MULTIUSER DETECTION USING RANDOM-SET THEORY

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ABSTRACT

In mobile multiple-access communications, not only the location of active users, but also their number varies with time. In typical analyses, multiuser detection theory has been developed under the assumption that the number of active users is constant and known at the receiver, and coincides with the maximum number of users entitled to access the system. This assumption is often overly pessimistic, since many users might be inactive at any given time, and detection under the assumption of a number of users larger than the real one may impair performance.

This paper assumes a dynamic environment where users are continuously entering and leaving the system, and undertakes a general approach to the problem of identifying active users and estimating their parameters and data. Our goal is to lay the foundation of multiuser detection theory in an environment where the number and the parameters of active users are unknown at the receiver, and in addition may change from one observation time to the next following a known dynamic model.

Using Random-Set Theory, we develop the tools that are needed for data detection in addition to parameter estimation, whereby a dynamic model for the evolution of parameters has been selected. Application of this theory allows Bayesian-filter equations to be written, which describe the evolution with time of the optimum causal multiuser detector.

We illustrate this theory through a simple example of application, consisting of the detection of the number and identity of active interferers and of the data they carry.

1. INTRODUCTION

We assume signal transmission over a common channel (specifically, we consider CDMA for simplicity's sake). Let $s(\mathbf{x}_t^{(0)})$ denote the signal transmitted by the reference user at discrete time t, $t = 1, 2, \ldots$, and $s(\mathbf{x}_t^{(i)})$, $i = 1, \ldots, K-1$, the signals that may be transmitted at the same time by $K-1$ interferers. Each signal has in it a number of known parameters, reflected by the known function $s(\cdot)$, and a number of random parameters, summarized by $\mathbf{x}_t^{(i)}$. The index i reflects the identity of the user, and is typically associated with its signature. The observed signal at time t is a sum of the users active at time t, and of a stationary random noise $\mathbf{z}_t$, i.e.:

$$\mathbf{y}_t = \sum_{\mathbf{x}_t^{(i)} \in \mathbf{X}_t} s(\mathbf{x}_t^{(i)}) + \mathbf{z}_t \tag{1}$$

where $\mathbf{X}_t$ is a random set.

The tool we use in our analysis is *random set theory* (RST). Using this tool, which was previously applied in the context of multitarget tracking and identification (see, e.g., [1–3, 5]), the whole set of users is modeled as a single entity. RST develops a probability theory over finite sets whose randomness is both in the number of their elements and in the values they take on. Since users, along with their parameters, are elements of a finite random set, RST provides a natural approach to multiuser detection in a dynamic environment. RST unifies in a single step two steps that would be taken separately without it, viz., detection of active users and estimation of their parameters. In the random set framework, the multiuser state is a set comprising a random number of single user states, each of which is a random vector. The basic problem amounts to computing the a posteriori density of this set-valued quantity.

A motivation for the development presented in this paper can be obtained by glancing over Fig. 2. This refers to a 3-user system, and compares two receivers. One does maximum-likelihood (ML) detection of users' data under the assumption that all interferers are active, while the other detects at the same time the number of interferers and the users' data. Several activity factors α are considered. It is seen that the latter receiver performs better whenever α is not close to one, and is more robust to the variations of α (whose value is assumed unknown to the receivers). A further improvement in performance can be obtained by providing the receiver with side information about the users' activity factor, and about their behavior in terms of appearance, disappearance, and movement from one observation interval to the next. The balance of this paper is devoted to showing how, using RST, this side information can be exploited by a receiver.

Random-set theory can be applied with only minimal (yet, nonzero) consideration of its theoretical foundations. Roughly speaking, a random set is a map $\mathbf{X}$ between a sample space (containing the outcomes of a random experiment) and a family of subsets of a space $\mathbb{S}$. This is the space of the unknown parameters of the active interferers. For example, we have $\mathbb{S} = \{0, \ldots, K-1\}$ if all parameters of the prospective users are known, except their number and their identities. Or we have $\mathbb{S} = \mathbb{R} \times \{0, \ldots, K-1\}$, $\mathbb{R}$ the set of real numbers, if one parameter (e.g., the interferer power) is also unknown in addition to the users' number and identities. We can also have $\mathbb{S} = \mathbb{R} \times \{0, \ldots, K-1\} \times \{\pm 1\}$ if the (binary antipodal) data are to be detected. In mathematical terms, $\mathbb{S}$ is generally a *hybrid space* $\mathbb{S} \triangleq \mathbb{R}^d \times U$, with U a finite discrete set and $d \geq 0$: in the rest of this paper we restrict our attention to the case $d = 0$, i.e. only the identities (and possibly the data) of the active users are unknown at the receiver end. It is worth noticing that in the above discussion it has been assumed that the reference user may be itself active or not: the situation that user ”0” is active with probability one, so that the other users represent intermittent interference, can be easily accounted for by modifying (1) as

$$\mathbf{y}_t = s(\mathbf{x}_t^{(0)}) + \sum_{\mathbf{x}_t^{(i)} \in \mathbf{X}_t} s(\mathbf{x}_t^{(i)}) + \mathbf{z}_t \tag{2}$$

where now the random set $\mathbf{X}_t$ varies in the space $\mathbb{S} = \{1,\ldots,K-1\}$ or $\mathbb{S} = \{1,\ldots,K-1\} \times \{\pm 1\}$ whether only user identities, or user identities and data are to be detected.

At the basis of RST is the concept of *belief function* of a random set $\mathbf{X}$. This is defined as

$$\beta_{\mathbf{X}}(\mathbf{C}) \triangleq \mathbb{P}(\mathbf{X} \subset \mathbf{C})$$

where $\mathbf{C}$ is a subset of an ordinary multiuser state space: $\mathbf{C} \subset \mathbb{S}$. The *density* of the belief function is defined as its “set derivative” (this is a generalized Radon-Nikodým derivative). Set derivatives can be computed by using an RST “toolbox,” and the resulting densities carry, from a practical viewpoint, the relevant properties of standard density functions of probability theory [3].

The ingredients necessary for application of RST to multiuser detection are the following:

① The belief density $f(\mathbf{y}_t \mid \mathbf{X}_t)$. This follows from the channel model and the measuring method.

② The belief density $f(\mathbf{X}_t \mid \mathbf{X}_{t-1})$. This follows from the model of the dynamics of the set of active users. The main assumption here is that $\{\mathbf{X}_t\}_{t=1}^{\infty}$ forms a random set sequence with the Markov property, i.e., such that $\mathbf{X}_t$ depends on its past only through $\mathbf{X}_{t-1}$.

Once the above densities are made available, they are used in the *Bayesian filter recursions*

$$\begin{aligned} & f(\mathbf{X}_t \mid \mathbf{y}_{1:t-1}) \\ & \quad = \int f(\mathbf{X}_t \mid \mathbf{X}_{t-1})\, f(\mathbf{X}_{t-1} \mid \mathbf{y}_{1:t-1})\, \delta\mathbf{X}_{t-1} \\ & f(\mathbf{X}_t \mid \mathbf{y}_{1:t}) \\ & \quad \propto f(\mathbf{y}_t \mid \mathbf{X}_t)\, f(\mathbf{X}_t \mid \mathbf{y}_{1:t-1}) \end{aligned} \tag{3}$$

which allow one to generate recursively the estimates of $\mathbf{X}_t$, for example in the form

$$\widehat{\mathbf{X}}_t = \arg\max_{\mathbf{X}_t} f(\mathbf{X}_t \mid \mathbf{y}_{1:t})$$

The integral in (3) is a “set integral,” the inverse of the set derivative.

With our channel model, the receiver detects only a superposition of interfering signals. Thus, the random set describing the receiver, denoted $\mathbf{y}_t$, has conditional density function

$$f(\mathbf{y}_t \mid \mathbf{X}_t) = f_{\mathbf{z}}(\mathbf{y}_t - \sigma(\mathbf{X}_t)) \tag{4}$$

where $f_{\mathbf{z}}(\cdot)$ is the density function of the additive noise, and

$$\sigma(\mathbf{X}_t) \triangleq \sum_{\mathbf{x}_t^{(i)} \in \mathbf{X}_t} s(\mathbf{x}_t^{(i)}) \tag{5}$$

Assuming the noise to be Gaussian, we have

$$f_z(\mathbf{y}_t - \sigma(\mathbf{X}_t)) \propto \exp\{-\|\mathbf{y}_t - \sigma(\mathbf{X}_t)\|^2/N_0\}$$

Thus, the estimate of $\mathbf{X}_t$ is performed by minimizing, over $\mathbf{X}_t$, the function

$$m(\mathbf{X}_t) \triangleq \|\mathbf{y}_t - \sigma(\mathbf{X}_t)\|^2 - \varepsilon(\mathbf{X}_t)$$

where $\varepsilon(\mathbf{X}_t) \triangleq N_0 \ln f(\mathbf{X}_t \mid \mathbf{y}_{1:t-1})$. The first term is the Euclidean distance between the observation and the sum of the interfering signals. This alone would be used in ML detection. The second term in the RHS of the above can be viewed as a correction term, coming from the uppermost step of iterations, and reflecting the influence on $\mathbf{X}_t$ of its past history. This plays the role of a priori information to be used in maximum a posteriori decisions, and its consideration and evaluation is the main point of this paper.

2. EXAMPLE OF APPLICATION: DETECTION OF ACTIVE USERS

Assume now the specific situation of a DS-CDMA system with signature sequences of length L and additive white Gaussian noise. At discrete time t, we may write, for the sufficient statistics of the received signal,

$$\mathbf{y}_t = \mathbf{R}\mathbf{A}\mathbf{b}_t(\mathbf{X}_t) + \mathbf{z}_t, \qquad t = 1,\ldots,T \tag{6}$$

where $\mathbf{X}_t$ is now the random set of all active users, $\mathbf{R}$ is the $L \times L$ correlation matrix of the signature sequences (assumed to have unit norm), $\mathbf{A}$ is the diagonal matrix of the users’ signal amplitudes, the vector $\mathbf{b}_t(\mathbf{X}_t)$ has nonzero entries in the locations corresponding to the active-user identities described by the components of $\mathbf{X}_t$, and $\mathbf{z}_t \sim \mathcal{N}(0,(N_0/2)\mathbf{R})$ is the noise vector, with $N_0/2$ the power spectral density of the received noise.

Throughout this paper we assume that the only unknown signal quantities may be the identities of the users and their data. Specifically, we may distinguish four cases in our context:

① *Static channel, unknown identities, known data.* This corresponds to a training phase intended at identifying users, and assumes that the user identities do not change during transmission. In this case we write $\mathbf{X}$ in lieu of $\mathbf{X}_t$.

② *Static channel, unknown identities, unknown data.* This may correspond to a tracking phase following ① above. We write again $\mathbf{X}$ in lieu of $\mathbf{X}_t$, and assume that $\mathbf{X}$ contains the whole transmitted data sequence.

③ *Dynamic channel, unknown identities, known data.* This corresponds to identification of users preliminary to data detection (which, for example, may be based on decorrelation).

④ *Dynamic channel, unknown identities, unknown data.* This corresponds to simultaneous user identification and data detection in a time-varying environment.

If we assume that, at every discrete time instant, only one binary antipodal symbol is transmitted, trained acquisition corresponds to the transmission of known bit streams, and hence to $\mathbb{S} = \{0,\ldots,K-1\}$, while in untrained acquisition user identification and data detection should be performed jointly, which corresponds to $\mathbb{S} = \{0,\ldots,K-1\} \times \{\pm 1\}$.

Consider now the construction of a dynamic model for $\mathbf{X}_t$. We assume that from $t-1$ to t some new users become active and some old users become inactive. We write

$$\mathbf{X}_t = \mathbf{S}_t \cup \mathbf{N}_t \tag{7}$$

where $\mathbf{S}_t$ is the set of *surviving* users still active from $t-1$, and $\mathbf{N}_t$ is the set of *new* users becoming active at t. The condition $\mathbf{X}_{t-1} \cap \mathbf{N}_t = \emptyset$ is forced, i.e., a user ceasing transmission at time $t-1$ cannot re-enter the set of active users at time t.

For the sake of clarity, here we limit ourselves to the construction of a dynamic model for $\mathbf{X}_t$ with trained acquisition. Suppose that there are n active users at $t-1$, with $\mathbf{X}_{t-1} = \{\mathbf{x}_{t-1}^{(1)}, \ldots, \mathbf{x}_{t-1}^{(n)}\}$. Then we may write, for the set of surviving users,

$$\mathbf{S}_t = \bigcup_{i=0}^{K-1} \mathbf{X}_t^{(i)} \tag{8}$$

where $\mathbf{X}_t^{(i)}$ denotes either an empty set (a user has become inactive) or the singleton $\{\mathbf{x}_t^{(i)}\}$. Let μ denote the "persistence" probability, i.e., the probability that a user survives from $t-1$ to t. We obtain, for the conditional density of $\mathbf{S}_t$ given that $\mathbf{X}_{t-1} = \mathbf{B}$:

$$f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) = \mu^{|\mathbf{C}|}(1-\mu)^{|\mathbf{B}|-|\mathbf{C}|}, \quad \mathbf{C} \subseteq \mathbf{B} \tag{9}$$

while it is 0 otherwise

For new users, we assume again a binomial birth process with parameter α. Since K is the maximum user number, we have:

$$f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) = \alpha^{|\mathbf{C}|}(1-\alpha)^{K-|\mathbf{C}|-|\mathbf{B}|}, \mathbf{C} \cap \mathbf{B} = \emptyset \tag{10}$$

Finally, assuming that births and deaths of users are conditionally independent given $\mathbf{X}_{t-1}$, the generalized convolution operation ruling the pdf of the union of independent random sets [1] becomes, under our assumptions:

$$f_{\mathbf{X}_t|\mathbf{X}_{t-1}}(\mathbf{C} \mid \mathbf{B}) \tag{11}$$
$$= \sum_{\mathbf{W} \subseteq \mathbf{C}} f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{W} \mid \mathbf{B})\, f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \setminus \mathbf{W} \mid \mathbf{B})$$
$$= f_{\mathbf{S}_t|\mathbf{X}_{t-1}}(\mathbf{C} \cap \mathbf{B}) f_{\mathbf{N}_t|\mathbf{X}_{t-1}}(\mathbf{C} \setminus (\mathbf{C} \cap \mathbf{B})) \tag{12}$$

Untrained acquisition can be dealt with similarly, provided that the conditional densities in (9)-(10) are multiplied by a factor depending on $|\mathbf{C}|$ to account for the data priors.

Two alternative strategies can at this point be conceived to estimate $\mathbf{X}_t$, whether trained or untrained situations are considered. The former one relies upon implementing the Bayes recursions outlined above, thus defining a causal set-sequence estimator. An alternative approach could be to consider the likelihood of the set sequence $\mathbf{X}_1, \ldots, \mathbf{X}_T$, i.e.:

$$f(\mathbf{X}_1, \ldots, \mathbf{X}_T \mid \mathbf{y}_{1:T}) \tag{13}$$
$$\propto f(\mathbf{y}_{1:T} \mid \mathbf{X}_1, \ldots, \mathbf{X}_T) f(\mathbf{X}_1) \prod_{t=2}^{T} f(\mathbf{X}_t \mid \mathbf{X}_{t-1})$$

where the Markov nature of the model has been exploited [1]. Since under both trained and untrained acquisition the unknown sets may take on a finite number of configurations, maximizing (13) simply amounts to determining a maximum-metric path in a trellis of depth T: the state space has cardinality 2^K for trained acquisition, and 3^K for untrained acquisition, and the maximization can be undertaken in both cases through a Viterbi algorithm.

[1]Implicit in the above is the need of assigning a density $f(\mathbf{X}_1)$, which obviously depends on the prior information as to the channel state at the beginning of the transmission.

3. RESULTS

We first illustrate the advantages of joint channel sensing and user demodulation by considering the case of a static CDMA channel with 7 users and processing gain 7; it is further assumed that user "1" is active with probability one, while the other users may be active or not, the number of active interferers being a uniform discrete random variable. Fig. 1 shows the bit error probability of a random-set-based detector. For comparison purposes, we also show the performance of a classical ML receiver assuming that all of the users are active, and the single-user bound: for all receivers, the spreading codes are $m-$sequences. The plots show that joint channel sensing and data demodulation prevents the performance impairment incurred by traditional multiuser systems under unknown channel occupancy.

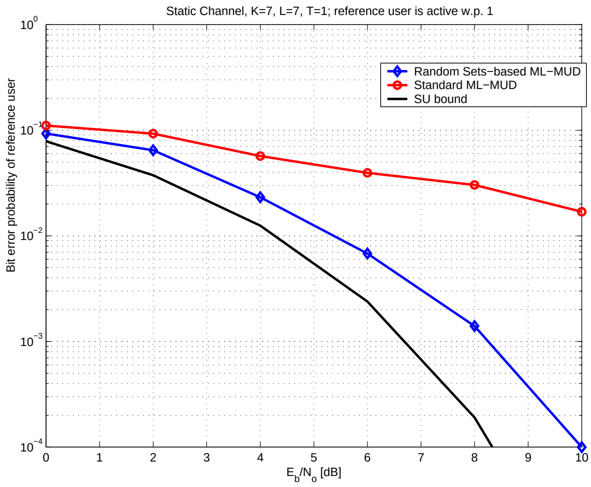

Figure 1: *Bit error probability of the reference user in a multiuser system with* 3 *users, independently active with probability* α.

Fig. 2 compares again "classic" ML multiuser detection [4], which assumes that all users are simultaneously active, and ML detection based on RST, which detects simultaneously the number of active users and the data of the reference user. The ordinate shows the bit error probability of the reference user in a system with 3 users transmitting binary antipodal signals, different active-user probabilities ($\alpha = 0.1$, 0.5, and 1), spreading sequences consisting of Kasami sequences with length 15, and perfect power control (and hence equal received powers from all users). The channel is Gaussian and static. The single-user bound is also shown as a reference. This figure was commented upon in Section 1.

We next consider the situation of a dynamic channel where both the user identities and their data are to be estimated. In this new situation no user is active with probability one, and the common persistence probability is $\mu = 0.8$, while $\alpha = 0.2$. We also assume a frame of $T = 10$ signaling intervals, and we consider the following situations: a) Estimation of the set $\mathbf{X}_1$, under both noncausal ("Viterbi") and causal ("Bayes") strategies; b) Estimation of the set $\mathbf{X}_{10}$, under both non-causal and causal strategies. The quantity on the vertical axis is the "bit sequence probability," i.e., the probability that at some t the estimated and the true bit stream

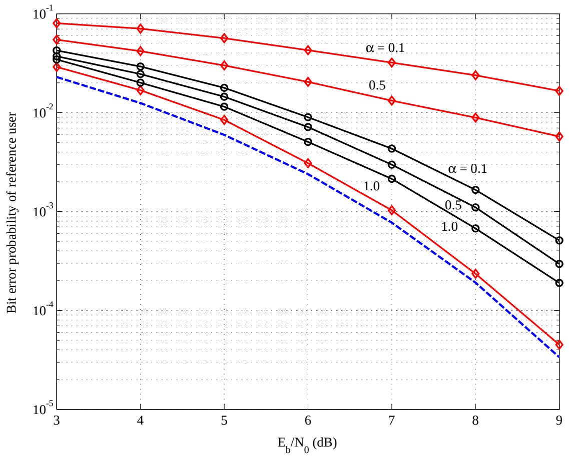

Figure 2: *Bit error probability of the reference user in a multiuser system with* 3 *users, independently active with probability* α. *Lines with diamond markers: Classic multiuser ML detection, assuming that all users are active. Line with circle markers: ML detection using RST. Dashed curve: Single-user bound.*

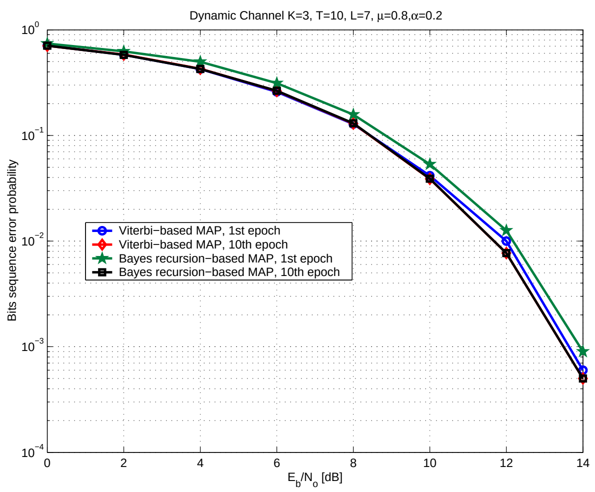

Figure 3: *Dynamic Channel.*

(transmitted by the active users, whose identities are in turn unknown) do not coincide. Notice that the causality constraint has some perceivable, yet minor, effect at epoch $t = 1$, while at time $t = 10$ the two algorithms yield equivalent performances, as they should. Additionally, some theoretical developments, not shown here for want of space, show that trained and untrained systems achieve close performances under a dynamic scenario.

4. CONCLUSIONS

We have described a technique for estimating the received-signal parameters in a CDMA system. Since the number of active interferers is itself a random variable, the set of parameters to be estimated has a random number of random elements. We have used a probability theory, called random-set theory, to develop multiuser detection tools under these conditions. In addition, we have developed Bayes-filtering equations that describe the evolution of the multiuser detector in a dynamic environment, and evaluated its performance also in comparison to a more traditional set-sequence estimator.

Our results show how joint channel sensing and user demodulation can be advantageous whenever the number of active users is unknown a priori. In addition, dynamic modeling of users' activity can provide further benefits. Thus, RST appears as a most promising tool to achieve fully adaptive receivers.

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APPENDIX

This appendix describes, mostly in a qualitative fashion, the fundamentals of Random-Set Theory. For a rigorous approach and for additional details, see [1–3, 5].

A *finite random set* is a mapping $\mathbf{X} : \Omega \to \mathcal{F}(\mathbb{S})$ from the sample space Ω to the collection of closed sets of the space $\mathbb{S}$, with $|\mathbf{X}(\omega)| < \infty$ for all $\omega \in \Omega$. Here the space $\mathbb{S}$ of finite random sets is assumed to be the *hybrid space* $\mathbb{S} = \mathbb{R}^d \times U$, the direct product of the d-dimensional Euclidean space $\mathbb{R}^d$ and a finite discrete space U. The elements of $\mathbb{S}$ characterize the users' parameters, which we categorize as continuous (d real numbers) and discrete (for example, the users' signatures and their information data). An element of $\mathbb{S}$ is the pair $(\mathbf{v}, u)$, $\mathbf{v}$ a d-dimensional real vector, and $u \in U$. The space $\mathbb{S}$ is endowed with a topology obtained as the product of the Euclidean topology in $\mathbb{R}^d$ and the discrete topology in U.

The *belief function* of a finite random set $\mathbf{X}$ is defined as

$$\beta_{\mathbf{X}}(C) \triangleq \mathbb{P}(\mathbf{X} \subseteq C) \tag{14}$$

where C is a closed subset of $\mathbb{S}$. The belief function characterizes the probability distribution of a random finite set $\mathbf{X}$, and allows the construction of a density function of $\mathbf{X}$ through the definition of a *set integral* and a *set derivative*.

Let $\mathcal{C}(\mathbb{S})$ denote the collection of closed subsets of $\mathbb{S}$. The set derivative of a set function $F : \mathcal{C}(\mathbb{S}) \to [0,\infty)$ at a point $\mathbf{x} \in \mathbb{S}$ is defined as

$$\frac{\delta F}{\delta \mathbf{x}}(S) \triangleq \lim_{\bar{m}(\Delta_{\mathbf{x}})} \frac{F(S \cup \Delta_{\mathbf{x}}) - F(S)}{\bar{m}(\Delta_{\mathbf{x}})}$$

where $\bar{m}(\cdot)$ denotes the hybrid Lebesgue measure, i.e. the product of the ordinary measure in $\mathbb{R}^d$ and of the counting measure. Thus, the belief density of the random set $\mathbf{X}$ is given by

$$f_{\mathbf{X}}(X) = \frac{\delta \beta_{\mathbf{X}}}{\delta X}(\emptyset) \tag{15}$$

Let f denote a function defined by

$$f(X) = \frac{\delta F}{\delta X}(\emptyset)$$

The set integral of f over the closed subset $S \subseteq \mathbb{S}$ is given by

$$\int_S f(X)\,\delta X = \tag{16}$$
$$f(\{\emptyset\}) + \sum_{k=1}^{\infty} \frac{1}{k!} \int_{S^k} f(\{\mathbf{x}_1,\ldots,\mathbf{x}_k\}) d\bar{m}(\mathbf{x}_1)\cdots d\bar{m}(\mathbf{x}_k)$$

where $f(\{\mathbf{x}_1,\ldots,\mathbf{x}_k\}) = 0$ if $\mathbf{x}_1,\ldots,\mathbf{x}_k$ are not distinct (and hence the set has less than k elements). Since we are dealing with *finite* random sets, the summation above contains only a finite number of terms.

The special case $d = 0$ (which corresponds to making $\mathbb{S}$ a discrete finite set) reduces the set integral to

$$f(\{\emptyset\}) + \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{\{\mathbf{x}_1 \neq \mathbf{x}_2 \ldots, \neq \mathbf{x}_k\} \subseteq S^k} f(\{\mathbf{x}_1,\ldots,\mathbf{x}_k\}) \tag{17}$$

since in this case the hybrid Lebesgue measure reduces to the counting measure, and the Lebesgue integrals in (16) become summations.

Set derivatives and set integrals turn out to be the inverse of each other. The following generalized fundamental theorem of calculus holds:

$$f(X) = \frac{\delta F}{\delta X}(\emptyset) \Longleftrightarrow F(S) = \int_S f(X)\,\delta(X) \tag{18}$$

By using the above result, belief functions and belief densities can be derived from one another.