback is considered for a practical scenario.

where $\mathbf{\Gamma} = \sum_{i=1}^{4} \sum_{j=1}^{n_R} \mathbf{\Theta}_i^* \mathbf{\Lambda}_{i,j}^* \mathbf{\Lambda}_{i,j} \mathbf{\Theta}_i$ and $\mathbf{A} = 2\sum_{j=1}^{n_R} \mathrm{Re}\{\mathbf{\Lambda}_{1,j}\mathbf{\Lambda}_{4,j}^*\mathbf{\Theta}_1\mathbf{\Theta}_4^* - \mathbf{\Lambda}_{2,j}\mathbf{\Lambda}_{3,j}^*\mathbf{\Theta}_2\mathbf{\Theta}_3^*\}$ are $N \times N$ diagonal matrices, or equivalently, $[\mathbf{\Gamma}]_{n,n} = \sum_{i=1}^{4} \sum_{j=1}^{n_R} |\lambda_{i,j,n}|^2 |\theta_{i,n}|^2$ and $[\mathbf{A}]_{n,n} = 2\sum_{j=1}^{n_R} \mathrm{Re}\{\lambda_{1,j,n}\lambda_{4,j,n}^*\theta_{1,n}\theta_{4,n}^* - \lambda_{2,j,n}\lambda_{3,j,n}^*\theta_{2,n}\theta_{3,n}^*\}$. Clearly, this representation for each subchannel is equivalent to the discussion in Section 2. Hence, the same QO-STBC orthogonalization algorithms can directly be applied to each frequency bin.

4. SIMULATIONS AND RESULTS

The simulation results are provided in Figure 4 in terms of the bit error rate (BER) performance of the proposed methods. QPSK is used as the modulation scheme. The performance of the proposed algorithms are compared to that of the open loop QO-STBC. For comparison reasons, the infinite precision performance of the phase rotation algorithm is also provided, however in order to simulate a practical application, a two bit feedback is considered for both the phase rotation and antenna selection algorithms. Note the negligible difference in performance between the infinite precision and two bit feedback phase rotation schemes.

However, there is a considerable difference between the receiver structures of the schemes. While for the antenna selection and infinite precision feedback phase rotation algorithms the matched filter output can directly be used for estimation of the transmitted symbols, for the two-bit feedback scheme the ML receiver provided in (3) and (4) has to be used. This is because, while the former algorithms can perfectly orthogonalize the QO-STBC, in the later algorithm some residual interference remains.

For the study of multipath fading channels, a 10-tap linear channel is used where the equal power taps are represented with zero-mean circularly-symmetric Gaussian random variables. The resolution of the FFT-IFFT pair is 64.

The curves for both frequency-flat fading and multipath fading scenarios overlap for all cases, therefore it can be concluded the MCM scheme is successful in combating the detrimental effect of multipath fading.

Remark: From the results, the antenna selection algorithm appears to be superior to the phase rotation algorithm. However, when a erroneous feedback channel is considered, especially with a medium-high rate of error ($P_e > 0.1$), this algorithm can perform worse than the open-loop QO-STBC. On the other hand, the worst case for the phase rotation algorithm achieves a performance equivalent to the open-loop QO-STBC. Hence, special care in selecting the algorithms has to be taken in a practical application. For details please see [6].

5. CONCLUSIONS

Multicarrier modulation based transmitter preprocessing algorithms with transmit antenna selection and transmitted signal phase rotation schemes are designed to combat the detrimental effects of both the frequency-selective fading channel and the coupling between the symbols in QO-STBC. It is demonstrated that the MCM scheme diagonalizes the channel yielding independent frequency-flat fading subchannels. Hence, the feedback algorithms for frequency-flat fading channels can directly be applied to the multipath fading case. A highly practical scenario with only two bit feedback per frequency bin is investigated in the simulations, and considerable gain over the open-loop QO-STBC is observed. While the open-loop QO-STBC can achieve a diversity order of two, the proposed methods can increase this order to four.

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