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SEPARATION OF INSTANTANEOUS MIXTURES OF CYCLOSTATIONARY SOURCES WITH APPLICATION TO DIGITAL COMMUNICATION SIGNALS

Pierre Jallon and Antoine Chevreuil

Equipe signal et communications
Institut Gaspard Monge - UMR 8049
Université de Marne la Vallée
5, boulevard Descartes
77454 Marne La Vallée Cedex 2

ABSTRACT

In this contribution, we provide a simple condition on the statistics of the source signals ensuring that the Comon algorithm [2], originally designed for stationary data, achieves the separation of an instantaneous mixture of cyclostationary sources. The above condition is analyzed for digital communications signals and is (semi-analytically) proved to be fulfilled.

1. INTRODUCTION

The Blind Source Separation problems (BSS) have given rise to a wide literature due to a large set of pratical scenarios involving multi-sensors and multi-sources. A classic model stands as :

$$\mathbf{y}(n) = \mathbf{H}\mathbf{s}(n) \tag{1}$$

where $\mathbf{y}(n)$ is the multivariate observed data on an array on M sensors, $\mathbf{s}(n) = [s_1(n), \cdots, s_K(n)]^T$ is the unobserved source signal and $\mathbf{H}$ is the unknown mixing matrix. In the following, we assume that $\mathbf{H}$ is a full rank $M \times K$ matrix (thus implying that the number of sensors is bigger than the number of sources). A key assumption of BSS is to assume that the components of $\mathbf{s}(n)$ are are mutually independent and one of the sources, at most, is Gaussian. BSS hence shifts to Independent Component Analysis (ICA): setting $\mathbf{r}(n) = \mathbf{G}\mathbf{y}(n)$ for any $K \times M$ matrix $\mathbf{G}$, the components of $\mathbf{r}(n)$ are mutually independent if and only if (see [2])

$$\mathbf{G}\mathbf{H} = \mathbf{P}\ \mathbf{D} \tag{2}$$

where $\mathbf{P}$ is a permutation matrix and $\mathbf{D}$ is a full rank diagonal matrix. The pioneering works assume that the source signals are stationary sequences. Several block methods have been proposed: for example, the algorithm SOBI [4] is based on the second-order statistics of $\mathbf{y}(n)$ and assumes that the source signals have pair-wise different auto-correlation functions. Cardoso [1] and Comon [2] proposed fourth-order cumulant based methods, leading to very efficient algorithms such as JADE. D.T. Pham [9, 10] proposed a method based on the mutual information. In [12, 11], a sequential approach is investigated (the so-called *deflation*): in this method, the inverse matrix $\mathbf{G}$ is computed row by row. Due to the well-known problem of cascading errors, we do not mention this kind of approach and we rather focus, in the sequel, on block methods.

Several works address the BSS of non-stationary data. The standard SOBI algorithm naturally extends to cyclostationary contexts and in [5], an improvement of SOBI consists in taking into account the cyclic statistics: it implicitly requires the knowledge of the cyclic frequencies which may not fit certain contexts; besides, the cyclic statistics may be numerically inconsistent (this is in particular the case for digital communication sources [13]). In [6], a second-order based method is presented which achieves the separation of mixture of general non-stationary data (not necessarily cyclostationary). This method however requires that 1) the source signal statistics do not vary too fast in regard of time 2) the variations of the statistics are numerically significant. Again, this is not the case for digital communication signals. More interestingly, [7] investigates the behavior of the JADE algorithm when the data are cyclostationary; a sufficient condition is given which ensures that JADE achieves the separation: when the source signals do not share the same non-null cyclic frequencies, this condition is automatically fulfilled; in the general case, however, the condition provided by the authors is not explicit (it depends on the eigenvalues of a certain Hermitian operator).

In this contribution, we focus on the Comon algorithm [2] when the data are cyclostationary (ECG signals, digital communication signals, etc...). In Section 2, we provide a simple condition under which the Comon algorithm, as in a stationary environment, performs BSS. In Section 3, we specialize our analysis to digital communication sources and prove, semi-analytically, that the above condition is satisfied, making the Comon achieve BSS. Simulation results (Section 4) corroborate the above theoretical points.

2. THE COMON ALGORITHM IN A CYCLOSTATIONARY CONTEXT

For a given source index k, $(s_k(n))_{n\in\mathbb{Z}}$ is by assumption a cyclostationary sequence; in particular, its auto-correlation sequence evolves almost periodically as a function of time: $\mathbb{E}\{s_k(n+m)s_k(n)^*\} = \sum_{\alpha\in I_k} \mathbf{R}_{s_k}^{(\alpha)}(m)e^{2i\pi\alpha n}$ where I_k is the set of (second-order) cyclic frequencies of s_k (hence containing 0). Due to the mutual independence of the sources, we deduce that

$$\mathbb{E}\{\mathbf{y}(n+m)\mathbf{y}^\dagger(n)\} = \sum_{\alpha\in I} \mathbf{R}_{\mathbf{y}}^{(\alpha)}(m)e^{2i\pi\alpha n} \tag{3}$$

where $\mathbf{y}^\dagger(m) = [y_1^*(n), \cdots, y_K^*(n)]$ and $I = \cup_k I_k$. Moreover,

$$\mathbf{R}_{\mathbf{y}}^{(\alpha)}(m) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}\{\mathbf{y}(n+m)\mathbf{y}^\dagger(n)\}e^{-2i\pi\alpha n} \tag{4}$$

is the cyclic correlation coefficient of $\mathbf{y}(n)$ at the cyclic frequency α and at lag m. The first step of the Comon algorithm consists in whitening the data, i.e. left-multiplying $\mathbf{y}(n)$ by a certain $K \times M$ matrix such that the resulting time-series $\mathbf{x}(n)$ verifies $\mathbf{R}_{\mathbf{x}}^{(0)}(0) = \mathbf{I}_K$ (if $K = M$, $\mathbf{x}(n) = R_y^{(0)}(0)^{-1/2}\mathbf{y}(n)$ goes). In other words, we may consider instead of (1) that the model of the data is rather

$$\mathbf{x}(n) = \mathbf{U}\mathbf{s}(n) \quad (5)$$

where $\mathbf{U}$ is a unitary matrix and the components of $\mathbf{s}(n)$ all have normalized average power, i.e. $\mathbf{R}_{s_k}^{(0)}(0) = 1$ for all indices k.

2.1 First approach

The BSS problem is to find the unitary matrix $\mathbf{U}$ or its inverse $\mathbf{U}^\dagger$ up to indeterminacies. In [2], the function $\sum_i |c_4(r_i(n))|^2$, where $\mathbf{r}(n) = \mathbf{V}\mathbf{x}(n)$, and $c_4(r_i(n))$ is the fourth-order autocumulant of the random variable $r_i(n)$, is introduced and its maximization over the unitary matrices $\mathbf{V}$ is discussed. In contrast, the cyclostationary assumption on the data makes the quantities $c_4(r_i(n))$ depend on the time-lag n, and its maximization at every lag n would involve a time-varying system. Now, we want a method that achieves the "inversion" of equation (5) by means of a constant matrix.

This requirement implies that the dependency of the statistics of the data on the time index has to be removed. For instance, we consider in this section

$$\Psi(\mathbf{r}) = \sum_{i=1}^{K} |\langle c_4(r_i(n))\rangle|^2. \quad (6)$$

This function coincides with the Comon objective one when the sources are stationary up to the fourth-order. The source signals being mutually independent, and the cumulant multilinear, (6) writes

$$\Psi(\mathbf{r}) = \sum_{i=1}^{K} \left| \sum_{k=1}^{K} |f_{i,k}|^4 b_k \right|^2 \quad (7)$$

where $b_k = \langle c_4(s_k)\rangle$ and $\{f_{i,k}\}$ are the coefficients of the matrix $\mathbf{F} = \mathbf{V}\mathbf{U}$. We have

Result 1 *If* **at most** *one of the b_k is zero, then*

$$\textit{for any unitary } \mathbf{V},\ \Psi(\mathbf{V}\mathbf{x}) \le \Psi(\mathbf{s}) \quad (8)$$

with equality if and only if $\mathbf{V}$ is essentially equal to $\mathbf{U}^\dagger$.

Proof 1 *As similar arguments are needed in Section 2.2, we may recall briefly why this result holds. We have:*

$$\begin{aligned} \Psi(\boldsymbol{F}\boldsymbol{s}) &\le \sum_i \left(\sum_k |f_{i,k}|^4 |b_k| \right)^2 & (9)\\ &\le \sum_i \left(\sum_k |f_{i,k}|^2 \sqrt{|b_k|} \right)^4 & (10) \end{aligned}$$

The right hand side of (10) *may be rewritten as* $\| \mathbb{F}\mathbf{b} \|_4^4$ *where* $\mathbb{F}_{i,j} = |f_{i,k}|^2$ *and* $\mathbf{b}^{(1/2)} = \left[\sqrt{|b_1|}, \ldots, \sqrt{|b_K|}\right]^T$. *Since* $\mathbf{F}$ *is unitary, the matrix* $\mathbb{F}$ *is bi-stochastic. A consequence of the Birkhoff theorem is that* $||\mathbb{F}\mathbf{b}^{(1/2)}||_4 \le ||\mathbf{b}^{(1/2)}||_4$. *Hence* (8). *Trivially,* (8) *is an equality when* $\mathbf{F}$ *is a permutation. Conversely, if* (8) *is an equality, necessarily* (10) *is an equality. This implies that the cross-products terms* $f_{i,k_1} f_{i,k_2} \sqrt{|b_{k_1} b_{k_2}|}$ *are* 0 *for every* $i = 1, \ldots, K$ *and* $k_1 \neq k_2$. *If none of the* b_k *is zero, this latter condition makes the matrix* $\mathbf{F}$ *have only one non-null coefficient on each row. As* $\mathbf{F}$ *is unitary, this proves that* $\mathbf{F}$ *is a permutation times a diagonal matrix with entries of modulus* 1. *If one of the sources (say the first one) is such that* $b_1 = 0$, *then* $\mathbf{F}$ *has two coefficients at most on the i-th row - say at positions* 1 *and* $\sigma(i)$ *- and* $||\mathbb{F}\mathbf{b}^{(1/2)}||_4^4 = \sum_{i=2,\ldots,K} |f_{i,\sigma(i)}|^8 ||b_i|^2$. *This term being equal to* $\sum_i |b_i|^2$ *and as* $|f_{i,\sigma(i)}|^2 \le 1$, *it finally yields* $|f_{i,\sigma(i)}|^2 = 1$, *hence* $\mathbf{F}$ *has exactly one non-null component of modulus* 1 *on every row: this means that* $\boldsymbol{V}$ *is essentially equal to* $\boldsymbol{U}^\dagger$.

2.2 A more realistic approach

In practice, Result 1 is difficult to exploit. This is due to the requirement of finding a consistent estimate of function Ψ. Let us specify this point.

The averaged fourth order cumulant of $r_i(n)$ can be expanded as:

$$\langle c_4(r_i(n))\rangle = \langle \mathbb{E}\{|r_i(n)|^4\}\rangle - 2\langle \left(\mathbb{E}\{|r_i(n)|^2\}\right)^2 \rangle - \langle \left|\mathbb{E}\{(r_i(n))^2\}\right|^2 \rangle$$

The source signals being second-order complex circular, the third term of the right-hand-side of the above equation cancels out. From the symmetry $R_{r_i}^{(\alpha)}(0) = \left(R_{r_i}^{(-\alpha)}(0)\right)^*$ and the Parseval equality, it finally yields

$$\begin{aligned} \langle c_4(r_i(n))\rangle &= \langle \mathbb{E}\{|r_i(n)|^4\}\rangle - 2|R_{r_i}^{(0)}(0)|^2 - 4 \sum_{\alpha \in I_+^*} |R_{r_i}^{(\alpha)}(0)|^2 \\ &= \langle \mathbb{E}\{|r_i(n)|^4\}\rangle - 2\ - 4 \sum_{\alpha \in I_+^*} |R_{r_i}^{(\alpha)}(0)|^2 \quad (11) \end{aligned}$$

where I_+^* is the set of the strictly positive cyclic-frequencies of the sources. A consistent estimate of (11) is simply

$$\hat{\Psi}(\mathbf{r}) = \langle |r_i(n)|^4\rangle_N\ - 2\ - 4 \sum_{\alpha \in I_+^*} |\langle |r_i(n)|^2 e^{-i2\pi\alpha n}\rangle_N|^2 \quad (12)$$

where N denotes the number of data and $\langle u(n)\rangle_N = \frac{1}{N}\sum_{n=0}^{N-1} u(n)$. The second term of (12) clearly requires the knowledge of the cyclic frequencies - or at least an accurate estimate of them. We have specified in the Introduction that such an assumption is incompatible with some contexts. As a consequence, it is not possible in general to estimate Ψ consistently. An idea consists in merely dropping the third term in (12). In other words, one may rather consider the estimate

$$\hat{\Phi}(\mathbf{r}) = \sum_{i=1}^{K} \left|\langle |r_i(n)|^4\rangle_N - 2\right|^2$$

and run the maximization over unitary matrices $\mathbf{V}$. Now, $\hat{\Phi}(\mathbf{V}\mathbf{x})$ does not in general converges to $\psi(\mathbf{V}\mathbf{x})$ but to the function Φ defined by

$$\begin{aligned} \Phi(\mathbf{r}) &= \sum_{i=1}^{K} \left|\langle \mathbb{E}\{|r_i(n)|^4\}\rangle - 2\right|^2 \quad (13)\\ &= \sum_{i=1}^{K} \left| \langle c_4(r_i(n))\rangle + 4 \sum_{\alpha \in I_+^*} |R_{r_i}^{(\alpha)}(0)|^2 \right|^2 \end{aligned}$$

which only coincides with (6) when the sources are all wide-sense stationary. It remains to prove that the maximization of function Φ given by (13) is achieved if and only if $\mathbf{V}$ is essentially equal to $\mathbf{U}^{\dagger}$. We investigate this point.

Remark 1 *Running the Comon algorithm with our data (although the algorithm was originally designed for stationary data) is nothing different than implementing the maximization of function $\hat{\Phi}$. In other words, if the function $\Phi(\boldsymbol{r})$ is a shown to be contrast function in a cyclostationnary context, then the identification of the mixing matrix $\boldsymbol{U}$ can be done by running this algorithm.*

Taking (11) into account, Φ can be expanded as:

$$\Phi(\mathbf{Fs}) = \sum_{i=1}^{K} \left| \sum_{k=1}^{K} |f_{i,k}|^4 \zeta_k + 4 \sum_{k_1 \neq k_2} |f_{i,k_1}|^2 |f_{i,k_2}|^2 \lambda_{k_1,k_2} \right|^2 \quad (14)$$

where

$$\zeta_k = \langle c_4(s_k(n)) \rangle + 4 \sum_{\alpha \in I_+^*} |R_{s_k}^{(\alpha)}(0)|^2 \quad (15)$$

and

$$\lambda_{k_1,k_2} = \sum_{\alpha \in I_+^*} R_{s_{k_1}}^{(\alpha)}(0) R_{s_{k_2}}^{(-\alpha)}(0). \quad (16)$$

In general, Result 1 does not apply directly (compare (7) and (14)). We distinguish between two cases :

2.2.1 *The non-null cyclic-frequencies are pair-wise different*

Result 2 *if the sources have pair-wise distinct strictly positive cyclic frequencies; if* **at most** *one of the ζ_k given by* (15) *is zero, then*

$$\text{for any unitary } \boldsymbol{V},\ \Phi(\boldsymbol{Vx}) \leq \Phi(\boldsymbol{s}) \quad (17)$$

with equality if and only if $\mathbf{V}$ is essentially equal to $\mathbf{U}^{\dagger}$.

Indeed, in this context, the terms $R_{s_{k_1}}^{(\alpha)}(0) R_{s_{k_2}}^{(-\alpha)}(0)$, for $k_1 \neq k_2$ are zero; this makes Φ have the same expression as Ψ - see Eq. (7) - except that the b_k are merely replaced by the ζ_k.

2.2.2 *The general case*

The terms $\lambda_{k_1,k_2} = \sum_{\alpha \in I_+^*} R_{s_{k_1}}^{(\alpha)}(0) R_{s_{k_2}}^{(-\alpha)}(0)$ are are not null in general. We have:

Result 3 *if*

Condition 1 $\forall\, k_1 \neq k_2,\ 4|\lambda_{k_1,k_2}| < \sqrt{|\zeta_{k_1} \zeta_{k_2}|}$

then, for any unitary matrix $\boldsymbol{V}$,

$$\Phi(\boldsymbol{Vx}) \leq \Phi(\boldsymbol{s}) \quad (18)$$

with equality if and only if $\mathbf{V}$ is essentially equal to $\mathbf{U}^{\dagger}$.

If Condition 1 is satisfied, we have:

$$\begin{aligned} \Phi(\mathbf{Fs}) &= \sum_i \left(\sum_k |f_{i,k}|^4 |\zeta_k| + 4 \sum_{k_1 \neq k_2} |f_{i,k_1}|^2 |f_{i,k_2}|^2 |\lambda_{k_1,k_2}| \right)^2 \\ &\leq \sum_{i=1}^{K} \left(\sum_k |f_{i,k}|^2 \sqrt{|\zeta_k|} \right)^4 \quad (19) \\ &\leq \sum_{i=1}^{K} |\zeta_i|^2. \quad (20) \end{aligned}$$

Inequality (20) holds because the matrix $\mathbf{F} = \mathbf{VU}$ is unitary, and the equality is reached when $\mathbf{V}$ is essentially equal to $\mathbf{U}^{\dagger}$. Conversely, (20) is an equality requires that (19) is also an equality. This occurs iff for all $k_1 \neq k_2$, $|f_{k_1} f_{k_2}|^2 \left(4|\lambda_{k_1,k_2}| - \sqrt{|\zeta_{k_1} \zeta_{k_2}|}\right) = 0$. Due to Condition 1, this is equivalent to $\mathbf{F}$ having a single non-null coefficient on every row, *i.e.* $\mathbf{F}$ is the product of a permutation and a diagonal matrix with modulus one entries.

3. APPLICATION TO DIGITAL COMMUNICATION CONTEXTS

Results 2 and 3 are general and can be applied to any kind of instantaneous mixtures of cyclostationary sources. In this section we propose to apply these results in a digital communication context.

We assume that the source signals result from a linear modulation of i.i.d. sequences of complex circular symbols. Hence, the $k-th$ component of the signal $\mathbf{s}(n)$ can be written:

$$s_k(n) = \sum_{p \in \mathbb{Z}} a_k(p) g_{a,k}(nT_e - pT_k) \quad (21)$$

where $\{a_k(p)\}_p$ are the transmitted symbols such that $\mathbb{E}\{|a_k(p)|^2\} = 1$. We assume that these symbols are circular so that the signal $s_{a,k}$ is also circular. $g_{a,k}$ is assumed to be a square-root raised-cosine function. The signal $s_{a,k}(t)$ is hence band-limited to $[-\frac{1+\gamma_k}{2T_k}, \frac{1+\gamma_k}{2T_k}]$, where γ_k is the excess bandwidth factor excess $(0 < \gamma_k < 1)$. The sampling period T_e is supposed to satisfy the Shannon condition.

The discrete-time source signal $\mathbf{s}$ is then cyclostationary. Moreover, any component s_k of $\mathbf{s}$ has at most 3 distinct cyclic frequencies, i.e. $I_k = \{-\alpha_k, 0, \alpha_k\}$, where $\alpha_k = T_e/T_k$ [3]. The fact that α_k is not known means that the baudrate of the $k-th$ source is possibily unknown and that no prior estimation is needed to run the algorithm.

Remark 2 *The source signals may be corrupted by non zero frequency offsets. But since the statistics involved in Condition 1 only depends on the modulus of the source signals, the frequency offset has no impact on the following results.*

3.1 Sources with pair-wise different baudrates

This condition on the baud-rates implies that the non-null positive cyclic frequencies are pair-wise different; hence Result 2 applies, and the Comon algorithm is expected to converge to a certain matrix $\hat{\mathbf{V}}$, supposedly close to $\mathbf{U}^{\dagger}$ (up to the classical indeterminacies). We emphasize the fact that this result holds as long as at most one of the ζ_k is zero. This condition is analyzed in the next section, and we prove that it is satisfied in a digital communication context.

3.2 General configuration of the baudrates

Notice that condition 1 is not very tractable in general, we propose to replace it by a stronger condition :

Condition 2 $\min_k |\zeta_k| > 4\max_k |R_{s_k}^{(\alpha_k)}(0)|^2$

The fact that a source has at most one cyclic-frequency makes that λ_{k_1,k_2} in (16) has at most one term. Moreover, we have, for any k_1, k_2: $\lambda_{k_1,k_2} \leq \max_k |R_{s_k}^{(\alpha_k)}(0)|^2$. Hence Condition 2 implies Condition 1 and is much simpler to evaluate.

Condition 2 is naturally satisfied for a given configuration of the sources if we show that the upper bound of $4|R_{s_k}^{(\alpha_k)}(0)|^2$ over a class of modulations is less than the inferior bound of $|\zeta_k|$ over the same set of modulations. For each kind of signal of this set of modulations, the index of the source is irrelevant, and it is not mentioned in order to simplify the notations. According to the previous notations, we set for the source s :

g_a		its shaping filter
$\{a_p\}_{p\in\mathbb{Z}}$		its symbol sequence
T		its symbol rate
γ		its excess bandwith
λ	$=$	$|R_s^{(\alpha)}(0)|^2$
ζ	$=$	$\langle c_4(s(n))\rangle + 4\lambda = \langle \mathbb{E}|s(n)|^4\rangle - 2$

Lemma 1 *If*

$$T_e \notin \left\{T, \frac{T}{2}, \frac{T}{3}, \frac{2T}{3}\right\}, \quad (22)$$

the numbers ζ and λ do not depend on the sampling period T_e; moreover

$$\lambda = \left|\frac{1}{T}\int_0^T \mathbb{E}|s_a(t)|^2 e^{2i\pi\frac{t}{T}}dt\right|^2 = \left|\frac{1}{T}\int_{-\infty}^{\infty}|g_a(t)|^2 e^{2i\pi t/T}dt\right|^2 \quad (23)$$

and

$$\langle c_4(s(n))\rangle = \frac{1}{T}\int_0^T c_4(s_a(t))dt = c_4(a_n)\frac{1}{T}\int_{-\infty}^{\infty}|g_a(t)|^4 dt \quad (24)$$

We recall briefly why this result holds : $t \mapsto c_4(s_a(t))$ is a periodic function of period T, and can thus be developed in Fourier series:

$$c_4(s_a(t)) = \sum_l c_l e^{2i\pi l t/T}$$

Where c_l are the Fourier coefficients. As $s_a(t)$ is a band-limited signal, $|c_l| = 0$ if $|l| > 3$ [8]. Hence :

$$\langle c_4(s_a(nT_e))\rangle = \sum_{|l|\leq 3} c_l \langle e^{2i\pi n l T_e/T}\rangle$$

As long as T_e satisfies (22), $\langle e^{2i\pi n l T/T_k}\rangle = 0$ if $l \neq 0$. Hence, $\langle c_4(s_a(nT_e))\rangle$ simplifies to c_0. Thanks to (21), $c_4(s_a(t)) = c_4(a_n)\sum_{n\in\mathbb{Z}}|g_a(t-nT)|^4$ and $c_0 = c_4(a_n)\frac{1}{T}\int_{\mathbb{R}}|g_a(t)|^4 dt$. A similar proof holds for λ.

In the sequel, we assume that, T_e verify (22). Of course, this has no implication in a real scenario, since the probability of choosing such a sampling period is 1.

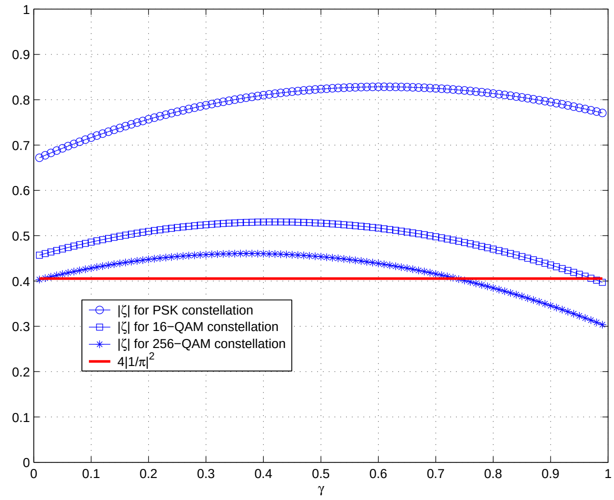

Figure 1: ζ for a set of linear modulation

As g_a is known (its a square-root raised-cosine function), we are able to compute λ analytically :

$$\lambda = \left(\frac{\gamma}{\pi}\right)^2 \leq \left(\frac{1}{\pi}\right)^2 \quad (25)$$

A similar way is possible to express ζ :

$$\zeta = c_4(a_p)\frac{1}{T}\int_{-\infty}^{\infty}|g_a(t)|^4 dt + 4\left(\frac{\gamma}{\pi}\right)^2 \quad (26)$$

We have not been able to express analytically ζ as a function of γ. Notice that as $g_a(\frac{t}{T})$ does not depend on T, ζ does not depend on T.

$|\zeta|$ is numerically computed for a set of modulation on figure 1:

- $|\zeta| > 0$ which ensure that the condition discussed in section 3.1 is satisfied
- $|\zeta| > \frac{4}{\pi^2}$ which ensures that Condition 2 (hence Condition 1) is satisfied for any mixture of PSK and QAM signals if there is no $4r^2$-QAM ($r > 2$) source with an excess band-width factor exceeding 70%.

For such mixtures, the maximization of the Comon objective function Φ achieves the separation of the sources.

3.3 Extension to mixture of CPM signals

This kind of modulation is widely used, due to the constant modulus property of the emitted signal. Indeed, $|s(n)| = 1$ for every $n \in \mathbb{Z}$. The assumption of circularity used along the paper is always satisfied for CPM signals except if the modulation index equals $1/2$ (or a multiple): this case is not considered.

The constant modulus of $s(n)$ makes $\mathbb{E}\{|s(n)|^2\} = 1$ whatever n (this does not mean that $s(n)$ is not a cyclo-stationary sequence since $\mathbb{E}\{s(n+m)\bar{s}(n)\}$ depends on the time index n in general). Due to (3), we simply deduce that $\lambda(s_k) = 0$ and $\zeta(s_k) = 1$. In particular, this means that for a mixture of non-filtered CPM signals, Condition 1 is always satisfied.

In a more realistic scenario, the transmitted CPM signals are filtered in order to enhance their spectral efficiency. Nevertheless, since the constant modulus property is a key property, the filtering slightly affects this property and hence the values of ζ and λ.

4. SIMULATION RESULTS

If $c_{k,l}(n)$ the contribution of the k-th source on the l-th sensor at time n, we may compute the estimated contribution $\hat{c}_{k,l}(n)$ thanks to the estimates of the sources provided by a BSS algorithm. The performance criterion for the k-th source is defined as:

$$
\begin{aligned}
C_q &= \sum_{k=1}^{K} \frac{\langle |\hat{c}_{q,k}(n) - c_{q,k}(n)|^2 \rangle_N}{\langle |y_k(n)|^2 \rangle_N} \\
&= \sum_{k=1}^{K} \frac{\langle |c_{q,k}(n)|^2 \rangle_N}{\langle |y_k(n)|^2 \rangle_N} \frac{\langle |\hat{c}_{q,k}(n) - c_{q,k}(n)|^2 \rangle_N}{\langle |c_{q,k}(n)|^2 \rangle_N}
\end{aligned}
$$

where N is the number of available samples. The global criterion of performance is defined as the mean of $C_1, \ldots, C_Q$. We recall that $y_k(n)$ is the observed sequence on sensor k.

We propose to average this criterion on 100 trials. The number of sources is $K = 4$ and the number of sensors $Q = 5$. For a given trial, the mixing matrix is randomly chosen. The sources are randomly chosen in the set $\{QPSK, 8-PSK\}$; the baud-rates are randomly chosen in $\{1, 4/3\}$ time-units, and the excess bandwidth factors are also randomly chosen in $(0\ ,\ 1)$. The duration of an experiment is 800 time-units. The sampling period T_e was chosen in accordance with the Shannon sampling condition. Figure 2 shows the repartition function of the performance criterion of different Comon algorithms :

- when the cyclic frequencies are unknown from the receiver. The curve obtained is denoted by "Maximisation of function Φ"
- when the temporal mean of the cumulants $\langle c_4(r_i(n)) \rangle$ estimated via Eq. (12) (the cyclic frequencies in this case are supposed to be known by the receiver: though not realistic, this gives a point of comparison). The curve obtained is denoted by "Maximisation of function Ψ".

The conclusion is that the knowledge of the cyclic-frequencies does not improve the results (i.e., taking into account the extra-terms in (12) seems to impair the performance).

5. CONCLUSION

We have given a simple condition on the statistics of the (cyclostationary) sources which ensures that the maximization of the Comon function (13) performs BSS. This condition does not depend on the number of sources and is semi-analytically shown to be fulfilled in the context of digital communications (mixtures of linear/CPM sources with any baud-rates).

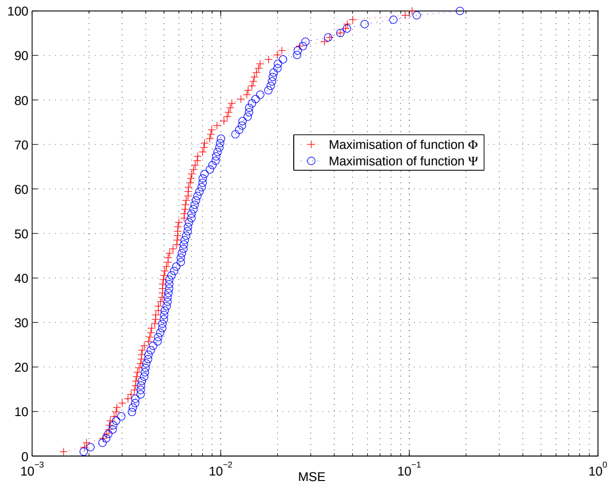

Figure 2: Performances of different versions of the Comon algorithm

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