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CLOSED-FORM APPROXIMATION FOR THE OUTAGE CAPACITY OF OFDM-STBC AND OFDM-SFBC SYSTEMS

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ABSTRACT

The combination of orthogonal frequency-division multiplexing (OFDM) and space-time or space-frequency block coding (STBC or SFBC) has been shown to be a simple and efficient means to exploit the spatial diversity in frequency-selective fading channels. From a general broadband multiple-input-multiple-output (MIMO) channel model, we derive a tight analytical approximation for the outage capacity of such systems assuming that the channel is known at the receiver and unknown at the transmitter. This expression is a simple function of the channel and system parameters. Numerical results are provided to demonstrate the excellent accuracy of the derived approximation in all cases.

1. INTRODUCTION

The straightforward application of space-time and space-frequency block coding (STBC and SFBC) into orthogonal frequency-division multiplexing (OFDM) systems is a simple way to exploit the inherent spatial diversity of multiple-input-multiple-output (MIMO) systems in broadband channels [1], [2], [3]. In OFDM-STBC the orthogonal code is applied across a number of consecutive OFDM symbols whereas in OFDM-SFBC the orthogonal code is applied across a number of neighbouring OFDM tones. Unlike other more complex space-time-frequency coding techniques, the referred OFDM-STBC and OFDM-SFBC are not able to exploit the multi-path diversity of the channel [4], but on the other hand, the receiver is significantly simpler.

OFDM-STBC and OFDM-SFBC configurations can be concatenated with outer codes providing frequency diversity and enhancing the system performance [5]. In this context, the channel capacity is a crucial performance measure to investigate the capacity-approaching capabilities of the overall system. The capacity of OFDM-based spatial multiplexing MIMO systems was analyzed in [6]. This work focuses on the capacity of OFDM-STBC and OFDM-SFBC systems when the channel is unknown at the transmitter and known at the receiver.

From a general broadband MIMO channel model we derive tight and simple approximations for the mean and variance of the mutual information (also called instantaneous capacity). Assuming that the mutual information is Gaussian distributed, we obtain a simple closed-form expression for outage capacity of OFDM-STBC and OFDM-SFBC systems. The fact that the mutual information can be well approximated by a Gaussian distribution was observed in [7] and [8] for general MIMO narrowband channels, and in [9] for MIMO-STBC narrowband channels. Here we show that this approximation is excellent also for OFDM-STBC and OFDM-SFBC systems, even for low number of antennas and channel taps. To the author's best knowledge, this is the first time that a closed-form expression for the outage capacity of OFDM-STBC and OFDM-SFBC systems is proposed.

In general, the closed-form expressions are useful in two ways. They can be used to generate performance curves (in this case outage capacity curves) without resorting to time-consuming Monte Carlo simulations. Second and more important, they can reveal the influence of the channel and system parameters on the system performance. Based on the derived expression we can easily analyze the dependence of the outage capacity on the spatial correlation, power delay profile (PDP), number of antennas, signal-to-noise ratio (SNR), code rate and number of OFDM tones.

2. BROADBAND MIMO CHANNEL MODEL

Consider a MIMO channel with n_T transmit antennas and n_R receive antennas. The MIMO channel transfer function (frequency response) is given by

$$\mathbf{H}\left(\mathrm{e}^{j2\pi\theta}\right) = \sum_{n=0}^{L-1} \mathbf{F}_n \exp\left(-j2\pi n\theta\right),\ 0 \le \theta < 1, \quad (1)$$

where $\mathbf{F}_n$ is a $n_R \times n_T$ matrix denoting the n-th tap of the discrete-time MIMO fading channel impulse response, and L is the number of channel taps. The entries of each matrix $\mathbf{F}_n$ are assumed to be circular symmetric complex Gaussian random variables. In general, they are spatially correlated according to a specific covariance matrix $\mathbf{R}_n = E\left[\mathrm{vec}\left(\mathbf{F}_n\right) \mathrm{vec}^H\left(\mathbf{F}_n\right)\right]$, whose entries are given by

$$\rho_n^{ij,ks} = E\left[f_n^{ij}\left(f_n^{ks}\right)^*\right],\quad {\scriptstyle i,k=1,\dots,n_R,\ \ j,s=1,\dots,n_T} \quad (2)$$

where f_n^{ij} is the entry of $\mathbf{F}_n$ corresponding to the jth transmit and the ith receive antennas. Note that the diagonal terms of the covariance matrices determine the PDP's of the channel. The nth term of the discrete-time PDP between the jth transmit and the ith receive antennas is given by $p_n^{ij} = \rho_n^{ij,ij}$. In general, there will be different PDP's for the different pairs of transmit-receive antennas. We assume, without loss of generality, that the channel is normalized so

$$\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{n=0}^{L-1} p_n^{ij} = \sum_{n=0}^{L-1} \mathrm{Tr}\left(\mathbf{R}_n\right) = n_R n_T. \quad (3)$$

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Since the $\mathbf{F}_n$ are zero-mean Gaussian matrices, the power correlation between the elements of $\mathbf{F}_n$ can be expressed as follows [10]

$$E\left[\left|f_n^{ij}\right|^2\left|f_n^{ks}\right|^2\right]=\left|\rho_n^{ij,ks}\right|^2+\rho_n^{ij,ij}\rho_n^{ks,ks}. \qquad (4)$$

We assume that the matrices $\mathbf{F}_n$ at different taps are uncorrelated. Therefore

$$E\left[f_n^{ij}f_m^{ks}\right]=0,\ n\neq m. \qquad (5)$$

Although the above assumption is not exact due to the finite bandwidth of the receiver, this is commonly accepted in discrete-time broadband channel models [5], [6], [11]. Since the f_n^{ij} are Gaussian, they are independent. Then

$$E\left[\left|f_n^{ij}\right|^2\left|f_m^{ks}\right|^2\right]=\rho_n^{ij,ij}\rho_m^{ks,ks},\ n\neq m. \qquad (6)$$

Considering (1), the channel frequency response at the OFDM subcarriers will be

$$\mathbf{H}_k=\mathbf{H}\left(\mathrm{e}^{j2\pi k/K}\right)\ k=0,\ldots,K-1, \qquad (7)$$

where K is the number of OFDM tones. Let γ_k denotes the squared Frobenius norm of the channel response at the kth tone: $\gamma_k=\|\mathbf{H}_k\|_F^2$. Considering (7) and (5), it is straightforward to show that the γ_k's are identically distributed. From (7), considering (6), (4) and (2), we obtain the following expressions for the mean, and covariance of the γ_k's

$$\mu_\gamma=E\left[\gamma_k\right]=\sum_{n=0}^{L-1}\mathrm{Tr}\left(\mathbf{R}_n\right)=n_Rn_T, \qquad (8)$$

$$\sigma_\gamma^{k,s}=\sum_{n=0}^{L-1}\sum_{m=0}^{L-1}\mathrm{Tr}\left(\mathbf{R}_n\mathbf{R}_m^H\right)e^{-j2\pi(k-s)\,(n-m)/K}. \qquad (9)$$

In particular, setting $k=s$, the variance of the γ_k's is given by

$$\sigma_\gamma^2=\mathrm{var}\left[\gamma_k\right]=\sum_{n=0}^{L-1}\sum_{m=0}^{L-1}\mathrm{Tr}\left(\mathbf{R}_n\mathbf{R}_m\right)=\|\mathbf{R}_S\|_F^2, \qquad (10)$$

where $\mathbf{R}_S=\sum_{n=0}^{L-1}\mathbf{R}_n$ is the sum of the correlation matrices at the channel taps.

3. OUTAGE CAPACITY

Since the channel is unknown at the transmitter, the total available power is allocated uniformly across all the transmit antennas and the OFDM subchannels. We assume that the OFDM uses a cyclic prefix with adequate length. In OFDM-STBC the orthogonal space-time code is applied across a number of consecutive OFDM symbols [1], [3]. If the channel remains constant during the transmission of the OFDM symbols involved in each STBC block, the MIMO-OFDM channel can be decomposed into a set of K effective uncloupled scalar channels with signal-to-noise ratio (SNR) given by

$$SNR_k=\frac{E_s\gamma_k}{\sigma_n^2n_TR}, \qquad (11)$$

where R is the code rate, E_s is the total transmitted energy per symbol time, and σ_n^2 is the power of the additive white Gaussian noise at the receive antennas. Under the channel normalization of (3), the average signal-to-noise ratio (SNR) at the receive antennas will be $\rho=E_s/\sigma_n^2$.

In OFDM-SFBC the orthogonal space-frequency code is applied across a number of neighbouring OFDM tones [2]. If the channel response at these tones is identical, the MIMO-OFDM channel can be decomposed into a set of K effective scalar channels with SNR also given by (11). Then, the instantaneous capacity of both OFDM-STBC and OFDM-SFBC systems can be expressed as follows

$$C=\sum_{k=0}^{K-1}C_k=\frac{R}{K}\sum_{k=0}^{K-1}\log_2\left(1+\frac{\rho\,\gamma_k}{n_TR}\right). \qquad (12)$$

The transmission of the cyclic prefix symbols results in a penalty in the transmission rate and in the SNR (a fraction of the available energy at the transmitter have to be expended on the cyclic prefix symbols). Since the number of information symbols is usually significantly higher than the number of cyclic prefix symbols, these penalties are neglected here.

Since the wireless channel is random, the γ_k's are random and the instantaneous capacity of (12) is a random variable. Assuming quasi-static fading channels, the random process of the channel is non-ergodic. In this case, the outage capacity is used as performance measure [12].

Since the γ_k's are identically distributed, the C_k's will be identically distributed. To derive a closed-form approximation for the average mutual information we consider a second-order Taylor series expansion of $C_k(\gamma_k)$ about $\gamma_K=\mu_\gamma$. Then, from (12)

$$\begin{aligned}C_k\ &\approx\ \frac{R}{K}\log_2\left(1+\frac{\rho\mu_\gamma}{Rn_T}\right)+\frac{R\rho\log_2 e}{K\left(n_TR+\rho\mu_\gamma\right)} \qquad (13)\\ &\times\ \left(\gamma_k-\mu_\gamma\right)-\frac{\rho^2R\log_2 e}{2K\left(Rn_T+\rho\mu_\gamma\right)^2}\left(\gamma_k-\mu_\gamma\right)^2,\end{aligned}$$

where e is the neper's number. Applying the expectation operator to (13) and considering (8) and (10) we obtain the following approximation for the average mutual information

$$\mu_C\approx R\log_2\left(1+\frac{\rho\,n_R}{R}\right)-\frac{R\rho^2\log_2 e}{2n_T^2\left(R+\rho n_R\right)^2}\ \|\mathbf{R}_S\|_F^2. \qquad (14)$$

Note that μ_C is independent on the number of OFDM tones. In $\|\mathbf{R}_S\|_F^2$ we can distinguish two contributions. First, a fixed contribution due to the diagonal terms, which equals $n_R\,n_T$ because of the channel normalization of (3). The second contribution comes from the non-diagonal terms of $\mathbf{R}_S$ which, in general, will be significant when the channel taps are spatially correlated. But, since the $\mathbf{R}_n$ are complex, high spatial correlation at the individual channel taps does not always leads to low μ_C.

Since the mutual information is a non-linear function of the γ_k's, its variance can be approximated, as a function of the covariances of the γ_k's, as follows [13]

$$\sigma_C^2\approx\sum_{k=0}^{K-1}\sum_{s=0}^{K-1}\frac{\partial C}{\partial\gamma_k}\,\frac{\partial C}{\partial\gamma_s}\,\sigma_\gamma^{k,s},$$

where the partial derivatives are calculated at μ_γ. Considering (12) and (9), and assuming that $K \geq L$, the variance can be written as follows

$$\sigma_C^2 \approx \left(\frac{R\rho \log_2 e}{n_T(R+\rho n_R)}\right)^2 \sum_{n=0}^{L-1} \|\mathbf{R}_n\|_F^2 \,. \qquad (15)$$

Therefore, according to this approximation, σ_C^2 does not depend on the number of OFDM tones. Unlike the average mutual information, high spatial correlation at the channel taps always leads to high values of σ_C^2.

From (14) and (15) we obtain a Gaussian approximation of the cumulative distribution function (CDF) of the mutual information $F_C(x)$. The q%-outage capacity (C_q) is defined as the transmission rate that is guaranteed for $1-q/100$ of the channel realizations. Then the q% outage capacity can be approximated as follows

$$C_q = F_C^{-1}(q) \approx \mu_C + \sigma_C\sqrt{2}\,\mathrm{erfc}^{-1}\left(2-\frac{q}{50}\right), \qquad (16)$$

where $\mathrm{erfc}(x)$ is the complementary error function. Since $\mathrm{erfc}^{-1}(x)$ is negative for $x>1$, the second term in (16) is always negative. Therefore, the higher variance of C, the lower outage capacity. According to (16), the outage capacity does not depend on the number of OFDM tones.

3.1 Channel with a common correlation matrix

In the case of spatially balanced channels with identical correlation matrix for all the taps: $\mathbf{R}_n = p_n\mathbf{R}$, where $\mathbf{R}$ is the common spatial correlation matrix with unit entries in its main diagonal. This situation tipically arises when the antennas are very close at the transmit and/or receive array, and when the transmitter and/or receiver are surrounded by local scatterers, so the angular spectrums are omnidirectional for any tap. In this case,

$$\|\mathbf{R}_S\|_F^2 = \|\mathbf{R}\|_F^2 \;,\; \sum_{n=0}^{L-1}\|\mathbf{R}_n\|_F^2 = \|\mathbf{R}\|_F^2 \sum_{n=0}^{L-1} p_n^2.$$

Note that the spatially uncorrelated channel can be viewed as a particular case where $\mathbf{R}$ is a diagonal matrix. In this case $\|\mathbf{R}\|_F^2 = n_R n_T$ because of the channel normalization. In general, $\|\mathbf{R}\|_F^2 \geq n_R n_T$, therefore μ_C will be always lower than in the corresponding uncorrelated channel. On the contrary, σ_C^2 will be higher than in the corresponding uncorrelated channel. Therefore, as it is expected, the spatial selectivity improves the outage capacity.

In the case of uniform PDP's, the variance reduces to

$$\sigma_C^2 \approx \left(\frac{R\rho \log_2 e}{n_T(R+\rho n_R)}\right)^2 \|\mathbf{R}\|_F^2 \;\frac{1}{L}. \qquad (17)$$

This is the lower variance for all the possible PDP's of length L. The variance for a two-rays PDP is obtained by setting $L=2$, regardless the delay between the two taps. By setting $L=1$ we obtain the variance for a one-ray PDP which corresponds to a channel with frequency flat response. In this case, the σ_C^2 coincides with the variance of the instantaneous capacity in narrowband spatially correlated MIMO-STBC channels with Rayleigh fading [9].

3.2 One-side spatially correlated channels

We first focus on channels spatially correlated in reception and uncorrelated in transmission. This situation usually arises in the uplink of a typical NLOS urban outdoor channel when the transmitter is surrounded by local scatterers and the receiver is not obstructed by local scatterers. Assuming that the correlation at the receiver array does not depend on the transmit antenna, the correlation matrices at the channel taps

$$\mathbf{R}_n = \mathbf{R_T}_n^T \otimes \mathbf{R_R}_n = \mathbf{I} \otimes \mathbf{R_R}_n,$$

where $\otimes$ denotes the Kronecker product, the superscript $(\cdot)^T$ denotes the matrix transpose operator, $\mathbf{R_R}_n$ is the $n_R \times n_R$ receive correlation matrix for the n-th tap and $\mathbf{R_T}_n$ is the $n_T \times n_T$ transmit correlation matrix for the n-th tap which, in this case, equals the identity matrix $\mathbf{I}$. Now,

$$\|\mathbf{R}_S\|_F^2 = n_T\,\|\mathbf{R_R}_S\|_F^2 \;,\; \sum_{n=0}^{L-1}\|\mathbf{R}_n\|_F^2 = n_T \sum_{n=0}^{L-1}\|\mathbf{R_R}_n\|_F^2\,, \qquad (18)$$

where $\mathbf{R_R}_S = \sum_{n=0}^{L-1}\mathbf{R_R}_n$ is the sum of the correlation matrices in reception. Substituting (18) in (14) and (15) we obtain the corresponding expressions for the mean and variance of the instantaneous capacity. Analogous expression are obtained when the channel is spatially uncorrelated in reception and correlated in transmission.

4. SIMULATION RESULTS

Now, to show the tightness of our approximation, we compare the analytical predictions of (16) with Monte Carlo simulations for a variety of channel conditions and system parameters. In all cases the analytical predictions are represented by solid lines and the Monte Carlo values are represented by markers. In every simulation, 20000 independent Monte Carlo runs have been performed.

Figure 1 shows the outage capacity for different MIMO configurations as a function of the delay between the channel taps, assuming an equal-gain two-rays channel. The channel is spatially uncorrelated. The average SNR at the receiver antennas is ρ = 10 dB in all cases. The number of OFDM tones is K = 512. It can be observed that the outage capacity is insensitive to the tap spacing. The graph shows that the analytical approximation (solid lines) closely matches the outage capacity (markers). The maximum relative approximation error is lower than 2.5% in all cases.

La figure 2 shows results of outage capacity versus outage probability for a 3×3 MIMO channel with $K=64$ OFDM tones. The code rate is $R=3/4$. The different curves corresponds to uniform PDP's with different lengths (L), assuming that the average SNR is $\rho = 10$ dB in all cases. There is common correlation matrix $(\mathbf{R})$ for all taps. It was obtained from the Jakes correlation model [14] as a function of the antenna spacing in the transmit and receive arrays. This model assumes uniform angular spectrum at both the transmitter and the receiver for all the channel taps. Also, it is assumed that the antennas are identical and single-polarized, at each array. According to this model the entries of $\mathbf{R}$ are given by

$$\rho^{ij,ks} = J_0\left(2\pi s_{ik}\right) J_0\left(2\pi s_{js}\right),$$

where $J_0(x)$ is the zero-order Bessel function of the first kind and s_{ik} and s_{js} are the distances (in wavelengths) between the

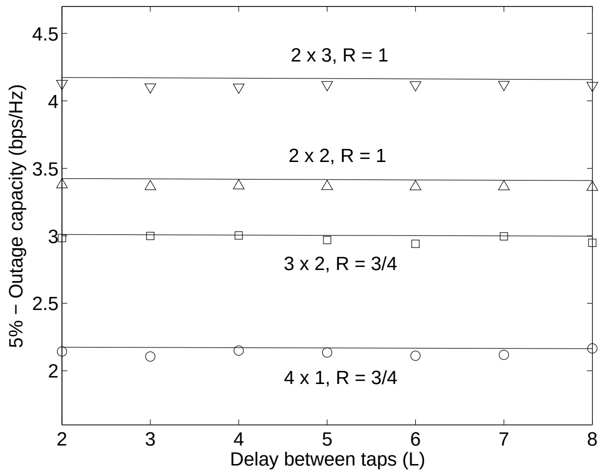

Fig. 1. Outage capacity as a function of the delay between the channel taps for different MIMO configurations and code rates.

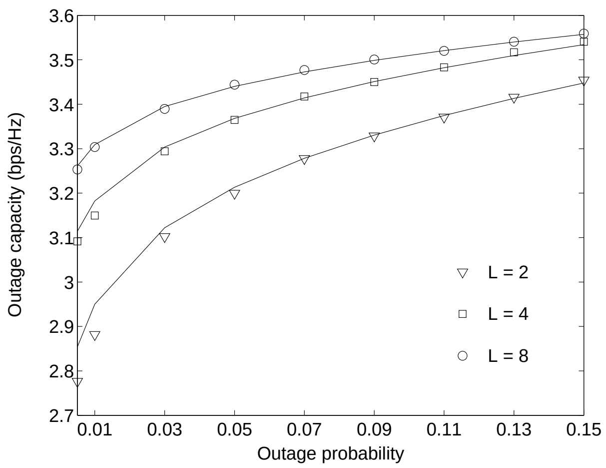

Fig. 2. Outage capacity of a 3×3 OFDM-STBC system, as a function of the outage probability, for different channel lengths.

corresponding antennas in the receive and transmit arrays, respectively. In the simulations we assume linear arrays with uniform antenna separations equal to $\lambda/5$ in both arrays. The figure shows that the proposed approximation is quite tight for any channel length and outage probability. The relative maximum approximation errors were 2.8%, 1.1% and 0.24% for the the cases $L = 2$, $L = 4$ and $L = 8$, respectively. The curves also show the dependency of the outage capacity with the channel length. Note that the variance of the capacity is inversely proportional to L, as (17) shows.

Now, we consider channels spatially correlated in reception and uncorrelated in transmission, where the spatial correlation matrix is different at the channel taps. We consider the channel model used in [6] [11]. This model is suitable for the uplink of a typical cellular suburban channel where the transmitter is surrounded by local scatterers and the receiver is not obstructed by local scatterers. The model assumes that each channel tap is due to the waves arriving from a scatterer cluster, where the waves from a given cluster experience the same delay. The model also assumes a linear array at the receiver with identical single-polarized antennas. For each cluster/tap the angle of arrival of the incoming waves (with respect the array axis) are Gaussian distributed around a mean value $(\bar{\theta}_n)$ with standard deviation σ_n^θ. In practice, this standard deviation depends on the scattering radius of the cluster and its distance to the receiver. Under these assumptions and for small angular spreads, the entries of the receive correlation matrices $\mathbf{R}_{\mathbf{R}n}$ can be expressed as follows [6], [11]

$$\rho_{R\ n}^{i,k} \approx p_n \exp\left[-j2\pi s_{ik}\cos\bar{\theta}_n - 2\left(\pi s_{ik}\,\sigma_n^\theta \sin\bar{\theta}_n\right)^2\right]. \quad (19)$$

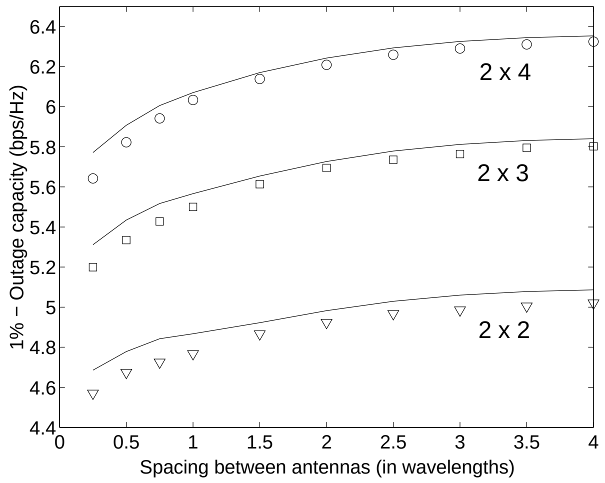

Fig. 3. 1%-Outage capacity of different OFDM-STBC systems as a function of the spacing between the receive antennas.

Note that, unlike in previous results, there are different correlation matrices for each channel path. Figure 3 shows results of 1% - outage capacity as function of the spacing between the receive antennas for different OFDM-STBC systems with two transmit antennas and variable number of receive antennas. The code rate is $R = 1$, the number of OFDM tones is $K = 128$ and the average SNR at the receiver branches is ρ = 15 dB, in all cases. We consider $L = 6$ taps /clusters with mean angles of arrival given by $(n+6)\pi/16$, $n = 0, \ldots, L-1$. That is the clusters are uniformly distributed around an arc of $5\pi/16$ radians. We also assume uniform PDP and identical angular standard deviation for all the clusters: $\sigma_n^\theta = (\pi/36)$. The mean and variance of the outage capacity are obtained considering (19) and (18). Once again, the analytical approximation (solid lines) closely matches the outage capacity (markers), with an relative approximation error lower than 2.8% in all cases.

In figure 4 we shows the 2%-Outage capacity versus the SNR at the receiver branches (ρ) when the spacing between adjacent receive antennas is $s_{ik} = \lambda$. The rest of channel and system parameters is identical than in the previous case (figure 3). In this case, the maximum relative approximation error is 2.7%.

5. CONCLUSIONS

In this work we have derived a tigth closed-form approximation for the outage capacity of OFDM-STBC and OFDM-

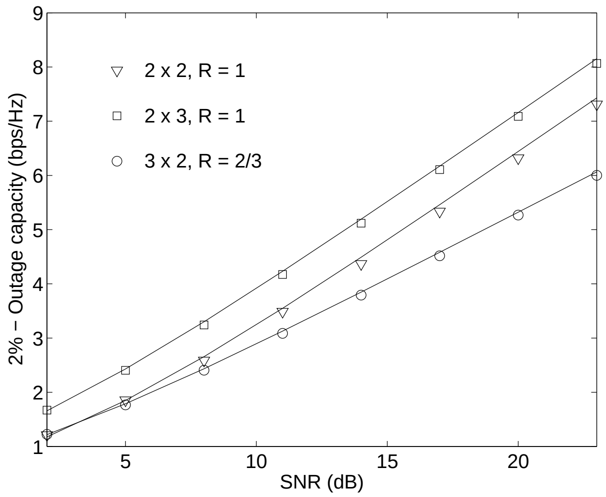

Fig. 4. 2%-Outage capacity for different correlated MIMO channels as a function of the average SNR at the receiver branches.

SFBC systems. The derived expression only depends on the spatial covariance matrices of the MIMO channel at the channel taps and on the system parameters. The accuracy of the expression reveals that these covariance matrices are the only channel statistics needed for a tight estimation of the outage capacity. The expression is quite accurate for any spatial correlation conditions, power delay profiles, SNR and system parameters. It clearly shows the dependency of the outage capacity on the system and channel parameters.

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