A FAMILY OF SPATIOTEMPORAL CHAOTIC SEQUENCES OUTPERFORMING GOLD ONES IN ASYNCHRONOUS DS-CDMA SYSTEMS

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ABSTRACT

In this work, a new family of spatiotemporal chaotic codes is proposed as an alternative to the Gold codes conventionally used in asynchronous DS/CDMA systems. In addition to their synchronization advantage over the temporal-only chaotic codes, these new codes are shown to have improved performance compared to the Gold codes. The performance criteria used in this study are Multiple Access Interference (MAI), Signal to Noise Ratio (SNR) and Bit Error Probability.

1. INTRODUCTION

Chaotic systems have been widely investigated in the field of secure communications and more particularly in spread-spectrum communications as code generators [1-6]. Compared to the conventional ones, the chaotic codes have showed to have more advantageous features**.** However, the chaotic synchronization techniques which have been published to date are sensitive to both noise and distortion in the channel [2]. For this reason, we are interested in spatiotemporal chaotic systems for which synchronization is more efficient [5, 6]. In particular, the spatiotemporal system that will be investigated in this paper offers more advantageous behavior than the conventional Coupled Map Lattices (CML).

In this paper we evaluate the performance of the sequences generated by such a system as spreading codes in asynchronous DS/CDMA systems with a conventional receiver. The performance criteria used are: aperiodic-correlation functions, multiple-access interference (MAI), signal to noise ratio (SNR) and bit error probability. We show that we can generate a large set of sequences which outperform Gold ones.

In the second section of this paper, we propose a new family of spatiotemporal chaotic system and show that it is more advantageous than the conventional CML, in terms of chaotic synchronization and correlation properties. Next, in section 3, we present the criteria used for the performance evaluation of the so generated sequences in asynchronous DS/CDMA systems. Numerical results are, then, reported and discussed in section 4. At last, some conclusions are drawn in section 5.

2. SPATIOTEMPORAL CHAOTIC SYSTEMS AND SEQUENCE GENERATION

The conventional Coupled Map Lattices (CML) are defined as follows:

$$\begin{cases} x_i(k+1) = (1-\varepsilon) f\left[x_i(k)\right] + \varepsilon f\left[x_{i-1}(k)\right] \\ x_0(k) = x(k) \end{cases} \quad (1)$$

where:

- i is the space index, i=1,…, M, M the system dimension,
- k is the time index, k=1,…,N
- ε is the coupling coefficient.
- $f(.)$ is a one dimensional chaotic map
- $x_0(k)$ is the key sequence which is chosen to be a series of uniformed distributed values in [0,1].

It has been shown in [5, 8] that the synchronization of two identical CML maps (1) having different initial conditions $x_i(0)$ can be obtained, for some regions of values of the coupling strength ε, only by driving the two systems with the same key sequence. Moreover, we have shown in [8] that the behavior of such a system, in terms of chaotic synchronization and correlation properties, depends on both ε and f.

Now, let's consider a new family of spatiotemporal chaotic systems defined by:

$$\begin{cases} x_i(k+1) = (\alpha_i x_i(k) + \beta\ x_{i-1}(k)) \bmod(1) \\ x_0(k) = x(k) \end{cases} \quad (2)$$

where α_i and β are the system parameters. We choose in this study $\alpha_i = 0.005^i$ and $\beta = 2$.

As the conventional CML (1), the synchronization of this system can also be obtained by using the same key sequence.

Moreover, this system is more advantageous than the first one (1), since it can offer better chaotic-synchronizing time and better correlation levels. Indeed, this result is illustrated in the two following figures. In fig.1 we present the time needed for synchronizing each lattice i for the new spatiotemporal system (2) and the conventional CML (1) based on logistic map ($f(x)=4x(1-x)$), for a coupling strength ε =*0.97* (for which the CML shows the best behavior).

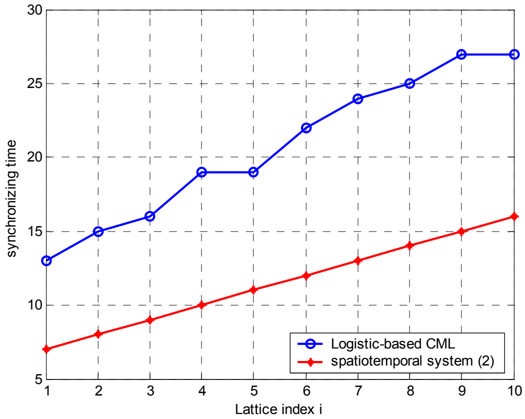

Figure 1: Synchronizing time versus the lattice index.

We can sea clearly that the synchronization of the new spatiotemporal system (2) is faster than the Logistic-based-CML one.
In figure 2 we present the correlation maxima of the sequences generated by the systems (1), (2) and the Logistic map.
These maxima are determined as follows: first, we calculate the maximum of the autocorrelation function of each generated sequence with the formula:

$$C_i^{\max} = \max_{\tau \neq 0} C_{ii}(\tau) \quad (3)$$

where

$$C_{ii}(\tau) = \hat{C}_{ii}(\tau) \big/ \hat{C}_{ii}(0), \quad (4)$$

and

$$\hat{C}_{ii}(\tau) = \frac{1}{N} \sum_{n=1}^{N} [x_i(n) - \bar{x}_i][x_i(n+\tau) - \bar{x}_i]^T . \quad (5)$$

$\bar{x}_i$ is the average value of $x_i(n)$, N = 100 is the sequence length. Then,

$$C^{\max} = \max_{i=1:M} C_i^{\max}, \quad (6)$$

M =10 being the number of sequences.
The plotted values are obtained by averaging the results over 100 runs.
The cross-correlation maxima, are obtained in the same way by using the formula:

$$C_{ij}(\tau) = \hat{C}_{ij}(\tau) \Big/ \sqrt{\hat{C}_{ii}(0)\hat{C}_{jj}(0)} \quad (7)$$

$$\hat{C}_{ij}(\tau) = \frac{1}{N} \sum_{n=1}^{N} [x_i(n) - \bar{x}_i][x_j(n+\tau) - \bar{x}_j]^T . \quad (8)$$

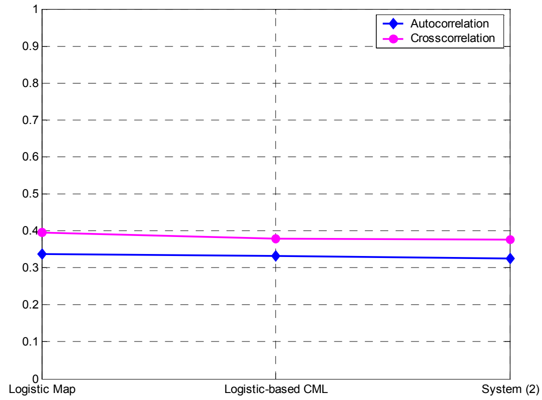

Figure 2: Auto- and cross-correlation maxima.

We can notice that sequences generated by the new spatiotemporal system (2) can show better correlation levels compared to both Logistic map and Logistic-based CML.
In the following sections we propose to evaluate the performance of an asynchronous DS/CDMA system based on sequences generated by the new spatiotemporal system (2), as spread-spectrum codes.
For this purpose, we first need to proceed to a quantization of the so generated sequences as follows:

$$s_i = Q(x_i),\ i=1 \ldots M.$$

Such as:

$$\begin{cases} s_i(j) = 1 & if \quad x_i(j) \geq 0.5 \\ s_i(j) = -1 & if \quad x_i(j) \leq 0.5 \end{cases} \quad (9)$$

3. PERFORMANCE CRITERIA

3.1. Aperiodic correlation parameters

The aperiodic auto and cross-correlation functions of the spreading sequences are of crucial importance in the evaluation of the communication performance for asynchronous DS/CDMA systems, in terms of multiple access interference (MAI), signal to noise ratio (SNR) and bit error rate probability.
The aperiodic cross-correlation function AC_{ij} of two N-period sequences s_i and s_j is defined [7] as :

$$AC_{ij} = \begin{cases} \sum_{0}^{N-l-1} x_i x_{j+l} & if \quad 0 \leq l \leq N-1 \\ \sum_{0}^{N+l-1} x_{i-l} x_j & if \quad 1-N \leq l \leq -1 \\ 0 \quad if \quad |l| \geq N \end{cases} \quad (10)$$

the auto-correlation AC_i is then

$$AC_i = AC_{ii} . \quad (11)$$

The peak aperiodic correlation parameters are defined as:

$$AC_a = \max\{\ AC_k(l),\ ,\ 0<l<N-1,\ 1<k<M\} \quad (12)$$

$$AC_c = \max\{\ AC_{ij}(l)\ ,\ 0<l<N-1,\ 1<i,j<M\} \quad (13)$$

$$AC_{max} = max(AC_a, AC_c) \quad (14)$$

The lower AC_{max}, the better the communication system performance.

3.2. Multiple Access Interference (MAI)

Given a set of N-period sequences $\{S_i\}$, $i=1,\dots,k$, the Multi Access Interference MAI of the i^{th} sequence is defined as :

$$MAI_i = \frac{1}{6N^3} \sum_{\substack{j=1 \\ j \neq i}}^{K} r_{ij} \quad (15)$$

The average MAI of a set of K sequences is

$$MAI = \sum_{i=1}^{K} \frac{MAI_i}{K} \quad (16)$$

where $$r_{ij} = 2\mu_{ij}(0) + \mu_{ij}(1) \quad (17)$$

and $$\mu_{ij}(n) = \sum_{l=1-N}^{N-1} AC_i(l) AC_j(l+n) \quad (18)$$

For the conventional receiver, the lower MAI, the higher the signal to noise ratio.

3.3. Signal to noise ratio (SNR)

This criterion is very important as a performance measure of the CDMA communication systems.[7]
For a conventional receiver in an asynchronous DS/CDMA system, the SNR of the i^{th} user, is given by [7] as:

$$\mathrm{SNR}_i = \left(\frac{1}{6N^3} \sum_{j \neq 1}^{K} \left[\sum_{l=1-N}^{N-1} AC_i(l) AC_j(l) + \sum_{l=1-N}^{N-1} AC_i(l) AC_j(l+1) \right] + \frac{N0}{2E} \right)^{-1/2} \quad (19)$$

where $E/N0$ is the AWGN-channel SNR and AC is the aperiodic correlation function.
We can notice that the SNR_i can be written as follows:

$$SNR_i = \left(MAI_i + \frac{N0}{2E} \right)^{-1/2} \quad (20)$$

Thus, we should minimize the MAI parameter in order to improve the SNR.

3.4. Bit Error Probability

By virtue of the Standard Gaussian approximation (SGA), the Bit Error Probability of the i^{th} sequence is given by:

$$P_i = \frac{1}{2} erfc\left(\sqrt{\frac{SNR_i}{2}} \right) \quad (21)$$

4. NUMERICAL RESULTS

4.1. Peak aperiodic-correlation parameter

We consider a set of 100 spatiotemporal chaotic sequences for which we compute aperiodic correlation functions according to the formula (10) and (11).
In figure 3, we present the peak correlation parameter $\boldsymbol{AC_a}$, of the considered set of chaotic sequences and the Gold ones, for a spreading factor N = 31.

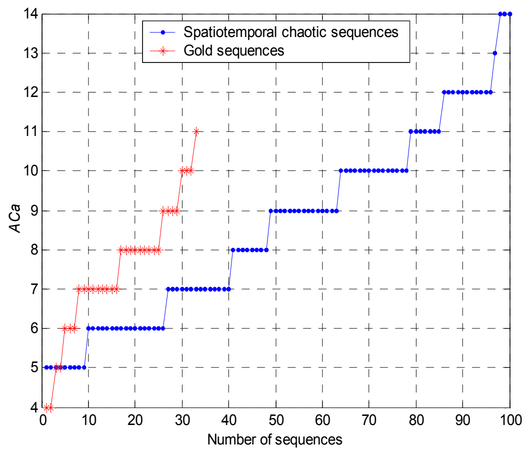

Figure 3 Peak aperiodic correlation parameter versus the number of sequences.

The figure shows clearly that the spatiotemporal chaotic sequences are not only more numerous, but also have better peak aperiodic-correlation parameter than Gold ones.

4.2. Multiple-access-interference parameter (MAI)

We present in figure 4 the average MAI parameter of the studied sequences (Chaotic and Gold). For the chaotic sequences we present the best selected ones according to the MAI parameter.

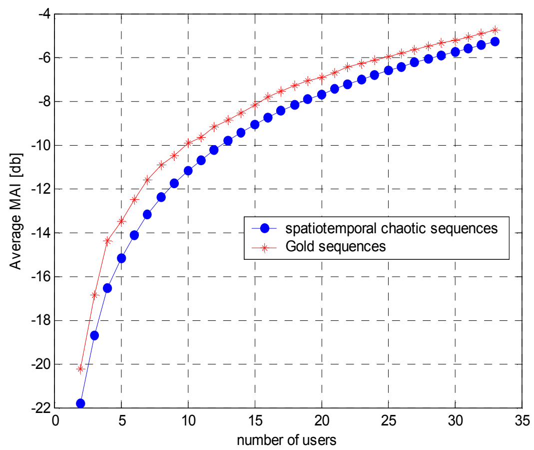

Figure 4: Average MAI parameter versus the number of sequences.

We can notice from this figure, that the spatiotemporal chaotic system (2) can generate a great number of sequences having better MAI parameter than the Gold ones.

4.3. Signal to noise ratio (SNR)

Figure 5 shows the average SNR of the conventional receiver based on spatiotemporal chaotic and Gold sequences. The spatiotemporal chaotic sequences are those selected according to the MAI parameter. It is clear that the use of spatiotemporal chaotic sequences improves the SNR of the conventional receiver, compared to the use of Gold ones. Indeed, this result is expected when using sequences that minimize the MAI parameter.

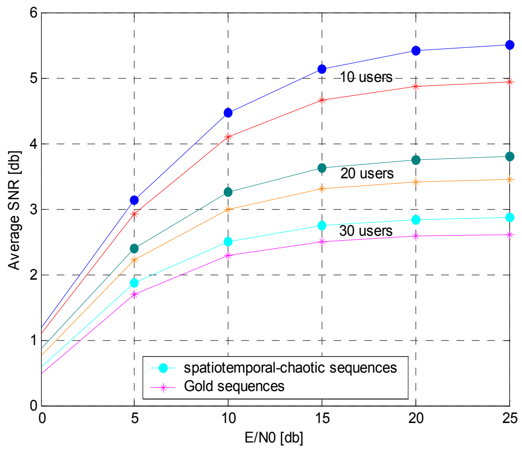

Figure 5 Average *SNR* of the conventional receiver with 10, 20 and 30 users, respectively.

4.4. Bit Error Probability

Since there has been an improvement in the *SNR* values, the *bit error probability* would be, consequently, improved, according to the expression (21). This result is illustrated in figure 6, which reports the average bit-error-probability of an asynchronous CDMA system with a conventional receiver based on spatiotemporal-chaotic and Gold sequences, respectively.

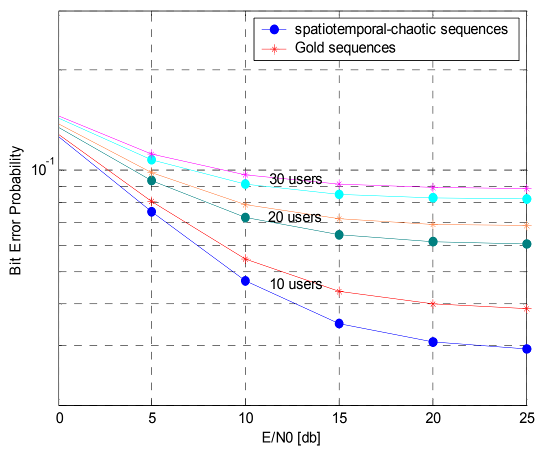

Figure 6: Bit Error Probability of the conventional receiver with 10, 20 and 30 users, respectively

5. CONCLUSIONS

We have evaluated the performance of an asynchronous DS/CDMA system based on new spatiotemporal chaotic sequences in terms of multiple-Access Interference (MAI), Signal to Noise Ratio (SNR) and Bit Error Probability.
The simulation results showed that these sequences improve the DS/CDMA-system performance, compared to the Gold ones. Thus, the proposed spatiotemporal chaotic system considered is not only advantageous in terms of chaotic- synchronization, but can also generate sequences that outperform the Gold ones in asynchronous DS/CDMA systems.

REFERENCES

[1] L.M. Pecora, T.L. Carroll, "Synchronization in chaotic systems", *Physical review letters,* 64, pp. 821-824, Feb. 1990.
[2] G.Setti, M.P.Kennedy, R.Rovatti. *Chaotic Electronics in Telecommunications*. CRC Press, 2000.
[3] Z. Ben Jamaa, S. Belghith, "Performances of Kneading Sequences in CDMA Application", *NOLTA 2000*, Dresden, Germany, September 17-21, 2000.
[4] M. Khanfouci, and S. Marcos, "PLM sequences for the performance optimization of linear multiuser detectors", *EUSIPCO 2005*, Antalya, Turkey, September 4-8, 2005.
[5] RJ.H. Xiao, G. Hu, Z. Qu, "Synchronization of spatio-temporal chaos and its application to multi-channel spread spectrum communication", *Physical review letters,* vol.77, pp. 4162-4165, Nov. 1996.
[6] X. Wang, M. Zhan, X. Gong, C. H. Lai and Y. C. Lai, "Spread-spectrum communication using binary spatiotemporal chaotic codes", Physics Letters A, 2004.
[7] M.B. Pursley. "Performance evaluation of phase coded SSMA part I : system analysis", *IEEE Transactions on communications*, vol. 25 : pp. 795-799, August 1977.
[8] S. Meherzi, S. Marcos and S. Belghith, "Synchronizing and correlation properties of spatiotemporal chaotic sequences", *ICECS 2005*, Gammarth, Tunisia, December 11-14, 2005, pp. 527-530.