

# CODE-AIDED FREQUENCY AMBIGUITY RESOLUTION AND CHANNEL ESTIMATION FOR MIMO OFDM SYSTEMS

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## ABSTRACT

This contribution deals with channel estimation and frequency ambiguity resolution in a MIMO OFDM context. Existing blind frequency-recovery algorithms for OFDM are able to provide a reliable estimate of the frequency offset up to an integer multiple of the subcarrier spacing. To resolve the remaining ambiguity, one can employ either pilot symbols or the unknown coded data symbols. Clearly, the latter method results in a higher bandwidth efficiency. Similar considerations hold for the estimation of a frequency-selective MIMO channel. In this contribution, we propose a code-aided technique to jointly estimate the channel and resolve the frequency ambiguity. The estimator is based on the expectation-maximization (EM) algorithm and exploits information from the unknown coded data symbols and only a small number of pilot symbols. A significant performance gain is observed compared to existing, solely pilot-based estimation techniques.

## 1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) for multiple-input multiple-output (MIMO) systems is promising due to its potential of achieving high data rates over frequency selective channels. Most receiver schemes are designed with the assumption that channel state information (CSI) is available. Estimation of the channel state in a MIMO OFDM context has been the topic of extensive research during the last few years. We can categorize the existing channel estimators into two categories: the so-called *data-aided* estimators that are based solely on the presence of pilot (or training) symbols within the space-time-frequency grid [1,2], and the so-called *code-aided* estimators exploiting (additional) information from the information-bearing coded data symbols. Typically, code-aided estimators are based on some iterative approach, where information between the detector and channel estimator is exchanged. The most convenient tool in this respect is the iterative expectation-maximization (EM) algorithm [3], and variations thereof. In the context of MIMO OFDM, we mention the following EM-based references: In [4] the EM algorithm was applied for CSI estimation in the context of uncoded systems with transmit diversity and in [5–7] a more general approach was taken for Space-Time Block Coded (STBC) systems.

Another important issue concerning OFDM is its sensitivity to carrier frequency offsets. An uncompensated frequency offset destroys the orthogonality between the dif-

ferent subcarriers, which deteriorates the detection performance. This makes frequency recovery a crucial task. Both data aided [8,9] and blind frequency synchronizers [10,11] have been proposed in the technical literature. Most of these estimators can only recover the fractional part of the normalized frequency offset. What remains is an integer ambiguity that needs to be resolved. In [9], a training-based scheme is used to resolve the frequency ambiguity (FA), while in [12], a FA resolver is proposed that exploits the presence of null subcarriers. All these estimators were designed for uncoded single-input single-output (SISO) systems and none of these estimators are sufficiently accurate at the low SNR values at which current state-of-the-art coding schemes operate. Hence, a code-aided estimator is called-upon.

As mentioned above, the EM algorithm is well-suited for the estimation of continuous parameters, such as the channel impulse response. For the estimation of a discrete parameter, such as the FA, a modification of the EM algorithm is required in order to avoid convergence problems. In [13,14], an extension to the EM algorithm is proposed to deal with phase and/or timing ambiguities in an AWGN context. In the present contribution, we adopt these concepts to derive a code-aided joint FA resolver and channel estimator.

**Notations:** Vectors will be underlined, while matrices will be represented in bold. For instance,  $\mathbf{I}_K$  denotes the  $K \times K$  identity matrix, while  $\underline{0}_K$  is a vector consisting of  $K$  zeros. Time-domain and frequency-domain quantities will be written in lower-case letters and capitals, respectively. That way the FFT of  $\underline{x}$  will be denoted  $\underline{X}$ . The operation  $\text{circ}^N(\underline{x})$  converts a length- $K$  vector  $\underline{x}$  (with  $K \leq N$ ) to a circular  $N \times N$  matrix with first column equal to  $[\underline{x}^T, \underline{0}_{N-K}^T]^T$ . The operation  $\text{shift}_\gamma(\underline{x})$  performs a  $\gamma$ -size cyclic shift on the length- $N$  vector  $\underline{x}$ , i.e.,  $\text{shift}_\gamma(\underline{x})$  corresponds to the vector obtained by taking the  $(\gamma + 1)$ <sup>th</sup> column of  $\text{circ}^N(\underline{x})$ . The operation  $\text{diag}(\underline{x})$  converts a length- $N$  vector to a  $N \times N$  diagonal matrix with  $\underline{x}$  on the diagonal. Finally, we introduce a shorthand notation for a set (containing vectors/matrices/...), e.g.,  $\underline{x}_{\{1:N\}} = \{\underline{x}_1, \dots, \underline{x}_N\}$  or  $\mathbf{X}^{\{1:N\}} = \{\mathbf{X}^{(1)}, \dots, \mathbf{X}^{(N)}\}$ .

## 2. SYSTEM MODEL

The transceiver is depicted in Fig. 1: a vector of information bits is encoded, interleaved and mapped onto a sequence of complex symbols, belonging to a unit energy signalling constellation  $\Omega$ . The coded symbols are then multiplexed over  $N_T$  transmit branches and the resulting  $N$  coded symbols on the  $m$ -th branch are denoted  $\underline{X}^{(m)} \in \Omega^N$ ,  $m = 1, \dots, N_T$ . This sequence is transformed by an  $N$ -point inverse FFT, yielding

$$\underline{x}^{(m)} = \mathbf{F}\underline{X}^{(m)} \quad (1)$$

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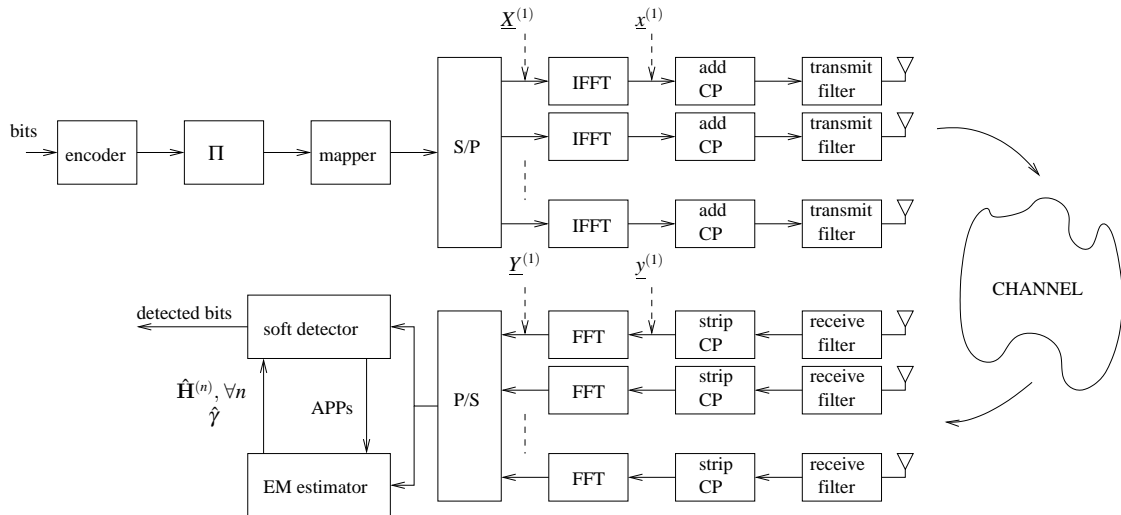


Figure 1: MIMO OFDM transceiver.

where  $\mathbf{F}$  is an  $N \times N$  matrix with  $F_{p,q} = \frac{1}{\sqrt{N}} e^{j2\pi pq/N}$ . A cyclic prefix (CP) of length  $N_g$  is added in order to avoid that the dispersive channel causes interference between successively transmitted OFDM symbols. The resulting  $N + N_g$  samples are applied (at a signaling rate  $1/T_s$ ) to a transmit filter and fed to the  $m$ -th transmit antenna.

The signal captured by the  $n^{\text{th}}$  ( $n = 1, \dots, N_R$ ) receive antenna is demodulated (using the same oscillator for the all receive antennas) and applied to a filter that is matched to the transmit filter. We adopt the common assumptions that all transmit/receive antenna pairs are affected by the same frequency offset [11] and that channel variations are negligible within one OFDM symbol. The CP is removed and an FFT is applied, resulting in the following  $N \times 1$  vector

$$\underline{Y}^{(n)} = \mathbf{F}^H \sum_{m=1}^{N_T} \Lambda(\Delta f N T_s) \text{circ}^N \left( \underline{h}^{(n,m)} \right) \underline{x}^{(m)} + \underline{W}^{(n)} \quad (2)$$

where the  $L \times 1$  vector  $\underline{h}^{(n,m)}$  denotes the sampled impulse response (including transmit filter, physical channel and receive filter) between the  $m^{\text{th}}$  transmit- and  $n^{\text{th}}$  receive antenna,  $\Lambda(x) \doteq \text{diag} \left( [1, e^{j2\pi \frac{x}{N}}, \dots, e^{j2\pi(N-1) \frac{x}{N}}]^T \right)$  and  $\Delta f N T_s$  denotes the normalized (w.r.t. the subcarrier spacing) frequency offset. The  $N \times 1$  vector  $\underline{W}^{(n)}$  contains independent white Gaussian noise samples with variance  $\sigma^2$  per real dimension. We decompose the normalized frequency offset as follows

$$\Delta f N T_s = \gamma + \nu, \quad (3)$$

where  $\gamma$  is an integer and the fractional part  $\nu$  is restrained to  $|\nu| < 0.5$ . In this paper, we will assume that the fractional part has been recovered (e.g., using the blind estimators from [10, 11]) and compensated for, such that  $\nu = 0$ . This allows us to rewrite (2) as

$$\begin{aligned} \underline{Y}^{(n)} &= \sum_{m=1}^{N_T} \mathbf{F}^H \Lambda(\gamma) \mathbf{F} \text{diag} \left( \underline{H}^{(n,m)} \right) \underline{X}^{(m)} + \underline{W}^{(n)} \\ &= \sum_{m=1}^{N_T} \text{shift}_{\gamma} \left( \text{diag} \left( \underline{X}^{(m)} \right) \underline{H}^{(n,m)} \right) + \underline{W}^{(n)}. \end{aligned} \quad (4)$$

where the  $N \times 1$  vector  $\underline{H}^{(n,m)}$  denotes the sampled channel frequency response between the  $m^{\text{th}}$  transmit- and  $n^{\text{th}}$  receive antenna, and is defined as

$$\underline{H}^{(n,m)} = \mathbf{G}^H \underline{h}^{(n,m)},$$

with  $\mathbf{G}$  an  $L \times N$  matrix containing the first  $L$  rows of  $\mathbf{F}$  multiplied by  $\sqrt{N}$  such that  $G_{p,q} = e^{j2\pi pq/N}$ . Considering a particular subcarrier  $k$ , and defining the  $N_R \times 1$  vector  $\underline{Y}_k = [y_k^{(1)}, \dots, y_k^{(N_R)}]^T$ , we can write

$$\underline{Y}_k = \mathbf{H}_{|k+\gamma|_N} \underline{X}_{|k+\gamma|_N} + \underline{W}_k \quad (5)$$

where  $|\cdot|_N$  denotes the modulo- $N$  operator, and where  $\mathbf{H}_k$  denotes the  $N_R \times N_T$  channel matrix corresponding to subcarrier  $k$ , i.e., the  $n, m$ -th entry of  $\mathbf{H}_k$  corresponds to the  $k$ -th entry of  $\underline{H}^{(n,m)}$ . Similarly, the  $N_T \times 1$  vector  $\underline{X}_k$  contains the  $k$ -th subcarrier frequency domain signals transmitted from the different transmit antennas. It is apparent from (5), that the FA gives rise to a cyclic shift of  $\gamma$  subcarriers in the frequency domain.

### Data detection

As is done in many state-of-the-art error-correcting coding schemes, we apply a 'soft' detector that computes a posteriori probabilities (APPs) of the coded bits and symbol vectors. This may be achieved as follows: in order to perform soft demapping and decoding (jointly: soft detection), the variables  $\underline{Y}_k$  need to be converted to probabilities. We denote the symbol vector likelihood sent from the FFT block to the soft detector by  $\{p^{(1)}(\underline{X}_k)\}$ ,  $k = 0, \dots, N-1$ , with

$$p^{(1)}(\underline{X}_k = \underline{\omega}) = C \exp \left( -\frac{1}{2\sigma^2} \left| \underline{Y}_{|k-\gamma|_N} - \mathbf{H}_k \underline{\omega} \right|^2 \right) \quad (6)$$

where  $\underline{\omega}$  is a  $N_T \times 1$  vector which elements belong to the signaling constellation  $\Omega$  and  $C$  is a normalizing constant. These symbol vector likelihoods are then converted to bit likelihoods and fed to an (iterative) soft-in soft-out detector. The latter returns the APPs of the coded bits and coded

(frequency-domain) symbol vectors. The symbol vector APPs are given by

$$p(\underline{X}_k | \underline{Y}_{\{1:N\}}, \mathbf{H}_{\{1:N\}}, \gamma) = C' p^{(1)}(\underline{X}_k) p^{(2)}(\underline{X}_k) \quad (7)$$

for some normalizing constant  $C'$ . In turbo-processing parlance,  $p^{(2)}(\underline{X}_k)$  is the *extrinsic* probability of symbol vector  $\underline{X}_k$ , as computed within the soft detector. Many other coded schemes, e.g. based on space-time or space-frequency block codes, allow a similar computation of the symbol vector APPs. In any case, it is clear that the detector requires knowledge of both the channel and the frequency shift  $\gamma$  in order to evaluate (7).

### 3. CODE-AIDED ESTIMATION USING THE EM ALGORITHM

We resort to the EM algorithm [3] to tackle our estimation problem. We start with a brief outline of the EM algorithm and the so-called *discrete* EM algorithm. As we will elaborate, the latter is required to deal with discrete parameters.

#### 3.1 EM algorithm - principle

The EM algorithm is an *iterative* technique to acquire the ML estimates when evaluation of the likelihood is cumbersome due to some unknown data. The original ML problem involves the estimation of a parameter(set)  $\theta$  from an observation  $r$ , by maximizing the likelihood function  $p(r|\theta)$ . In the presence of unknown data (e.g. unknown transmitted symbols), finding the ML solution can be very difficult. The main idea behind the EM algorithm is to define so-called missing (or unobserved) data  $a$ , such that, if the missing data were known, estimating  $\theta$  would be easy, i.e. maximizing  $p(r|\theta, a)$  is feasible. However, since we do not know the missing data, an iterative approach is called upon. We denote the iteration index by  $i$ . Starting from an initial estimate  $\hat{\theta}(0)$ , we iteratively apply the following two steps:

1. E-step:

$$Q(\theta | \hat{\theta}(i)) = \int \log p(r | \theta, a) p(a | r, \hat{\theta}(i)) da \quad (8)$$

2. M-step:

$$\hat{\theta}(i+1) = \arg \max_{\theta} Q(\theta | \hat{\theta}(i)). \quad (9)$$

The EM algorithm terminates when the estimate has converged or a certain stopping criterion has been met. We denote the final estimate by  $\hat{\theta}(+\infty)$ . It can be shown that for continuous parameters the final estimate  $\hat{\theta}(+\infty)$  equals the ML estimate under some mild restrictions w.r.t. the likelihood function and as long as the initial estimate is sufficiently accurate. When dealing with discrete parameters, convergence to the ML estimate is no longer guaranteed [15].

#### 3.2 Discrete EM algorithm - principle

To avoid the convergence problems of the EM algorithm associated with discrete parameters, we adopt the modification proposed in [13, 14]. The idea is to split the parameter set  $\theta$  into a discrete subset  $\theta^d$  and continuous subset  $\theta^c$ , i.e.  $\theta = [\theta^c, \theta^d]$ . Assume that  $\theta^d$  can take on values from a finite set  $S$ . We then evaluate the EM-algorithm for a fixed value of  $\theta^d$ , say  $\theta^d = \hat{\theta}^d$ . The M-step becomes:

$$\hat{\theta}^c(i+1; \hat{\theta}^d) = \arg \max_{\theta^c} Q([\theta^c, \hat{\theta}^d] | [\hat{\theta}^c(i; \hat{\theta}^d), \hat{\theta}^d]). \quad (10)$$

where  $Q(\cdot)$  (E-step) is defined in (8). After performing  $I_d$  iterations between the E-step and M-step, we obtain  $\hat{\theta}^c(I_d; \hat{\theta}^d)$ . We repeat the previous steps for all possible values of  $\theta^d$ . Afterwards, we make a decision w.r.t. the discrete parameter:

$$\hat{\theta}^d = \arg \max_{\theta^d} Q([\hat{\theta}^c(I_d; \theta^d), \theta^d] | [\hat{\theta}^c(I_d; \theta^d), \theta^d]). \quad (11)$$

The discrete subset of  $\theta$  has now been resolved. To further refine the estimates related to the continuous subset, we may perform some additional iterations. At the end, the final estimate of the parameter  $\theta$  is given by:

$$\hat{\theta}(+\infty) = [\hat{\theta}^d, \hat{\theta}^c(+\infty; \hat{\theta}^d)] \quad (12)$$

where  $\hat{\theta}^d$  is was found through (11).

#### 3.3 Joint channel estimation and frequency ambiguity resolution

Let us now turn to our original problem: the estimation of the channel coefficients  $\mathbf{H}_{\{1:N\}}$  and the FA  $\gamma$ . Due to the invariance property of ML estimation, the ML estimate of the channel frequency response can be computed from the corresponding ML estimate of the channel impulse response

$$\hat{\underline{H}}_{ML}^{(n,m)} = \mathbf{G}^H \hat{\underline{h}}_{ML}^{(n,m)}. \quad (13)$$

where  $\mathbf{G}$  is defined in section 2. From now on we focus on the estimation of the channel impulse response  $\underline{h}^{(n,m)}$ . Introducing the  $N_T L \times 1$  vector  $\underline{h}^{(n)} = [(\underline{h}^{(n,1)})^T, \dots, (\underline{h}^{(n,N_T)})^T]^T$ , we apply the discrete EM algorithm by setting:  $[\underline{h}^{\{1:N_R\}}, \gamma] \rightarrow \theta$ ,  $[\underline{Y}^{\{1:N_R\}}] \rightarrow r$ , and  $[\underline{X}^{\{1:N_T\}}] \rightarrow a$ . Evidently,  $\underline{h}^{\{1:N_R\}}$  corresponds to the continuous subset  $\theta^c$  and  $\gamma$  corresponds to the discrete subset  $\theta^d$  of the parameterset  $\theta$ . The likelihood function required for the computation of (8) is given by

$$\begin{aligned} \log p(\underline{Y}^{\{1:N_R\}} | \underline{X}^{\{1:N_T\}}, \underline{h}^{\{1:N_R\}}, \gamma) \\ = \sum_{n=1}^{N_R} 2Re \left[ \text{shift}_{-\gamma}(\underline{Y}^{(n)})^H \mathbf{X} \underline{h}^{(n)} \right] - \underline{h}^{(n)H} \mathbf{X}^H \mathbf{X} \underline{h}^{(n)} \end{aligned} \quad (14)$$

where we defined the  $K \times LN_T$  matrix

$$\mathbf{X} = [ \text{diag}(\underline{X}^{(1)}) \mathbf{G}^H \quad \dots \quad \text{diag}(\underline{X}^{(N_T)}) \mathbf{G}^H ]. \quad (15)$$

Inserting (14) in (8) and (10) yields the following closed-form solution for the M-step:

$$\hat{\underline{h}}^{(n)}(i+1; \gamma) = \left( \overline{\mathbf{X}^H \mathbf{X}}(i; \gamma) \right)^{-1} \overline{\mathbf{X}}(i; \gamma)^H \text{shift}_{-\gamma}(\underline{Y}^{(n)}) \quad (16)$$

where

$$\overline{\mathbf{X}}(i; \gamma) = \int \mathbf{X} p(\underline{X}_{\{1:N\}} | \underline{Y}_{\{1:N\}}, \hat{\mathbf{H}}_{\{1:N\}}(i; \gamma), \gamma) d\underline{X}_{\{1:N\}} \quad (17)$$

and

$$\overline{\mathbf{X}^H \mathbf{X}}(i; \gamma) = \int \mathbf{X}^H \mathbf{X} p(\underline{X}_{\{1:N\}} | \underline{Y}_{\{1:N\}}, \hat{\mathbf{H}}_{\{1:N\}}(i; \gamma), \gamma) d\underline{X}_{\{1:N\}} \quad (18)$$

are a posteriori expectations. It is readily seen that the computation of  $\overline{\mathbf{X}^H \mathbf{X}}(i; \gamma)$  requires second order symbol moments. However, it can easily be shown that only second

order moments of symbols belonging to the *same* subcarrier are required. This implies that  $\bar{\mathbf{X}}(i; \gamma)$  and  $\bar{\mathbf{X}}^H \bar{\mathbf{X}}(i; \gamma)$  can be computed based on the *marginal* symbol vector APPs  $p(\underline{\mathbf{X}}_k | \underline{\mathbf{Y}}_{\{1:N\}}, \hat{\mathbf{H}}_{\{1:N\}}(i; \gamma), \gamma)$ , rather than the joint symbol vector APP  $p(\underline{\mathbf{X}}_{\{1:N\}} | \underline{\mathbf{Y}}_{\{1:N\}}, \hat{\mathbf{H}}_{\{1:N\}}(i; \gamma), \gamma)$ . Fortunately, these marginal APPs are exactly the quantities provided by the detector as described in section 2. Hence, the channel estimator can be easily implemented in any practical system as long as a soft detector is available.

For every possible value of the discrete parameter  $\gamma$ , we perform  $I_d$  channel estimation iterations (16). Afterwards we make a decision with respect to  $\gamma$  according to criterion (11). For the evaluation of (11), we insert (14) into (8). We have verified through simulations, that dropping the second term in (14) does not affect the performance of the FA resolver. Hence, if we ignore this term, our criterion becomes

$$\hat{\gamma} = \arg \max_{\gamma} \sum_{n=1}^{N_R} \text{Re} \left[ \text{shift}_{-\gamma} \left( \underline{\mathbf{Y}}^{(n)} \right)^H \bar{\mathbf{X}}(I_d; \gamma) \hat{\mathbf{h}}^{(n)}(I_d; \gamma) \right]. \quad (19)$$

Again, implementation is feasible since the operation of the FA resolver is based on soft information, (i.e., marginal symbol vector APPs) provided by the detector.

Once the ambiguity  $\gamma$  has been resolved, further iterations may be performed to refine the corresponding channel estimate. After convergence, we obtain the final channel estimate  $\hat{\mathbf{h}}^{(n)} = \hat{\mathbf{h}}^{(n)}(+\infty; \hat{\gamma})$ .

### 3.4 Initialization and complexity considerations

#### Initialization

The EM algorithm requires an initial estimate of the channel. Therefore we have to insert a few pilot subcarriers to provide this initial channel estimate. It was shown in [2], that the minimal number of pilot subcarriers required to estimate the channel is given by  $N_p = L \times N_T$  (per transmit antenna). In this contribution, we aim at a minimal pilot overhead, hence we choose the number of pilot symbols equal to  $L \times N_T$ . We further adopt the optimal placement and optimal values of the pilot symbols from [2]. The pilot symbols are also included in the EM algorithm: as their values are known to the receiver, their APPs are Dirac distributions.

#### Complexity considerations

The EM algorithm is iterative and, generally, the detector tends to be iterative as well (e.g., when performing iterative demapping or when deploying a turbo-code). Hence, in case of iterative detection, EM estimation becomes hugely complex: each time the estimates are updated, APPs have to re-computed, requiring many iterations within the detector. The resulting complexity will scale as the number of detector iterations times the number of EM iterations. As this may be prohibitive, we resort to the concept of *embedded* estimation [16]: each time the channel estimate is updated, only a single iteration within the detector is performed. Furthermore, extrinsic probabilities are not reset from one EM iteration to the next. This allows for a huge saving in computational complexity.

Furthermore, the discrete EM algorithm requires that for every possible value of the discrete parameter  $\gamma$ , we carry out  $I_d$  iterations. Thus, denoting by  $d_{max}$  the number of possible

values of  $\gamma$ , the total number of iterations required for the discrete EM algorithm is  $I_d d_{max}$ . We can limit the computational complexity by restricting the value of  $I_d$ . As we will show in the next section, the algorithm performs well for  $I_d = 1$  or  $I_d = 2$ , depending on the considered SNR. So the computational load required for FA resolution can be reduced to a strict minimum.

## 4. NUMERICAL RESULTS

We present some simulation results to illustrate the performance of the proposed joint channel estimation and FA resolution technique. We consider a  $N_T = 2$ ,  $N_R = 2$  MIMO system with a channel impulse response length of  $L = 4$ . The number of subcarriers is  $N = 256$ . We make use of a rate 1/2 recursive convolutional code with octal generator polynomials  $(31, 37)_8$ , along with  $\{-1, 1\}$  BPSK signaling. A random interleaver separates the encoder from the spatial multiplexer and symbol mappers, to fully exploit the spatial and frequency diversity of the channel. As explained in section 3.4, we apply a minimal number of pilot subcarriers to provide initial channel estimates; this number is equal to  $N_p = L \times N_T = 8$ . The remaining subcarriers are used for coded data transmission. Each codeword corresponds to 248 information bits. Performing rate 1/2 encoding, interleaving, spatial multiplexing and symbol mapping, we retain 248 coded symbols. We also compare our estimator to data-aided estimators using more than 8 pilots. In this case, we reduced the number of information bits to allow additional pilot subcarriers.

The channel was generated according to the following model: for each transmit- and receive antenna pair, the taps  $\{h_k^{(n,m)}\}_{k=0,\dots,L-1}$  are zero mean iid complex Gaussian random variables, normalized such that the expected energy per subcarrier (per transmit- and receive antenna pair) is equal to 1 (i.e., 1/2 per real dimension). The set of possible frequency ambiguity values is chosen equal to  $S = [-2, -1, 0, +1, +2]$ , such that  $\gamma$  can take on 5 different values.

A convenient measure to evaluate the performance of the FA resolver is the FA error rate (FAER), being the fraction of OFDM symbols for which the frequency ambiguity recovery fails. The latter has a direct relation the overall BER. Denoting the BER with perfect FA recovery and perfect CSI by  $BER_p$ , we can approximate the actual BER performance with FA errors as

$$\begin{aligned} BER &\simeq BER_p (1 - FAER) + \frac{1}{2} FAER \\ &= BER_p \left( 1 + \frac{1}{2} \frac{FAER}{BER_p} \right) \end{aligned}$$

since roughly 50% of the bits will be erroneous when a FA error occurs. Hence, we understand that the impact of FA errors on the overall BER is marginal as long as  $\frac{FAER}{BER_p} \ll 1$ . Let us examine the estimators with respect to this measure. A few remarkable results are observed in Fig. 2. First, the ratio  $\frac{FAER}{BER_p}$  (yet also the BER degradation caused by FA errors) increases with increasing  $E_b/N_0$  for the ML data-aided estimators. The opposite behavior is observed for the EM code-aided estimator. This also translates into the BER plots from Fig. 3, where we observe a flooring effect for the data-aided estimators, while no such phenomenon is observed with the

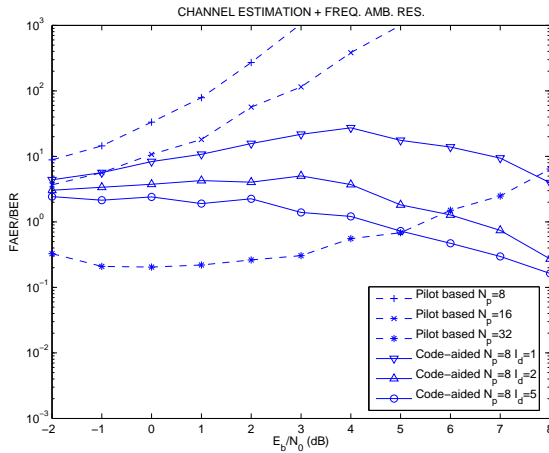


Figure 2: Normalized frequency ambiguity error rate (FAER) comparison. ( $\sigma^2 = \frac{N_0}{2E_s} = \frac{N_0}{E_b}$ )

code-aided estimators. It is also apparent from Fig. 2 and Fig. 3, that the number of iterations required to make a reliable decision w.r.t. the FA, can be limited to  $I_d = 1$  or  $I_d = 2$ . Hence, the computational overhead caused by the FA resolver is minimal. The performance of our code-aided estimator is very close to the performance of a receiver that has perfect channel state information and perfect knowledge of the FA. All these arguments advocate the code-aided algorithm as an appealing alternative for its data-aided counterpart.

## 5. CONCLUSIONS

We have proposed a joint channel estimation and frequency ambiguity resolution algorithm for coded MIMO OFDM systems. By associating one codeword to each OFDM symbol, the channel estimation and frequency recovery can be carried out from the observation of a single OFDM symbol. The algorithm is based on the EM algorithm and it iterates between data detection and estimation. Because the EM algorithm generally fails to estimate discrete parameters (such as the frequency ambiguity), we resorted to a recently proposed EM extension. Computer simulations were run for a bit-interleaved coded modulation scheme with spatial multiplexing. The proposed code-aided estimator exhibits excellent performance results. Application to other (coded) configurations is conceptually straightforward, as long as a soft-information providing detector is available.

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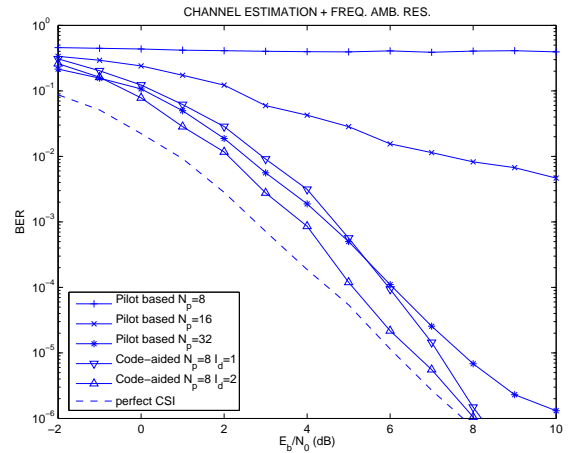


Figure 3: BER-performance of discrete EM algorithm for channel estimation and FA resolution compared to a conventional data-aided algorithm. ( $\sigma^2 = \frac{N_0}{2E_s} = \frac{N_0}{E_b}$ )

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