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ADAPTIVE QUADRATIC-METRIC PARALLEL SUBGRADIENT PROJECTION ALGORITHM AND ITS APPLICATION TO ACOUSTIC ECHO CANCELLATION

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ABSTRACT

Adaptive Projected Subgradient Method (APSM) serves as a unified guiding principle of various set-theoretic adaptive filtering algorithms including NLMS/APA. APSM asymptotically minimizes a sequence of non-negative convex functions in a real-Hilbert space. On the other hand, the exponentially weighted stepsize projection (ESP) algorithm has been reported to converge faster than APA in the acoustic echo cancellation (AEC) problem.

In this paper, we first clarify that ESP is derived by APSM in a real Hilbert space with *a special inner product*. This gives us an interesting interpretation that ESP is based on iterative projections onto the same convex sets as APA with *a special metric*. We can thus expect that a proper choice of metric will lead to improvement of convergence speed. We then propose an efficient adaptive algorithm named adaptive quadratic-metric parallel subgradient projection (AQ-PSP). Numerical examples demonstrate that AQ-PSP with a very simple metric achieves even better echo canceling ability than ESP, proportionate NLMS, and Euclidean-metric version of AQ-PSP, while keeping low computational complexity.

1. INTRODUCTION

Acoustic echo cancellation (AEC) is a key to design a hands-free system such as teleconferencing and car phone [1, 2]. A basic scheme of AEC is illustrated below.

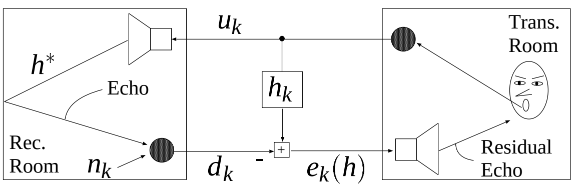

Figure 1: Acoustic echo canceling scheme.

Let $k \in \mathbb{N}$ be the time index, where $\mathbb{N}$ denotes the set of all non-negative integers. With a sequence of input signals $(u_k)_{k\in\mathbb{N}} \subset \mathbb{R}$, let $(\boldsymbol{u}_k)_{k\in\mathbb{N}} \subset \mathbb{R}^N$ be a sequence of input vectors defined as $\boldsymbol{u}_k := [u_k, u_{k-1}, \cdots, u_{k-N+1}]^T$. Here $\mathbb{R}$ denotes the set of all real numbers, $N \in \mathbb{N}^* := \mathbb{N}\backslash\{0\}$ the filter length, and the superscript T the transposition. For $r \in \mathbb{N}^*$, define $U_k := [\boldsymbol{u}_k, \boldsymbol{u}_{k-1}, \cdots, \boldsymbol{u}_{k-r+1}] \in \mathbb{R}^{N\times r}$ (usually $r \ll N$). Also define the noise vector $\boldsymbol{n}_k := [n_k, n_{k-1}, \cdots, n_{k-r+1}]^T \in \mathbb{R}^r$, $\forall k \in \mathbb{N}$, with $(n_k)_{k\in\mathbb{N}}$ being a sequence of additive noise process. With the echo impulse response $h^* \in \mathbb{R}^N$, we introduce the following linear model for the data process $(\boldsymbol{d}_k)_{k\in\mathbb{N}} \subset \mathbb{R}^r$: $\boldsymbol{d}_k := U_k^T h^* + \boldsymbol{n}_k$. The goal of the echo cancellation is to remove the echo part $U_k^T h^*$ from $\boldsymbol{d}_k$ by subtracting the output of adaptive (linear) filter $h_k \in \mathbb{R}^N$, $k \in \mathbb{N}$, as $\boldsymbol{d}_k - U_k^T h_k$. If $h_k \approx h^*$, the echo is successfully canceled, thus the problem can be interpreted as the system identification (i.e., identify an unknown system h^* by using input-output relations), which is also called the adaptive filtering.

In 2003, a unified method to derive a variety of adaptive filtering algorithms has been proposed, which is called *adaptive projected subgradient method (APSM)* [3, 4]. APSM is successfully extended in [5, 6], and it has been proved to be a promising method to derive excellent algorithms for applications to the stereo echo cancellation [7], blind multiple access interference suppression in DS/CDMA systems [8, 9], and robust adaptive beamforming [10, 11] problems. The classical normalized least mean square (NLMS) algorithm and the affine projection algorithm (APA) [1] are derived by APSM from the cost functions of distances to a certain hyperplane and a certain linear variety, respectively (their constrained versions are also derived in simple ways [3, 4]). A more efficient adaptive algorithm, named adaptive parallel subgradient projection (adaptive PSP) [12], is derived from the cost function of a convex combination of distances to multiple half-spaces. All algorithms that have so far been shown to be derived by APSM are associated with the Euclidean (or standard) metric defined by the (standard) inner product $\langle a, b\rangle = a^T b$ for any $a, b \in \mathbb{R}^N$. In the AEC problem, it is known that the room impulse responses decay exponentially on average, based on which the exponentially weighted stepsize projection (ESP) algorithm has been proposed and reported to be more effective than APA [13, 14]. This special structure encourages us to explore a more reasonable metric for AEC.

The contribution of this paper is twofold. We first clarify that ESP can be derived by APSM from a cost function similar to APA but with a different metric. We then propose a fast echo canceling algorithm, named adaptive quadratic-metric parallel subgradient projection (AQ-PSP), which is based on the adaptive PSP technique with an effective metric. The proposed algorithm enjoys robustness against noise and $O(N)$ computational complexity (see Remark 1). Numerical examples demonstrate that the proposed algorithm exhibits better echo cancellation performance than ESP, proportionate NLMS [15, 16], and adaptive PSP with the Euclidean metric.

2. EXTENDED ADAPTIVE PROJECTED SUBGRADIENT METHOD

Throughout the paper, the following notation is used. A real Hilbert space $\mathcal{H}$ equipped with an inner product $\langle\cdot,\cdot\rangle$ will be denoted by $(\mathcal{H}, \langle\cdot,\cdot\rangle)$. Its induced norm is given by $\|x\| := \langle x, x\rangle^{1/2}$, $\forall x \in \mathcal{H}$. Finite dimensional Hilbert spaces such as $\mathbb{R}^N$ ($N \in \mathbb{N}^*$) are also called Euclidean spaces, which are often the stages in real-world applications.

A set $C \subset \mathcal{H}$ is said to be *convex* if $\nu x + (1-\nu)y \in C$, $\forall x, y \in C$, $\forall \nu \in (0,1)$. A function $\Theta : \mathcal{H} \to \mathbb{R}$ is said to be *convex* if $\Theta(\nu x + (1-\nu)y) \le \nu\Theta(x) + (1-\nu)\Theta(y)$, $\forall x, y \in \mathcal{H}$, $\forall \nu \in (0,1)$.

Given a mapping $T : \mathcal{H} \to \mathcal{H}$, the fixed point set of T is denoted and defined as $\mathrm{Fix}(T) := \{y \in \mathcal{H} : T(y) = y\}$. A mapping T is said to be *nonexpansive* if $\|T(x) - T(y)\| \le \|x - y\|$, $\forall x, y \in \mathcal{H}$. If, in addition, $\mathrm{Fix}(T) \ne \emptyset$ and there exists $\eta > 0$ s.t. $\eta\,\|x - T(x)\|^2 \le \|x - f\|^2 - \|T(x) - f\|^2$, $\forall x \in \mathcal{H}$, $\forall f \in \mathrm{Fix}(T)$, then T is said to be *strongly* or *η-attracting nonexpansive*. The identity mapping $I : \mathcal{H} \to \mathcal{H}$, $x \mapsto x$, can be considered as an η-attracting nonexpansive mapping for an arbitrary $\eta > 0$ with $\mathrm{Fix}(I) = \mathcal{H}$. Given

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a nonempty closed convex set $C \subset \mathscr{H}$, the mapping that assigns every point in $\mathscr{H}$ to its unique nearest point in C is called *metric projection* onto C and is denoted by P_C. Mathematically, one can state $P_C : \mathscr{H} \to C$, $x \mapsto P_C(x) \in \arg\inf_{y\in C} \|x-y\|$. P_C has the following properties: $\mathrm{Fix}(P_C) = C$; P_C is 1-attracting nonexpansive; $\|x - P_C(x)\| = d(x,C) := \inf_{y\in C} \|x-y\|$, $\forall x \in \mathscr{H}$.

Given a continuous convex function $\Theta : \mathscr{H} \to \mathbb{R}$, the *subdifferential* of Θ at any $y \in \mathscr{H}$, the set of all the *subgradients* of Θ at y; $\partial\Theta(y) := \{a \in \mathscr{H} : \langle x-y, a\rangle + \Theta(y) \le \Theta(x), \forall x \in \mathscr{H}\}$, is nonempty. Let $\Theta_k : \mathscr{H} \to [0,\infty)$, $k \in \mathbb{N}$, be a continuous convex function and $\partial\Theta_k(y)$ the subdifferential of Θ_k at y. Also let $T : \mathscr{H} \to \mathscr{H}$ denote an η-attracting nonexpansive mapping. The following scheme, an extension of the scheme in [3, 4], provides a vector sequence that minimizes asymptotically the sequence of functions $(\Theta_k)_{k\in\mathbb{N}}$ over $\mathrm{Fix}(T)$.

Scheme 1 *(Extended Adaptive Projected Subgradient Method* [5, 6]*) For an arbitrary given $h_0 \in \mathscr{H}$, generate a sequence $(h_k)_{k\in\mathbb{N}} \subset \mathscr{H}$ by*

$$h_{k+1} := \begin{cases} T\left(h_k - \lambda_k \frac{\Theta_k(h_k)}{\|\Theta_k'(h_k)\|^2}\Theta_k'(h_k)\right), \\ \qquad \text{if } \Theta_k'(h_k) \neq 0, \\ T(h_k), \quad \text{otherwise,} \end{cases}$$

where $\Theta_k'(h_k) \in \partial\Theta_k(h_k)$, $\lambda_k \in [0,2]$, $\forall k \in \mathbb{N}$, and 0 is the zero vector. The sequence $(h_k)_{k\in\mathbb{N}}$ enjoys great features; monotone approximation, asymptotic optimality, and strong convergence (see Appendix A).

Replacing T with a metric projection operator, Scheme 1 is reduced to the original APSM [3, 4].

In the following, to specify an inner product and its induced norm, we respectively use $\langle a,b\rangle_G := a^T Gb$, $\forall a,b \in \mathscr{H}$, and $\|a\|_G := \sqrt{\langle a,a\rangle_G}$, $\forall a \in \mathscr{H}$, where $G \in \mathbb{R}^{N\times N}$ is a positive definite matrix (which is denoted as $G \succ 0$). In the real Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle_G)$, the distance between arbitrary two elements is given by $d_G(a,b) := \|a-b\|_G$, $\forall a,b \in \mathscr{H}$. Similarly, the distance between an arbitrary element $a \in \mathscr{H}$ and a closed convex set C is given by $d_G(a,C) := \inf_{b\in C} \|a-b\|_G$, and the projection of $h \in \mathscr{H}$ onto C is given as $P_C^{(G)}(h) := \arg\inf_{b\in C} d_G(a,b)$.

3. EFFECTIVE METRIC FOR ACOUSTIC ECHO CANCELLATION

In this section, we present an effective metric for the AEC problem. Following the derivation of the ESP algorithm [13, 14] by APSM, we propose an efficient AEC algorithm derived also by APSM. Hereafter we let $\mathscr{H} := \mathbb{R}^N$.

3.1 A Novel Interpretation of ESP Algorithm

In [13, 14], it has experimentally been shown that room impulse responses and its variations decay by the same exponential ratio on average[1]. (The ratio can be measured in advance, since it is not variable under fixed acoustic conditions of a room; e.g., size, absorption coefficient etc.) This motivates us to give step sizes, proportional to the expected mismatch levels, to filter coefficients. Aiming at exponentially decaying step sizes, define [13, 14]

$$(\mathbb{R}^{N\times N} \ni) A := \mathrm{diag}(\alpha_1, \alpha_2, \cdots, \alpha_N) \succ 0,$$

where $\alpha_i := \alpha_0\gamma^{i-1}$ with a positive constant $(\mathbb{R} \ni)\alpha_0 > 0$ and the exponential ratio $\gamma \in (0,1)$ (NOTE: α_0 is of no importance because it will be canceled out in the algorithm). Then, the ESP algorithm is given by (superscript †: the Moore-Penrose pseudoinverse [18])

$$h_{k+1} := h_k + \lambda_k A U_k (U_k^T A U_k)^\dagger e_k(h_k),\ \forall k \in \mathbb{N}, \tag{1}$$

[1]This fact is also verified theoretically in [17].

where $\lambda_k \in [0,2]$ and e_k is the error (or residual) function; $e_k : \mathscr{H} \to \mathbb{R}^r$, $h \mapsto U_k^T h - d_k$. The equivalence of (1) to the ESP algorithm [14] is straightforward (see also [12, Appendix B]).

Consider here the real Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle_{A^{-1}})$. Given $r \in \mathbb{N}^*$, define a sequence of data-dependent linear varieties $(V_k)_{k\in\mathbb{N}}$ as

$$V_k := \left\{h \in \mathscr{H} : e_k(h) = U_k^T h - d_k = 0\right\},\ \forall k \in \mathbb{N}.$$

Let $\Theta_k(h) := d_{A^{-1}}(h, V_k) = \left\|h - P_{V_k}^{(A^{-1})}(h)\right\|_{A^{-1}}$. Then, $\partial\Theta_k(h) \ni \Theta_k'(h) = \dfrac{h - P_{V_k}^{(A^{-1})}(h)}{d_{A^{-1}}(h,V_k)}$, if $h_k \notin V_k$, $\Theta_k'(h) = 0$, otherwise. Applying $\Theta_k(h)$ and $K := \mathscr{H}$ to Scheme 1 yields

$$h_{k+1} = \begin{cases} h_k + \lambda_k\left(P_{V_k}^{(A^{-1})}(h_k) - h_k\right), & \text{if } h_k \notin V_k, \\ h_k, & \text{otherwise.} \end{cases} \tag{2}$$

The equivalence of (2) to (1) is proved by the following observation.

Observation 1 *Given any positive definite matrix $G \succ 0$,*

$$P_{V_k}^{(G^{-1})}(h) = h + GU_k(U_k^T GU_k)^\dagger e_k(h),\ \forall h \in \mathscr{H}. \tag{3}$$

Proof: See Appendix B.

The above argument verifies that the ESP algorithm [13] is derived by APSM with the metric $d_{A^{-1}}$ while it has been shown in [3, 4] that the APA algorithm [1] is derived with the Euclidean metric d_I. This interpretation implies that ESP as well as APA is endowed with great features of APSM (see Appendix A). Indeed, it has been reported that ESP converges faster than APA [13]. It is thus expected that $d_{A^{-1}}$ is an effective metric and that a more efficient algorithm can be derived from a different sequence of cost functions $(\Theta_k)_{k\in\mathbb{N}}$ with this kind of metric based on the exponentially decaying structure of room impulse responses. We present an efficient AEC algorithm based on parallel subgradient projection with an effective metric below.

3.2 Proposed Echo Canceling Algorithm

Consider now the real Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle_{G^{-1}})$, where $G \succ 0$ is an appropriate positive definite matrix such as A or

$$(\mathbb{R}^{N\times N} \ni) B := \begin{bmatrix} I & O \\ O & \gamma^{N/2} I \end{bmatrix} \succ 0. \tag{4}$$

Here $I \in \mathbb{R}^{N/2\times N/2}$ and $O \in \mathbb{R}^{N/2\times N/2}$ denote the identity and zero matrices, respectively, and $\gamma \in (0,1)$ is introduced in Sec. 3.1. (I and O will be used for any size of matrix.) The matrix B is defined based on essentially the same idea as A, but is much simpler and requires fewer arithmetic operations in the algorithm (see Remark 1).

It is easy to see that the true echo impulse response h^* belongs to $V_k^* := \left\{h \in \mathscr{H} : e_k(h) = U_k^T h - d_k = n_k\right\}$, $\forall k \in \mathbb{N}$, hence h^* is most likely out of V_k in noisy environments. This unfortunately causes sensitivity, to noise, of APA-based algorithms including ESP (For details, see [12]). We thus introduce the following stochastic property set (closed convex set)

$$C_k(\rho) := \left\{h \in \mathscr{H} : g_k(h) := \|e_k(h)\|^2 - \rho \le 0\right\},\ \forall k \in \mathscr{H},$$

where $\rho \ge 0$ determines the membership probability that $h^* \in C_k(\rho)$. Note that ρ should be designed by taking into account the noise information [12, Ex. 1]. The direct projection onto $C_k(\rho)$ requires high computational cost in general, thus we introduce an approximation of the projection; i.e., projection onto the closed half-space $H_k^-(h) := \{x \in \mathscr{H} : \langle x-h, \nabla g_k(h)\rangle_{G^{-1}} + g_k(h) \le 0\} \supset C_k(\rho)$, whose boundary hyperplane separates the current estimate h_k and

$C_k(\rho)$ if $h_k \notin C_k(\rho)$. Note that $\partial g_k(h) = \{\nabla g_k(h)\}$ in this (differentiable) case. We stress now that we are considering $(\mathscr{H}, \langle\cdot,\cdot\rangle_{G^{-1}})$. The projection onto $H_k^-(h)$ has the following simple closed-form expression:

$$P_{H_k^-(h)}^{(G^{-1})}(h) = \begin{cases} h - \dfrac{g_k(h)}{\|\nabla g_k(h)\|_{G^{-1}}^2}\nabla g_k(h), & \text{if } h \notin H_k^-(h), \\ h, & \text{otherwise.} \end{cases}$$

It is easy to verify, from the definition of subdifferential (see Sec. 2), that $\nabla g_k(h) = 2GU_k e_k(h)$. We remark [12] that $P_{H_k^-(h)}^{(G^{-1})}(h) \cong P_{C_k(\rho)}^{(G^{-1})}(h)$; and $P_{H_k^-(h)}^{(G^{-1})}(h)$ requires only $O(N)$ complexity.

Given $q \in \mathbb{N}^*$, define the control sequence $\mathscr{I}_k := \{\iota_1^{(k)}, \iota_2^{(k)}, \cdots, \iota_q^{(k)}\}$, $\forall k \in \mathbb{N}$. The control sequence indicates the closed half-spaces to be processed at time k. Also define a weight to each half-space as $w_\iota^{(k)} \in (0,1]$, $\iota \in \mathscr{I}_k$, $k \in \mathbb{N}$, satisfying $\sum_{\iota\in\mathscr{I}_k} w_\iota^{(k)} = 1$. We define a sequence of cost functions $(\Theta_k)_{k\in\mathbb{N}}$ as, $\forall k \in \mathbb{N}$,

$$\Theta_k(h) := \begin{cases} \frac{1}{L_k}\displaystyle\sum_{\iota\in\mathscr{I}_k} w_\iota^{(k)} d_{G^{-1}}[h_k, H_\iota^-(h_k)] d_{G^{-1}}[h, H_\iota^-(h_k)], \\ \text{if } L_k := \sum_{\iota\in\mathscr{I}_k} w_\iota^{(k)} d_{G^{-1}}[h_k, H_\iota^-(h_k)] \neq 0, \\ 0, \text{ otherwise.} \end{cases}$$

Note that, by the factor $d_{G^{-1}}[h_k, H_\iota^-(h_k)]$, a large weight is given to the set that is 'far' from h_k in the sense of the metric $d_{G^{-1}}$. Also note that $d_{G^{-1}}[h, H_\iota^-(h_k)] = \left\| h - P_{H_\iota^-(h_k)}^{(G^{-1})}(h) \right\|_{G^{-1}}$. For the function $f(h) := d_{G^{-1}}[h, H_\iota^-(h_k)]$, $\forall h \in \mathscr{H}$, we have

$$\partial f(h) \ni f'(h) = \begin{cases} \dfrac{h - P_{H_\iota^-(h_k)}^{(G^{-1})}(h)}{d_{G^{-1}}[h, H_\iota^-(h_k)]}, & \text{if } h \notin H_\iota^-(h_k), \\ 0, & \text{otherwise.} \end{cases}$$

We denote the early and tale parts of any vector $x \in \mathscr{H}$ as $x_{(\mathrm{e})} \in \mathbb{R}^{N/2}$ and $x_{(\mathrm{t})} \in \mathbb{R}^{N/2}$, respectively; i.e., $x =: [x_{(\mathrm{e})}^T x_{(\mathrm{t})}^T]^T$. We then introduce the following two constraint sets that respectively restrict the energy of early and tale parts of h_k:

$$K_{\mathrm{e}} := \left\{ \begin{bmatrix} h_{(\mathrm{e})} \\ h_{(\mathrm{t})} \end{bmatrix} \in \mathscr{H} : \left\| \begin{bmatrix} h_{(\mathrm{e})} \\ 0 \end{bmatrix} \right\|_{G^{-1}}^2 \leq \varepsilon_{\mathrm{e}} \right\},$$
$$K_{\mathrm{t}} := \left\{ \begin{bmatrix} h_{(\mathrm{e})} \\ h_{(\mathrm{t})} \end{bmatrix} \in \mathscr{H} : \left\| \begin{bmatrix} 0 \\ h_{(\mathrm{t})} \end{bmatrix} \right\|_{G^{-1}}^2 \leq \varepsilon_{\mathrm{t}} \right\}.$$

Here $\varepsilon_{\mathrm{e}}, \varepsilon_{\mathrm{t}} > 0$ should be designed based on an estimate of α_0 and the exponential ratio γ. Focusing only on the early part, K_{e} is, with the Euclidean metric, an ellipsoid that has a large radius in an axis corresponding to a large component of the echo impulse response h^*. However, with the metric $d_{G^{-1}}$, K_{e} is a sphere with its radius $\sqrt{\varepsilon_{\mathrm{e}}}$. Moreover, K_{e} has no constraint on the tale part. Thanks to this simple structure, the projection onto K_{e} is simply given as follows:

$$\forall h = \begin{bmatrix} h_{(\mathrm{e})} \\ h_{(\mathrm{t})} \end{bmatrix} \in \mathscr{H},\ P_{K_{\mathrm{e}}}^{(G^{-1})}(h) = \begin{cases} \begin{bmatrix} \dfrac{\sqrt{\varepsilon_{\mathrm{e}}}}{\alpha(h)} h_{(\mathrm{e})} \\ h_{(\mathrm{t})} \end{bmatrix}, & \text{if } h \notin K_{\mathrm{e}}, \\ h, & \text{otherwise,} \end{cases}$$

where $\alpha(h) := \|[h_{(\mathrm{e})}^T 0^T]^T\|_{G^{-1}}$. The projection onto K_{t} can be computed in a similar way. Application of Scheme 1 to Θ_k with $T := P_{K_{\mathrm{e}}}^{(G^{-1})} P_{K_{\mathrm{t}}}^{(G^{-1})}$, which is a 1/2-attracting mapping with $\mathrm{Fix}(T) = K_{\mathrm{e}} \cap K_{\mathrm{t}}$ [5, 6], derives the proposed algorithm as below.

Algorithm 1 [Adaptive Quadratic-Metric Parallel Subgradient Projection (AQ-PSP) Algorithm] *For an arbitrary initial vector $h_0 \in \mathscr{H}$, generate a sequence of adaptive filtering vectors $(h_k)_{k\in\mathbb{N}} \subset \mathscr{H}$ as*

$$h_{k+1} := P_{K_{\mathrm{e}}}^{(G^{-1})} P_{K_{\mathrm{t}}}^{(G^{-1})} \left\{ h_k + \lambda_k \mathscr{M}_k \left[\sum_{\iota\in\mathscr{I}_k} w_\iota^{(k)} P_{H_\iota^-(h_k)}^{(G^{-1})}(h_k) - h_k \right] \right\},$$

$\forall k \in \mathbb{N}$, where $\lambda_k \in [0,2]$ is the step size and

$$\mathscr{M}_k := \begin{cases} \dfrac{\displaystyle\sum_{\iota\in\mathscr{I}_k} w_\iota^{(k)} \left\| P_{H_\iota^-(h_k)}^{(G^{-1})}(h_k) - h_k \right\|_{G^{-1}}^2}{\left\| \displaystyle\sum_{\iota\in\mathscr{I}_k} w_\iota^{(k)} P_{H_\iota^-(h_k)}^{(G^{-1})}(h_k) - h_k \right\|_{G^{-1}}^2}, & \text{if } h_k \notin \displaystyle\bigcap_{\iota\in\mathscr{I}_k} H_\iota^-(h_k), \\ 1, & \text{otherwise.} \end{cases}$$

Algorithm 1 is endowed with great features of APSM (see Appendix A). A remark on complexity of Algorithm 1 is given below.

Remark 1 *(Overall complexity of Algorithm 1) In the update equation of Algorithm 1, each projection in the summation can be computed independently, thus the algorithm has the inherently parallel structure. In fact, the algorithm not only can be implemented with parallel processors but also has a fault tolerance nature; i.e., a trouble in one or some processors does not seriously affect the overall performance of the algorithm (which is not true for the other major adaptive algorithms).*

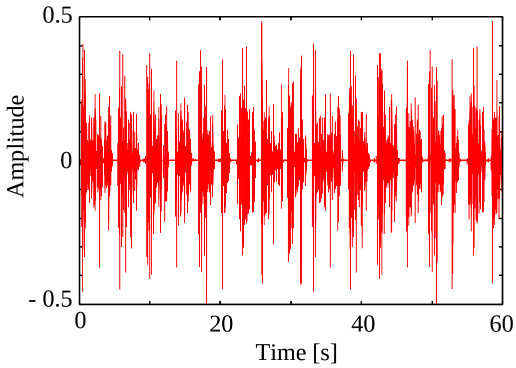
For $G = B$, with q concurrent processors, the order of the number of multiplications imposed on each processor at each iteration is approximately $(2r+4)N$ [19]*, which is the same as the adaptive PSP algorithm with the Euclidean metric* [12]*. The key to reduce the complexity is the following property: $a^T Ba = a_{(\mathrm{e})}^T a_{(\mathrm{e})} + \gamma^{N/2} a_{(\mathrm{t})}^T a_{(\mathrm{t})}$, $\forall a = [a_{(\mathrm{e})}^T, a_{(\mathrm{t})}^T]^T \in \mathscr{H}$. This implies that Algorithm 1 can significantly raise, by increasing q, convergence speed while keeping low time consumption, which is very important for real-time applications including AEC.*

4. NUMERICAL EXAMPLES

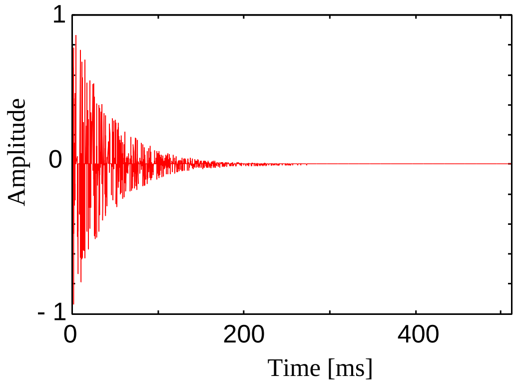
To verify the efficacy of the proposed AQ-PSP algorithm, simulations are performed with an English-native-male's speech signal recorded at sampling rate 8 kHz (see Fig. 2). To consider a noisy situation with a model mismatch, we use $h^* \in \mathbb{R}^{4096}$ and $h_k \in \mathbb{R}^{1024}$, $\forall k \in \mathbb{N}$, with Signal to Noise Ratio (SNR) $:= 10\log_{10}(E\{z_k^2\}/E\{n_k^2\}) = 10$ dB, where $z_k := u_k^T h^*$ denotes *pure* echo.

Throughout this section, $\|\cdot\|$ stands for the Euclidean norm of any size of vector. To measure the achievement level of echo path identification as well as that of echo cancellation, we evaluate the following two criteria: Echo Return Loss Enhancement (ERLE) [2] and system mismatch defined as $10\log_{10}(\|\widehat{h}^* - h_k\|^2/\|\widehat{h}^*\|^2)$ at kth iteration, where $\widehat{h}^* \in \mathbb{R}^{1024}$ is a sub-vector of h^* with its first 1024 components. To obtain smooth ERLE curves, after calculating $\mathrm{ERLE_{tmp}}\ (k) := 10\log_{10}[z_k^2/(z_k - u_k^T h_k)^2]$, $\forall k \in \mathbb{N}$, we pass it through three times a smoothing filter with length 20000. For numerical stability against poor excitation of the speech input signals, certain regularization and threshold are utilized for all the algorithms.

In Fig. 3, AQ-PSP is compared with the adaptive PSP algorithm [12] with the Euclidean metric, which will be referred to as *adaptive Euclidean-metric PSP (AE-PSP)*. For AQ-PSP and AE-PSP, we use the common setting; $r = 1$, $q = 8, 16$, $\rho = \rho_3 (= 0)$ (ρ_3: the peak value of the probability density function of the random variable $\xi := \|n_k\|^2$ [12]), $\lambda_k = 0.6$ and $w_\iota^{(k)} = 1/q$, $\forall k \in \mathbb{N}$. For AQ-PSP, we simply set $G = B$ (In this simulation, there is no mismatch in γ). Moreover, to examine the pure effect of the newly introduced metric, we do *not* use the constraint sets K_{e} and K_{t}, which corresponds



(a) Input signal



(b) Room impulse response

Figure 2: (a) The input signal and (b) the room impulse response used in the simulations.

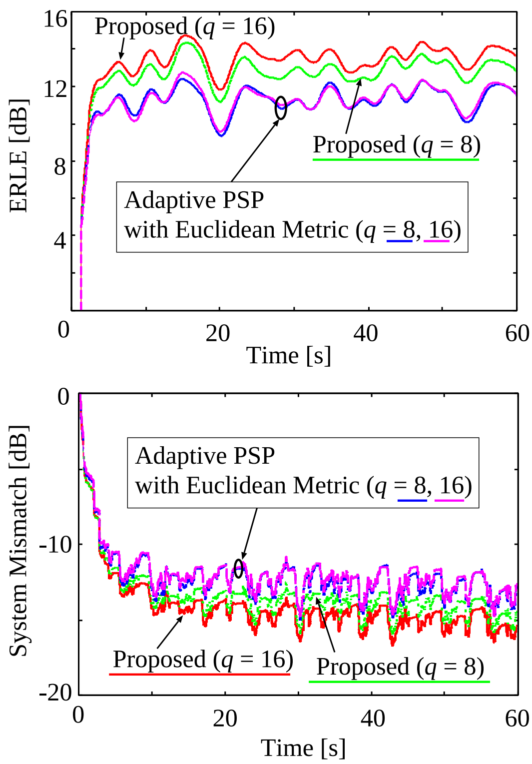


Figure 3: Proposed versus adaptive PSP algorithm with the Euclidean metric; i.e., $d(a,b) := d_I(a,b) = (a-b)^T(a-b)$. For the both algorithms, $q = 8, 16$ and $r = 1$. SNR = 10 dB.

to assigning very large values to ε_e and ε_t. Table 1 shows the steady state performance of AQ-PSP and AE-PSP, which is averaged over the last 10^5 samples (12.5 sec.).

Figure 4 draws a comparison of AQ-PSP with ESP [13] and the Proportionate NLMS[2] (PNLMS) [15, 16]. For AQ-PSP, the setting is the same as in Fig. 3 for $q = 16$. For ESP, we use (a) $r = 1$, $\lambda_k = 0.5$, $\forall k \in \mathbb{N}$, and (b) $r = 2$, $\lambda_k = 0.2$, $\forall k \in \mathbb{N}$. There is no mismatch in γ also for ESP. For PNLMS, we set $\lambda_k = 0.5$, $\forall k \in \mathbb{N}$. Table 2 shows the steady state performance of AQ-PSP, ESP and PNLMS. We see that the comparison in Figs. 3 and 4 is fair since the initial convergence speed in system mismatch is almost identical for all curves. The results are discussed below.

5. DISCUSSION AND CONCLUDING REMARKS

From Table 1, we see that AQ-PSP for $q = 16$ gains more than 2 dB compared with AE-PSP for $q = 16$. From Table 2, moreover, compared with ESP (b) and PNLMS, we see that AQ-PSP for $q = 16$ gains more than 4 dB in ERLE and almost 3 dB in system mismatch.

We conclude that the proposed algorithm has great advantages over the existing AEC algorithms even in highly noisy situations. We finally remark that the proposed algorithm can be extended to a time-varying metric (NOTE: PNLMS can be interpreted as a time-varying metric version of ESP), although the proof of several great features of APSM must further be considered in this case.

[2]PNLMS is based on a special structure of impulse responses like ESP, but the weighting matrix (A for ESP) is data-dependent and time-varying.

Table 1: Steady state performance of AQ-PSP for (a) $q = 16$ and (b) $q = 8$, and AE-PSP for (a) $q = 16$ and (b) $q = 8$ in ERLE and system mismatch.

Algorithm	AQ-a	AQ-b	AE-a	AE-b
ERLE	13.6	12.8	11.3	11.4
System Mismatch	−15.1	−14.3	−12.7	−12.8

Appendix A: Properties of (Extended) APSM

Scheme 1 has the following properties [5, 6].

(a) (Monotonicity)

$$\left\| h_{k+1} - h^{*(k)} \right\| \le \left\| h_k - h^{*(k)} \right\|, \ \forall k \in \mathbb{N},$$
$$\forall h^{*(k)} \in \Omega_k := \{h \in C : \Theta_k(h) = \inf_{x \in C} \Theta_k(x)\}.$$

(b) (Asymptotic minimization)
Suppose $(\Theta_k'(h_k))_{k\in\mathbb{N}}$ is bounded and $\exists N_0$ s.t. (i) $\inf_{x\in C}\Theta_k(x) = 0$, $\forall n \ge N_0$ and (ii) $\Omega := \bigcap_{k\ge N_0} \Omega_k \ne \emptyset$. Then, we have

$$\lim_{k\to\infty} \Theta_k(h_k) = 0.$$

Note that Θ_k' used to derive Algorithm 1 in Sec. 3.2 is automatically bounded [4].

(c) (Strong convergence)
Under some mild conditions, the sequence $(h_k)_{k\in\mathbb{N}}$ converges to a point $\widehat{h} \in T$.

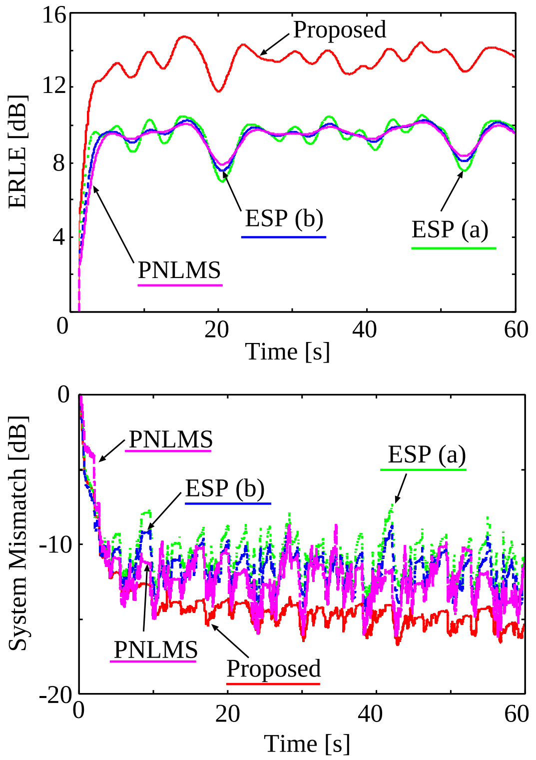


Figure 4: Proposed ($q = 16$, $r = 1$) versus ESP and Proportionate NLMS ($\lambda_k = 0.5$). For ESP, (a) $r = 1$, $\lambda_k = 0.5$ and (b) $r = 2$, $\lambda_k = 0.2$. SNR = 10 dB.

Table 2: Steady state performance of AQ-PSP for $q = 16$, ESP for (a) $r = 1$ and (b) $r = 2$, and Proportionate NLMS in ERLE and system mismatch.

Algorithm	AQ	ESP-a	ESP-b	PNLMS
ERLE	13.6	9.5	9.5	9.5
System Mismatch	−15.1	−11.0	−12.0	−12.4

Appendix B: Proof of Observation 1

First of all, $P_{V_k}^{(G^{-1})}(h)$ can be decomposed as

$$P_{V_k}^{(G^{-1})}(h) = P_{V_k}^{(G^{-1})}(0) - P_{M_k^{\perp(G^{-1})}}^{(G^{-1})}(h), \qquad \text{(B.1)}$$

where $M_k^{\perp(G^{-1})} := \{h \in \mathscr{H} : \langle h, x \rangle_{G^{-1}} = 0,\ \forall x \in M_k\}$ with $M_k := \{h \in \mathscr{H} : U_k^T h = 0\}$ being the translated subspace of V_k. It is not hard to see that $M_k^{\perp(G^{-1})} = \text{span}\{Gu_k, Gu_{k-1}, \cdots, Gu_{k-r+1}\}$, and that (see, e.g., [18])

$$P_{M_k^{\perp(G^{-1})}}^{(G^{-1})}(h) = GU_k(U_k^T GU_k)^{\dagger} U_k^T h_k. \qquad \text{(B.2)}$$

Moreover, by [18, Theorem2, p. 62], we obtain

$$P_{V_k}^{(G^{-1})}(0) = GU_k(U_k^T GU_k)^{\dagger} d_k,$$

which, with (B.1) and (B.2), yields (3).

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